Unit-1

CONSUMER BEHAVIOUR-I

Unit Structure:

1.0 OBJECTIVES

After going through this unit you will be able to explain the concepts of -

- Revealed Preference
- The feasible set
- The consumption decision
- The comparative statics of consumer Behaviour.
- Income and substitution effects.

1.1 INTRODUCTION

Samuelson's revealed preference theory is regarded as scientific explanation of consumer's behaviour as against the psychological explanation provided by Marshallian and Hicks-Allen theories of demand.

The indifference coordinal theory required less data about the consumer than the marginal utility (or cardinal) theory. In the indifference theory one did not have to know the quantities of utilities of goods. It was enough to know the rankings of preferences or consumers. However, to draw the indifference map one had to know all the possible combination of goods. This information had to be supplied by the consumer it the consumer did not supply if one could not construct his indifference map.

To meet this difficulty Prof. Paul Samuelson offered another theory to explain the consumer's behaviour in the market. According to it, the consumer need not supply data on his preference we could ourselves find out about his preferences by observing his behaviour by seeing what he buys and at what prices, provided his tastes do not change. With this information one may reconstruct his indifference map. This is known as the Revealed preference Theory. It may be explained as follows.

When consumer buys one set of goods as against other he may have reasons for doing so : (a) he likes that particular set more than the other, (b) that set is cheaper than the other And between two sets of goods A and B. Suppose the consumer is seen to buying A but not B, this may not mean that he necessarily prefers A to B. He have bought A because it is cheaper that B.

Indeed it is possible that even he might have liked B more than A and may reject that he cannot afford B. However if A and B cost same amount of money to the consumer and yet he has bought A and B, the reason could only be that cost he prefers A to B. Generally, if A is preferred to B, C, D etc. But B, C, D are just as expensive as A, we may say then that A is revealed preference to B, C, D or B, C, D are revealed to be inferior to A.

1.2 REVEALED PREFERENCE

We know that utility functions are convenient numerical representations of preferences and that neither they nor the consumer's preferences are directly observable. This subjectivity of the foundations of consumer theory stimulated interest in the development of a theory of demand based solely on observable and measurable phenomena, namely the bundles actually bought by a consumer and the prices and money incomes at which they were bought. The emphasis in this approach is on assumption about the consumer's behaviour, which can be observed, rather than on preferences, which cannot.

As in the utility theory, we assume that the consumer faces a given price vector, p. and has a fixed money income, M. Our first behavioral assumption is that the consumer spends all income.

The second assumption is that only one commodity bundle x is chosen by the consumer for each price and income situation. Confronted by a particular p vector and having a particular M, the consumer will always. Chose the some bundle.

The third assumption is that there exists one and only one price and income combination at which each bundle is chosen. For a given x there is some P, M situation in which x will be chosen by the consumer and that situation is unique.

The fourth and crucial as sumption is that the consumer's choices are consistent. By this we mean that, if a bundle x^0 is chosen and a different bundle x^1 could have been chosen, then when x^1 is chosen x^0 must no longer be a feasible alternative.

To amplify this, let P^0 be the price vector at which x^0 is chosen. Then if x^1 could have been chosen when x^0 was actually chosen, the cost of x^1, $P^0 x^1$ must be no greater than the cost of x^0, which is $P^0 x^1$. This latter is also the consumer's money income $M_0 \, P^0 x^0$ when x^0 is chosen.

Similarly, let P^1 be the price vector at which x^1 is chosen. Then x^0 could not have been available at price P^1 otherwise it would have been chosen. That is, its cost $P^1 x^0$ must exceed the cost of x^1 , P^1 , x^1, which equal the consumer's money income M_1 when x^1 is chosen. Hence this fourth assumption can be stated succinctly as

$$P^0 x^0 \geq P^0 x^1 \text{ implies } P^1 x^1 < P^1 x^0 \qquad [1.01]$$

when x^0 is chosen at P^0, M_0 *and* x^1 *at* P^1, M_1. If x^0 is chosen when x^1 is purchasable x^0 is said to be revealed preferred to x^1. The statement [1.01] is usually referred to as the weak around of revealed preference.

This set of mild behavioural assumptions generates au the utility based predictions concerning the consumer's demand functions. Consider first the sign of the substitution effect. Figure 1.04 shows the consumers initial budget line B_0 defined by price vector P^0 and money income M_0. The bundle chosen initially on B_0 *is* $x^0 . B$ is the budget line after a fall in P_1 with M unchanged, and x^1 the new bundle chosen on B_1. Our behavioural assumptions do not place any restriction on the location of x^1 on B_1. As in section 2D, it is useful to partition the price effect $\left(x^0 \text{ to } x^1\right)$ into a change in x due solely to relative price change C the substitution effect and a change due solely to a change in real income. Since we have forsworn the use of utility functions in this section we cannot use the indifference curve through x^0 to define a constant real income. Instead we adopt the constant purchasing power or slit sky definition of constant real income Accordingly the consumer's money income is lowered until, facing the new prices, the initial bundle x^0 can just be brought. In fig 1.01 the budget line is shifted inward parallel with B_1, until at B_2 it passes through x^0. The consumer confronted with B_2 will buy the bundle x^2 to the right of x^0. Therefore x^0 to x^2 is the substitution effect and x^2 to x^1 the income effect of the fall in P_1.

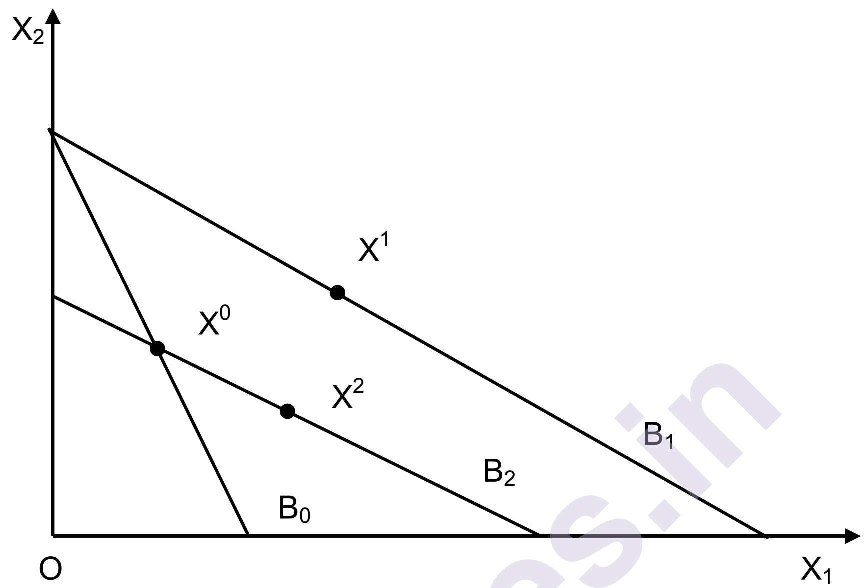


Fig. 1.01

We can now prove that if the consumer satisfies assumption 1.01 the substitution effect must always lead to an increase in consumption of the good whose price has fallen. This is easily done in the two-good example of fig $1.01.x^2$ must lie on B_2 (by the assumption that all income is spent) and hence there are three possibilities : x^2 can be to the left or the right of or equal to, x^0 x^2 cannot be to the left of x^0 on B_2 because these bundles are inside the consumer's initial feasible set and were rejected in favor of x^0, x cannot equal x^0 because the prices at which x^2 *and* x are chosen differ and, by our second assumption different bundles are chosen in different price income situation. Therefore x^2 must contain more x_1 than (c.e be to the right of) x^0.

This result can be extended to the n-good case, and the proof is instructive because similar arguments will be used to derive comparative statics predictions in the theory of the firm. We can generalize the steps in the analysis of Fig 1.01 as follows. p^0, x^0 are the initial price vector and consumption bundle, p^1, and x^1 are the new price vector and consumption bundle. The consumer's income in adjusted until at $M_2 x^0$ can just be purchased at the new price, p^1, so that $p^1 x^0 = M_2$. Faced with price vector p^1 and the compensated money income, M_2, the consumer chooses x^2, and because all income is spent we have that $p^1 x^2 = M_2$. Hence the compensating change in M ensures that

$$p^1x^0 = M_2 = p^1x^2 \qquad [1.02]$$

New x^2 is chosen when x^0 is still available (i.e. they are both on the same budget plane) so that by our consistency assumption 1.01 we have

$$p^0x^0 < p^0x^2 \qquad [1.03]$$

or : x^2 was not purchasable when x^0 was bought. Rearranging 1.02 gives

$$p^1x^0 - p^1x^2 = p^1\left(x^0 - x^2\right) = 0 \qquad [1.04]$$

and similarly 1.03 gives

$$p^0x^0 - p^0x^2 = p^0\left(x^0 - x^2\right) < 0 \qquad [1.05]$$

subtracting 1.05 from 1.04 gives

$$p^1\left(x^0 - x^2\right) - p^0\left(x^0 - x^2\right) = \left(p^1 - p^0\right)\left(x^0 - x^2\right) > 0$$

and multiplying by (-1) we have

$$\left(p^1 - p^0\right)\left(x^2 - x^0\right) < 0 \qquad [1.06]$$

This prediction applies irrespective of the number and direction of price changes, but in the case of a change in the ith price only, $p^1 \; and \; p^0$ differ only in pi and so 1.06 becomes

$$\sum_i \left(p_i^1 - p_i^0\right)\left(x_i^2 - x_i^0\right) = \left(p_i^1 - p_i^0\right)\left(x_i^2 - x_i\right) < 0 \qquad [1.07]$$

Hence when p_i changes the substitution effect $\left(x_i^2 - x_i\right)$ is of opposite sign to the price change. The constant purchasing power demand curve will therefore slope downwards.

We can also derive the slut sky equation from the behavoura assumption. Since $M_2 = p^1x^0 \; and \; M_0 = p^0x^0$ the compensating reduction in M is

$$\Delta M = M_0 - M_2 = p^0x^0 - p^1x^0 = \left(p^0 - p^1\right)x^0 = -\left(p^1 - p^0\right)x^0$$

and in the case of a change $(\Delta p;)$ in p_i only we have

$$\Delta M = -\Delta p_i x_i^0 \qquad [1.08]$$

The price effect of $p_i \; on \; x_i \; is \left(x_i^1 - x_i^0\right)$ and this can be partitioned into the substitution $\left(x_i^2 - x_i^0\right)$ and income $\left(x_i^1 - x_i^2\right)$ effects:

$$x_i^1 - x_i^0 = \left(x_i^2 - x_i^0\right) + \left(x_i^1 - x_i^2\right)$$

Dividing by Δp_i gives

$$\frac{x_i^1 - x_i^0}{\Delta p_i} = \frac{x_i^2 - x_i^0}{\Delta p_i} + \frac{x_i^1 - x_i^2}{\Delta P_i} \qquad [1.09]$$

But from [1.08] $\Delta p_i = -\Delta M / x_i^0$ and substituting this in the second term on the right hand side of [1.09] yield

$$\frac{x_i^1 - x_i}{\Delta p_i} = \frac{x_i^2}{\Delta p^i} - x_i^o . \frac{\left(x_i^1 - x_i^2\right)}{\Delta m} \left.\frac{\Delta x_i}{\Delta p_i}\right|_m = \left.\frac{\Delta x_i}{\Delta p_i}\right|_{px} - x_i^0 . \left.\frac{\Delta x_i}{\Delta M}\right|_p \qquad [1.10]$$

The 1_m notation indicates that money income is held constant in evaluating the rate of change of x_1 with respect to p_i and the similar notation on the right hand side that purchasing power px and price vector p are being held constant in evaluating the rate of change of x_i with respect to p_i and M [1.010] is the descrete purchasing power version of the slutsky equation of section.

It is possible to show that the utility maximizing theory of the consumer and the revealed preference theory are equivalent. All the predictions derived from the assumption about preferences in section 2A can also be derived from the assumption about behaviour made in this section. A consumer who satisfies the preference assumptions mill also satisfy these behavioral assumption. Similarly, if the consumer satisfies the behavioural assumptions, we can construct curves from observed choices which have all the properties of the indifference curves of section 2-1. The consumer acts as if possessing preferences satisfying the preference assumptions. (strictly the weak action needs to be strengthened slightly). Since the two theories are equivalent we will not consider more of the predictions of the theory of revealed preference but will instead use the theory of revealed preference but will instead use the theory to investigate some properties of price indices.

1.2.1 The feasible set:

We initially assume that the consumer has a given money income M, faces constant prices for all of the goods in the utility function and cannot consume negative quantities of any good. Then A Consumer's feasible set defined by these assumptions is the set of bundles satisfying

$$p_1x_1 + p_2x_2 + ... + p_nx_n - \sum p_i \times \leq m \qquad [1.1]$$

$$x_1 \geq 0, x_2 \geq 0, ... x_n \geq 0$$

where p_i is the price of good i.

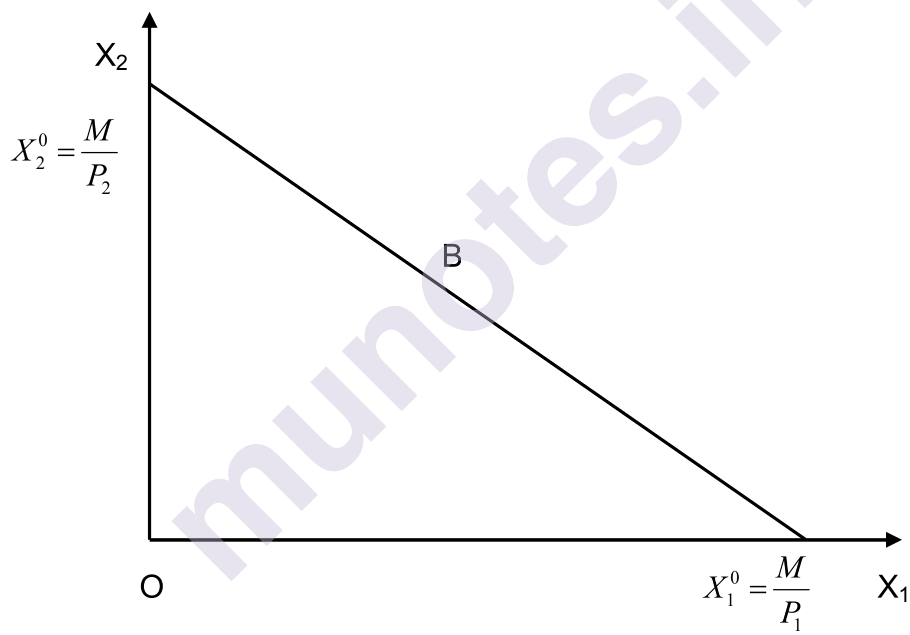


Fig. 1.02

The feasible set in the two good case is shown in Fig 1.02 as the triangular area $Ox_1^0\ x_2^0 = M / P_1$ is the maximum amount of x_1 that can be bought with income M at a price of $p_1 . x_2^0$ is analogously defined. The budget constraint is $p_1x_1 + p_2x_2 \leq M$ in this two-good case, or:

$$x_2 \leq \left(M - p_1x_1\right) / p_2 \qquad [1.2]$$

which is satisfied by all points on or below the line B from $x_1^0\ to\ x_2^0\ B$, the upper boundary of the feasible set, is known as the consumer's budget line and is defined by

$$x_2 = \left(M - p_1 x_1\right) / P_2 \qquad [1.3]$$

The slope of the budget line is

$$\left.\frac{dx_2}{dx_1}\right|_{M\ constant} = -\frac{P_1}{P_2} \qquad [1.4]$$

where the notation on the left-hand side is to remind us that this is the rate at which a consumer with fixed income can exchange $x_1\ for\ x_2$ on the market. A one unit reduction in purchase of x_1 reduces expenditure by P_1, and so, since 1 unit of x_2 costs P_2 . the consumer can buy p_1 / p_2 extra units of x_2. Therefore 1 unit of x_1 exchanges for p_1 / p_2 units of x_2.

The consumer's feasible set has a number of properties relevant for the analysis of the optimal consumption decision It is.

(a) bounded, from below by the non-negativity constraints on the x_i and from above by the budget constraint, provided that M is finite and no price is zero. If, for example, $p_1 = 0$ then the budget line would be a line parallel to the x_1 axis through the point $x_2^0 = M / p_2$ and the feasible set would be unbounded to the right : Since x_1 would be a free good the consumer would consumes as much of it as he wished;

(b) closed. Since any bundle on the budget line B or the quantity does is available;

(c) Convers, Since for any two bundles x^1 and x^{11} in the feasible set, any bundle $\bar{x}$ lying on a straight line between them will also be in the feasible set. Since $\bar{x}$ lies between $x^1\ and\ x^{11}$, and they both satisfy the non-negativity constraints, $\bar{x}$ will cost no more than the consumer's income lying between x^1 and x^{11} it must cost no more than the more expensive of them, say x^1. But since x^1 lies within the feasible set, so must $\bar{x}$ lies within the feasible set, so must $\bar{x}$. Hence $\bar{x}$ is in the feasible set.

(d) non-empty-provided that M > 0 and at least one price is finite the consumer can buy a positive amount of at last one good.

Consider the effects of changes in M and P_i on the feasible set, in preparation for section D where we examine their effects on the consumer's optimal choice. It money income increases from

$M_o \; to \; M_1$, the consumer's feasible set expands as the budget line moves outward parade with its initial position, as in fig 1.03 (a) with $M = M_0$ the intercepts of the budget line B. on the $x_1 \; and \; x_2$ axes respectively are $M_o / P_1 \; and \; M_0 / P_2$. A doubling of M for example, will double the value of the intercepts, Since $M_1 / P_2 = 2M_0 / P_2$ when $M_1 = 2M_0$. The slope of the budget line is - P_1 / P_2 and is unaffected by changes in M.

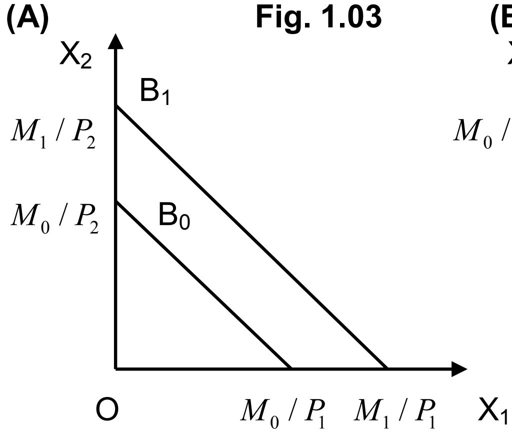
Consider next an increase in P_1, as shown in fig (1.03)(b) since M and P_2 are unchanged the budget line will still have the same M / P_2 intercept on the x_2 axis. An increase in P_1 will cause the budget line to pivot about M_0 / P_2 and become more steeply sloped as P_1 / P_2 becomes larger. In fig 1.03(b) a rise in $P_1 \; to \; P_1^1$ shifts the x_1 intercept from $M_0 / P_1 \; to \; M_0 / P_1^1$ where $M_0 / P_1 > M_o / P_1^1$ since $P_1 < P_1^1$.

Equal proportionate increases in all prices cause the budget line to shift inwards towards the origin as in fig 1.03 (c) suppose $P_1 \; and \; P_2$ increase from $P_1 \; and \; P_2 \; to \; KP_1 \; and \; KP_2 \; where \; k > 1.$ Then the slope of the new budget line is unchanged :

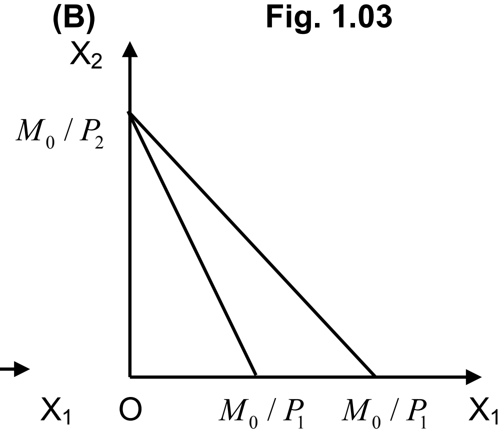
$-KP_1 / KP_2 = -P_1 / P_2$ and the new intercepts are
$M / KP_1 < M / P_1 \; and \; M / KP_2 < M / P_2$

Finally, if all prices and M change in the some proportion the budget lines is unchanged. The intercept on the1st axis after all price and M change by the factor K is $KM / KP_i = M / P_i$ so the intercept is unaffected, as is the slope, which is $-KP_1 / KP_2 = - P_1 / P_2$.

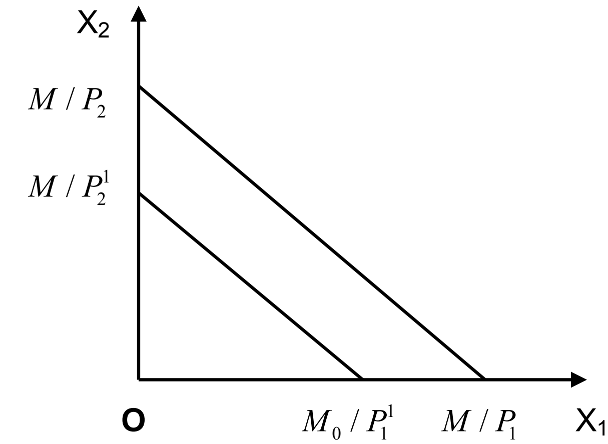
(A) **Fig. 1.03**



(B) **Fig. 1.03**



(C) Fig. 1.03



1.2.2 The consumption decision :

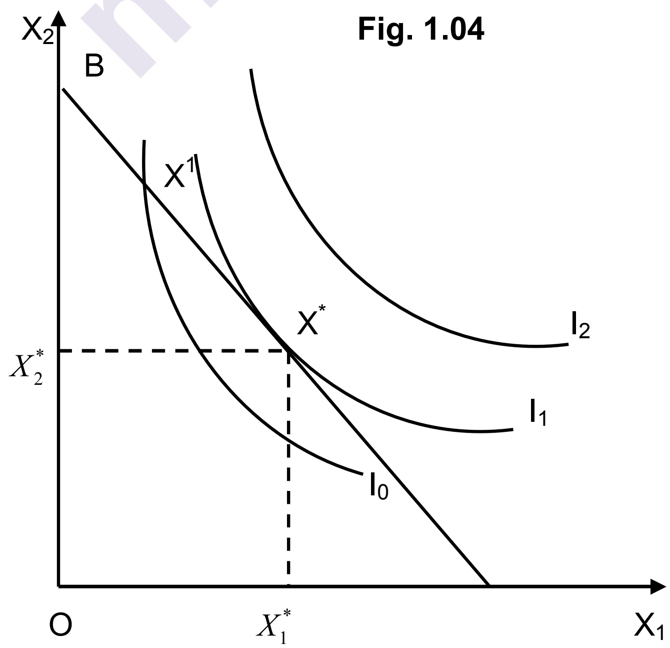
Given the assumption of the previous two sections, the consumer's problem of choosing the most preferred bundle from those available can be formally stated as

$$\max u(x_1, x_2 \ldots x_n)\; s.t. \sum_i P_i X_i \le M,\; X_i \ge 0,\; i = 1 \ldots n\; x_1, \ldots x_n \qquad [1.3]$$

We can derive the conditions which the solution to this problem must satisfy by a diagrammatic analysis of the two good case. We leave to the latter part of this section a brief confirmation of our results using more rigorous methods.

From the assumption of section A we can represent the consumer's preferences by a utility function which has indifference curves or contours like those of figure 1.04.

Fig. 1.04



All commodities are assumed to have positive marginal utility so that bundles on higher indifference curves are preferred to these on lower indifference curves. This assumption (a consequence of assumption 4 in section A). Also means that the consumer will spend all his income since he cannot be maximizing a if he can buy more of some good with positive marginal utility. The consumer will therefore choose a bundle on his budget line B.

In figure 1.04 there is a tangency solution where the optimal bundle x is such that the highest attainable indifference curve. It is tangent to the budget line and the consumer consumes some of both goods the slope of the indifference curve is equal to the slope of the budget line at the optimum.

$$\left.\frac{dx_2}{dx_1}\right|_{\mu\ constant} = \left.\frac{dx_2}{dx_1}\right|_{M\ constant}$$

The negative of the slope of the indifference curve is the marginal rate of substitution MRS_{21}; and the negative of the slope of the budget line is the ratio of the price of x_1 *and* x_2. Hence the consumer's equilibrium condition can be written as

$$MRS_{21} = \frac{\mu_1}{\mu 2} = \frac{P_1}{P_2} \qquad [1.4]$$

The consumer is in equilibrium (choosing on optional bundle) when the rate at which he can substitute one good for another on the market is equal to the rate at which he is just content to substitute one good for another.

We can interpret this property of the optimal choice is a some what different may. If the consumer spent an extra unit of money on x_1 he would be able to buy $1/P_1$ units of $x_1.\mu_1\Delta x_1$ is the gain in utility from an additional Δx_1 units of x_1 Hence $\mu 1/P_1$ is the gain in utility from spending an additional unit of money on x_1. $\mu 2/P_2$ has an along us interpretation. The consumer will therefore be maximizing utility when he allocates his income between x_1 and x_2 so that the marginal utility of expenditure on x_1 is equal to the marginal utility of expenditure on x_2.

$$\frac{\mu_1}{P_1} = \frac{\mu_2}{P_2} \qquad [1.5]$$

This is exactly the condition obtained by multiplying both sides of 1.4 by $\mu_2 / P_{1.}$

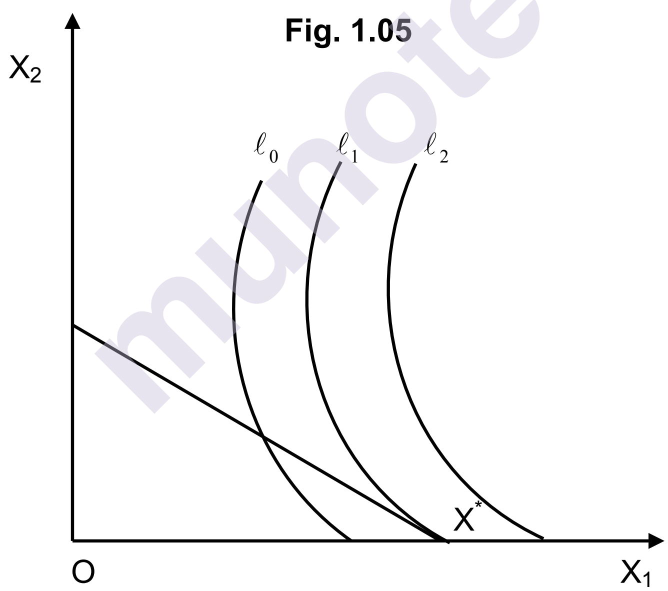
If the consumer's he would be indifferent between spending it on x_1 or x_2, in either case utility could rise by $\mu_1 / P_1 = \mu_2 / P_2$. Hence, if we call the rate at which the consumer's utility increase as income increases the marginal utility of income, densted by μ_m, we have

$$\frac{\mu_1}{P_1} = \frac{\mu_2}{P_2} = \mu_m \qquad [1.6]$$

A more plausible optimum when there are many goods would be a corner point solution, where the optimal bundle x does not contain positive amounts of all goods, as in Fig 1.05 where no x_2 is purchased. In this case the indifference curve at x is steeper than the budget line i.e has a small slope (remembering that the indifference curve and the budget line are negatively stopped). Hence

$$\left.\frac{dx_2}{dx_1}\right|_{\mu\, constant} < \left.\frac{dx_2}{dx_1}\right|_{M\, constant} \qquad [1.7]$$

Fig. 1.05



and therefore

$$\left.\frac{-dx_2}{dx_1}\right|_{\mu\, constant} = MRS_{12} = \frac{\mu_1}{\mu_2} > \frac{P_1}{P_2} = \left.\frac{-dx_2}{dx_1}\right|_{M\, constant}$$

Rearranging, this condition can be written

$$\mu m = \frac{\mu_1}{P_1} > \frac{\mu_2}{P_2} \qquad [1.9]$$

The marginal utility of expenditure on the good purchased, x_1, is greater than the marginal utility of expenditure on x_2, the good not purchased. Because of the higher marginal utility of expenditure on x_1 than on x_2 the consumer would like to move further down the budget line substituting x_1 for x_2 but is restrained by the fact that consumption of negative amounts of x_2 is not possible.

A more formal analysis

Since the consumer's preferences satisfy the assumptions of section A, the objective function in problem above is continuous and strictly quasi-concave From section B the feasible set for the problem, defined by the budget and non-negativity constraints, is non-empty, closed, bounded and convex. From the Existence, Local-Global and uniqueness Theorems. the consumer's optimization problem has a unique solution and there are no non global local solutions.

Since there is at least one good with positive marginal utility the consumer spends all income and hence the budget constraint can be written as an equality constraint. $M - \sum P_i X_i = 0$. If we assume that the solution will be such that some of all goods will be consumed $\left(x_i > 0 \left(i = 1 \ldots n\right)\right)$ where x_i is the optimal level of x_i, then the non-negativity constraints are non-binding and we have a problem to which can be applied the method of lagrange outlined in Appendix G. The lagrange function derived from is

$$L = \mu\left(x_1, \ldots, x_n\right) + \lambda\left[M - \sum P_i X_i\right] \qquad [1.10]$$

1.3 THE COMPARATIVE STATICS OF CONSUMER BEHAVIOUR

The solution to the consumer's optimization problem depends on the consumer's preferences, prices and money income. We can write the solution, which we call the demand for goods, as a function of prices and money income.

$$X_i = D_i\left(P_1, P_2, \ldots P_n, M\right) = D_i\left(P, M\right) i = 1, \ldots, n \qquad [1.11]$$

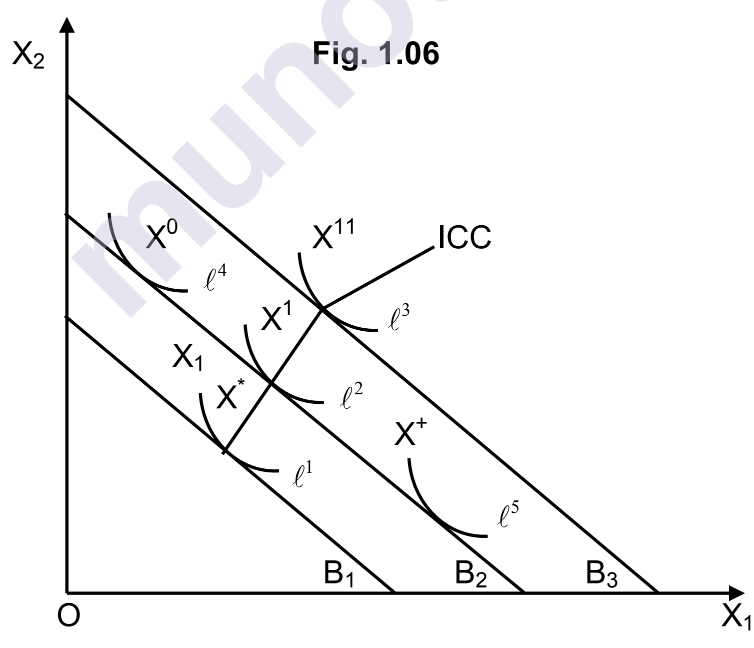
where $P = \left(P_1, P_2, \ldots P_n\right)$ is the vector of prices, and the form of the Marshallian demand function D_i depends on the consumer's references.

The properties of feasible sets and the objective function enable us to place restriction on the form of the demand functions first provided that p, M are finale and positive, the optimization problem must have a solution, since the requirements of the Existence theorem are satisfied, second, the differentiability of the indifference curves and the linearity of the budget constraint imply that the optimal bundle will vary continuously in response to changes in prices and income, and that the demand functions are differentiable. Third, the conditions of the uniqueness theorem are satisfied and so the demand relationships are functions rather than correspondences : a unique bundle is chosen at each (P, M) combination.

We now consider the comparative statics properties of the model. We investigate the effects of changes in the exogenous variables C prices, money income on the equilibrium value of the endogenous variables (the consumer's demand for goods). We want to predict what happens to the optimal bundle $X^* = \left(X_1^*, X_2^*, ... X_n^*\right) = \left(D_1, D_2, ... D_n\right)$ as the feasible set varies.

We consider first change in the consumer's money income. In figure 1.06, B_1 is the initial budget line, X^* the initial bundle chosen. An increase in M, with P_1, P_2 constant, will shift the budget line outward paroled with itself, say to B_2 where X^1 is chosen. A further increase in M will shift the budget line to B_3 where x^{11} is chosen

Fig. 1.06

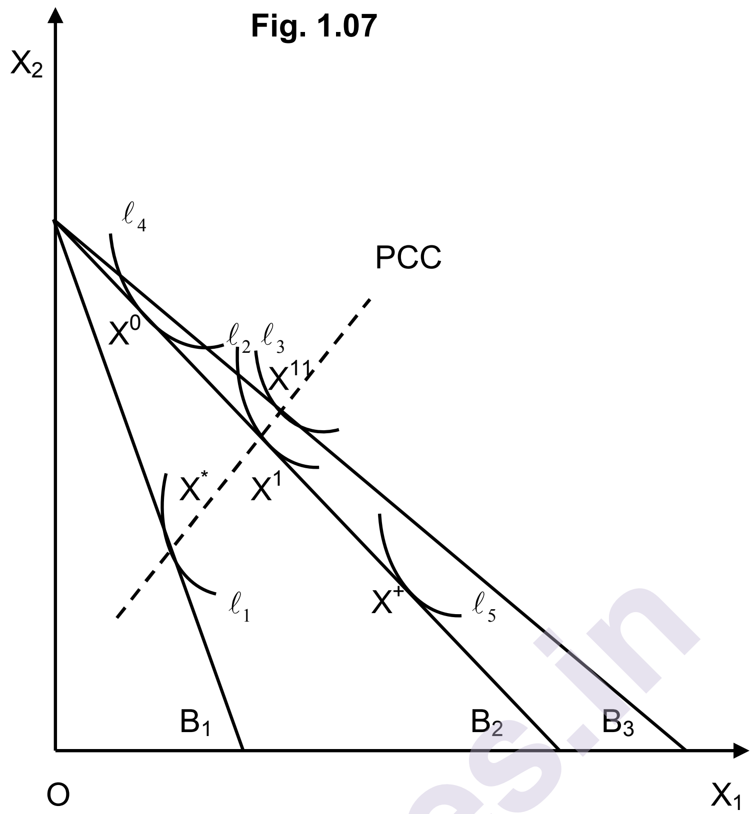


The income consumption curve is the set of optimal points trade out as income vaxies in this may, with prices constant. In the case illustrated both x_1 *and* x_2 are normal goods, for which demand increases as money income rises. However with different preferences the consumer might have chosen x^0 *or* x^1 *on* B_2. If x^0 had been chosen (If I^4 *and not* I^2 had been the consumer's indifference curve) then the demand for x_1 would have fallen as money income rose x_1 would than be known as an inferiat good. A rise in M may lead to a rise, a fall, or no change in the demand for a good. Without knowledge of preferences we cannot predict whether a particular good will be inferior or normal. The theory of consumer behavior cannot be tested by considering the effect of changes in M on the demand for a single good, since any effect is compatible with the theory.

The theory does predict, however, that all goods cannot be inferior. If the consumer reduces demand for all goods when income rises he will be behaving inconsistently. To show this, let x^* be the bundle chosen with an initial money income of M_1 *and* X^1 the bundle chosen when money income rises to M_2. $x^1 << x^* i.e.$ if the demand for all goods is reduced, then X^1 must cost less then x^* since prices are held constant x^1 was therefore available when x^* was chosen. But when x^1 was chosen x^* was still attainable (Since money income had increased). The consumer there form preferred x^* over x with a money income of M_1 *and* x^1 *over* x^* with money income $M_2 > M_1$. He is therefore inconsistent : his behaviour violates the translate assumption of section A, and our model would have to be rejected.

We now turn to the effects of changes in price on the consumer's demands. Fig 1.07 shows the implications of a fall in the price of x_1 with money income held constant. B_1 is the initial budget line, x^* the initial optimal bundle. A fall in P_1, say from P_1 *to* P_1, causes the budget line to shift to B_2, X^1 is the optimal bundle on B_2, X^{11} the optimal bundle on B_3, which results from a further fall in P_1 from P_1^1 *to* P_1^{11}. The price consumption curve (PCC) is traced out as the set of optimal bundles as goods increases as P_1 falls. However with different preferences the optimal bundle might have been x^0 *or* x^+ *on* B_2. If x^0 was the optimal bundle with $P_1 = P_1^1$ then x_1 would be a Griffin good, the demand for which falls as its price falls. We conclude that the demand for a good may fall, rise or remain unchanged as a result of change in a price facing the consumer once again the model yields no definite (refutable) prediction about the effect on a single endogenous variable (the demand for a good) of a change in one of the exogenous variables (in this case a price). If is again possible, however, to predict (by reasoning similar to that employed in the case of a change in M) that a fall in price will not bad to a reduction in demand for all goods, and the reader should supply the argument.

Fig. 1.07



1.4 INCOME AND SUBSTITUTION EFFECTS

The analysis of the effect of price changes on the consumer's demands (optimal choices) has suggested that demand for a good may increase, decrease or remain unchanged, when its price rises; in other words anything may happen we now examine the effect of a change in the price of good 1 in more detail in order to see if it is possible to make more definite (refutable) predictions. We proceed by making a conceptual experiment. All we can actually following a price change. However, we can carry out a hypothetical analysis which decomposes the overall demand change into two components. We then use this decomposition to say something more definite about consumer behauiour.

In Fig 1.08. it can be seen that the fall in price of good 1 does two things :

(a) it reduces the expenditure required to achieve the initial utility level I_1 allowing the higher utility level I_2 to be achieved with the some expenditure . There has been an increase in the consumers real income:

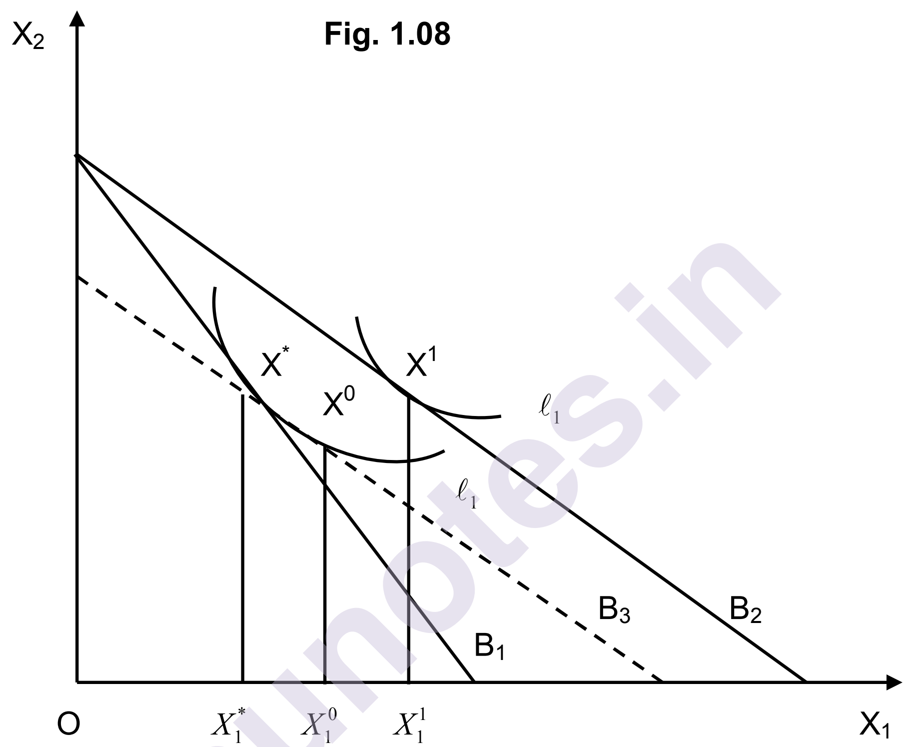
(b) it changes the relative prices facing the consumer.

In Fig 1.08 we accordingly break dawn the change in demand for x_1 into :

(a) the income effect, which is the change resulting solely from the change in real income with relative price held constant; and

(b) the own substitution effect, which result's salary from the change in P_1 with real income held constant.

Fig. 1.08



X^* *and* X^1 are the optimal bundles before and after the fall in P_1, B_1 *and* B_2 the corresponding budget lines. The compensating variation in money income is that change in M which will make the consumer just as well off after the price fall as he was before. In other words, there will be some reduction in M after the price fall which will 'cancel out' the real income gain and return the consumer to the initial indifference curve. I_1. The budget line is shifted inwards (reducing M) pazallel with the post price fall budget line B_2 until at B_3 it is just tangent to the original indifference curve I_1. The consumer confronted with this budget line would chose bundle x^0. The difference between x^* and x^0 is due to a change in relative prices with real income (utility) held constant. The difference between x^0 and x^1 is due to the change in money income with relative prices held constant. X_1^*, x_1^1 and x_1^0 are the amounts of x_1 contained in the bundles x^*, x^1, x^0 and

(a) $x_1^0 - x_1^*$ is the own substitution effect;

(b) $x_1^1 - x_1^0$ is the income effect;

(c) $\left(x_1^0 - x_1^*\right) + \left(x_1^1 - x_1^0\right) = x_1^1 - x_1^*$ is the total price effect.

The purpose of carrying out this experiment in hypothetical compensation is to demonstrate that the own substitution effect will always be positive in the case of a price fall and negative for a price rise. The absolute value of the slope of the indifference curve declines from left to right, i.e. as more x_1 *and less* x_2 is consumed the curve flattens. The fall in P_1 flattens the slope of the budget line, and hence the budget line B_3 must be tangent with I_1 to the right of x^*, i.e. at a bundle containing more x_1.

The income effect is positive in the particular case illustrated in Fig 1.08. The income effect reinforces the own substitution effect since x^1 contains more x_1 than x^0. If x_1 had been inferior the income effect of the price fall would have been negative and in the opposite direction to the own substitution effect, so that the price effect would be smaller than the own substitution effect, In fig 1.8 (a) the income effect partially offsets the substitution effect but the price effect is still positive; a fall in P_1 leads to a rise in the demand for x_1. In figure 1.8 (b) the negative income effect more than offsets the positive substitution effect and x_1 is a giffen good. Hence inferiority is a necessary, but not sufficient, condition for a good to be a Giffen good.

Fig. 1.8 (a)

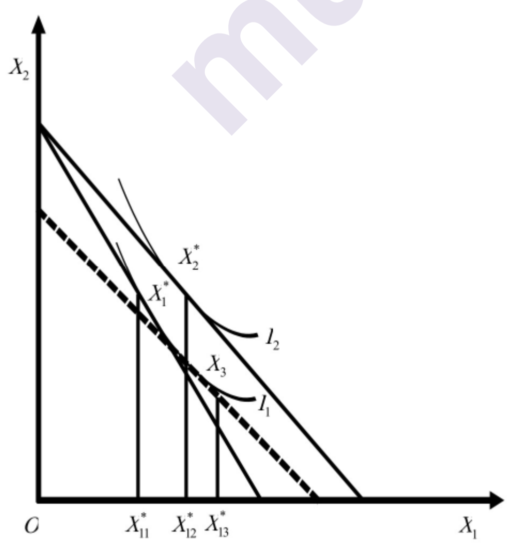
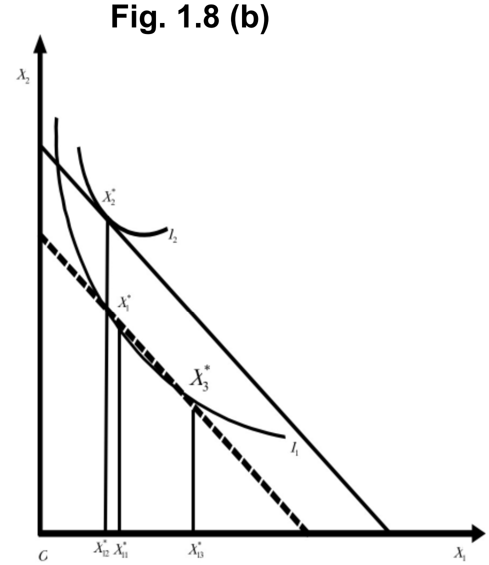


Fig. 1.8 (b)



This decomposition of the price effect has generated two further predictions :

1) A normal good cannot be a Giffen good. Hence if we observe that a consumer increases demand for a good when money income rise (other things including prices being held constant), we would predict that, if its prices should fall, he will want to buy more of it. If we then observe that he reduced his demand for the good when its price falls (and all other prices are constant and his money income is reasonably close to its original level), then the optimizing model of consumer behaviors has yielded a false prediction.

2) The own substitution effect is always of opposite sign to the price change.

The above decomposition of the price effect into an income and substitution effect is based on the definition, made by Hicks, of unchanged real income as an unchanged utility level. Stutsky suggested an alternative definition of a constant real income as the ability to purchase the bundle of goods bought before the price change. This constant purchasing power definition has the advantage that it does not require detailed knowledge of the consumer's indifference map.

Figure 1.9 reproduces Figure 1.6 with some additions to show the relationship between the Hicks and slutsky definitions of a constant real income. The budget line B_4 just enables the consumer to buy x^*, the initially optimal bundle, at the lower price of P_1. Confronted with this budget line, the consumer actually chooses x_1^+. The price effect has been decomposed into as income effect $\left(x_1^1 - x_3^+\right)$ and an own substitution effect $\left(x_1^+ - x_1^+\right)$. The income effect will again be positive, negative or zero depending on the form of the indifference map. The substitution effect will, as in the Hicksian case, always lead to a rise in demand for a good whose price has faller x^+ cannot lie to the left of x^+ on B_4 because this would mean that the consumer is new choosing x^+ when x^* is still available, having previously rejected x^+ in fauour of x^*. The transitivity assumption would be violated by such behaviour. The slutsky definition yields a prediction. (The sign of the substitution effect) which can be tested without specific knowledge of the consumer's indifference curves to cancel out the income effect.

Our consideration of the comparative static properties of the model has shown that it does not yield refutable predictions about the overall change in demand for individual goods induced by ceteris paribus change in a price or money income. In other words, Fig 1.9

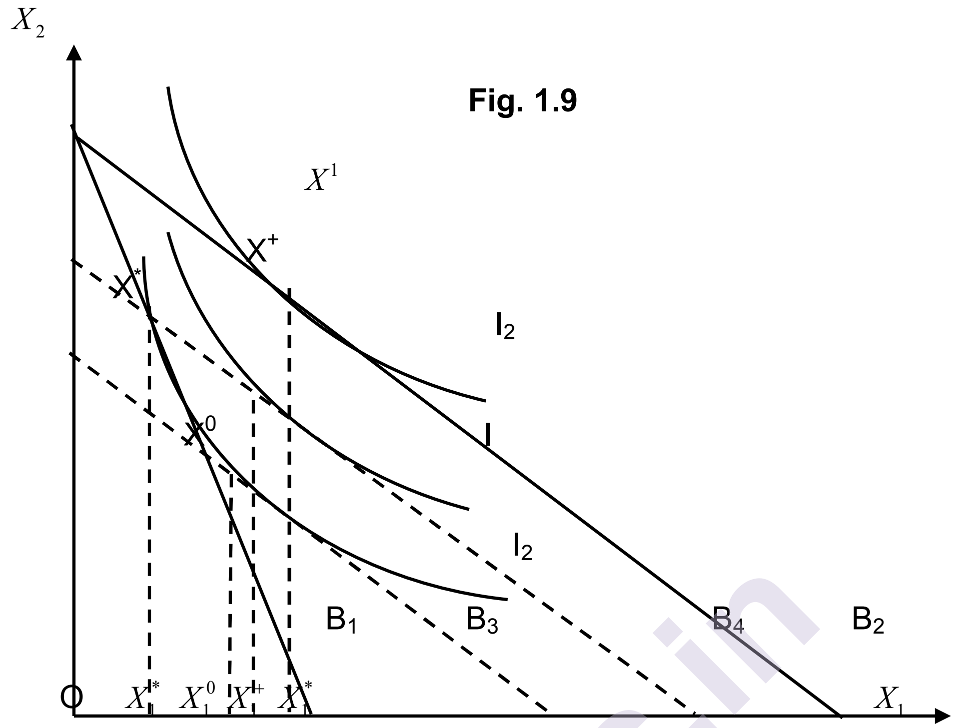


Fig. 1.9

$$\frac{\partial x\psi}{\partial pi} = \frac{\partial Di}{\partial Pi} \gtrless 0 \; i, \; j = 1, 2, ..., n$$

and $$\frac{\partial x\psi}{\partial M} = \frac{\partial Di}{\partial M} \gtrless 0 \;\; i = 1, 2, ..., n$$

for every good and price, only by considering the effect of changes in $P_1 \; or \; M$ on goods, or by considering the effect of changes in p_i and M on a single good or by making more specific assumptions about the consumer's preferences can definite predictions be generated.

Consider, however, the consequences of equal proportionate change in all prices and M. Suppose M increases to KM (K > 1) and price to KP_1 and KP_2. The slope of the budget line will be unaffected. The intercept on the x_1 axis is M / P_1 before the changes in M and prices and $KM / KP_1 = M / P_1$ after the change similarly for the intercept on the x_2 axis. Hence the equal proportionate changes in M and all prices alter neither the slope nor the intercepts on the budget line and so the feasible set is unaltered. If the feasible set is unchanged then so is the optimal bundle.

The model therefore predicts that the consumer will not suffer from money illusion; he will not alter his behaviour if his purchasing power and relative prices are constant, irrespective of the absolute level of prices and money income more formally, the demand function D_i for every commodity is homogeneous of degree zero in prices and money income, since we have

$$x_i^* = D_i\left(Kp, kM\right) = K^0 Di\left(p, M\right) = D_i\left(p, M\right) \quad [1.12]$$

1.5 SUMMARY

Revealed preference theory is the scientific explanation of consumer's behaviour. The feasible set is the set of bundles satisfying $\rightarrow \sum Pi \times i \leq M$.

Consumer chooses the optimal consumption bundle.

1.6 QUESTIONS FOR REVIEW

1. Critically examine the Revealed Preference Theory of consumer behaviour.

2. Examine the Revealed Preference theory and show how it is an improvement over the indifference curve analysis.

❖❖❖❖

Unit-2

CONSUMER BEHAVIOUR - II

Unit Structure :

2.0 OBJECTIVES

At the end of this lesson you will be able to explain -

- The expenditure function.
- The indirect utility function.
- The Roy's Identity
- The Slutsky Equation.
- Properties of demand function.
- Concept of choice under uncertainty.

2.1 INTRODUCTION

In the previous chapter we defined the consumer problem as that of choosing a vector x to solve the problem max $\mu(x)\, s.t.\, px = M$, where p is a price vector and M money income. From the solution we derived marshallian demand functions $x_i = D_i(p, M)(i = 1, \ldots n)$, which express demands as functions of prices and money income. We observed that we cannot place restrictions on the signs of the partial derivatives of these functions: $\partial D_i / \partial M \overset{\geq}{\underset{<}{-}} 0, \partial p_i / \partial M \overset{\geq}{\underset{<}{-}} 0 \left(i, j = \ell, ..., n\right)$, In particular the demand for a good does not necessarily vary inversely with its own price. However, as a result of a diagrammatic analysis, we were able to say that this will be trees of normal goods or of inferior goods whose income effects are weaker than their substitution effects. We know put this analysis on a more rigorous and general basis. We also consider the problem, central to many applications of consumer theory, of deriving a money measure of the costs and benefits incurred by a consumer as a result of price changes. In doing so, we develop the methods and concepts of duality theory, an approach to the analysis of optimization problems which permits an elegant and concise derivation of comparative static results.

2.2 THE EXPENDITURE FUNCTION

The expenditure function is derived from the problem of minimizing the total expenditure necessary for the consumer to achieve a specified level of utility is :

$$\min_{X_1 \ldots\ldots X_n} \sum p_i x_i \ s.t. \ (i) \ \mu\left(x_1 \ldots\ldots x_n\right) \geq \mu$$

$$(ii) \ x_i \geq 0, i = 1, \ldots. n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2.1)$$

If all prices are positive the first constraint in (2.1) will be satisfied as an equality in the solution, since if $\mu(x) < \mu$ expenditure can be reduced without violating the constraint. If it is further assumed that all x; are strictly positive in the solution, we can write the lagrange function for the problem (with μ as the lagrange multiplier) as

$$L = \sum p_i x_i + \mu\left[\mu - \mu\left(x_1 \ldots\ldots x_n\right)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (2.2)$$

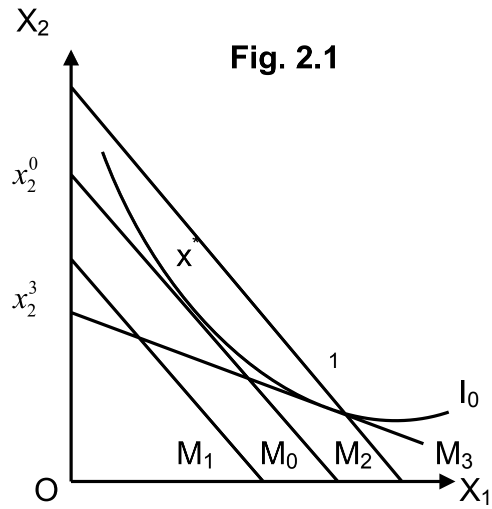
and the necessary conditions for a minimum of L, also the necessary conditions for a solution of [2.1] are

$$\frac{\partial L}{\partial x_i} = p_i - \mu\mu_i = 0 \qquad i = 1, \ldots. n \quad \ldots\ldots\ldots\ldots\ldots\ldots.. \ (2.3)$$

$$\frac{\partial L}{\partial \mu} = \mu - \mu = \left(x_i, \ldots\ldots x_n\right) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2.4)$$

The conditions on the x_i bear a striking resemblance to 1.10 in chapter 1 writing them as $p_i = \mu u_i$ and dividing the condition on x_i by the condition on x_i gives.

$$\frac{p_i}{p_i} = \frac{\mu_i}{u_i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \ (2.5)$$



The ratio of prices is equated to the marginal rate of substitution. This is not surprising as examination of the two-good case in figure 2.1 indicates. The indifference curve I_0 shows the combinations of x_1 and x_2 which give a utility level of u and the feasible set for the problem is all points on or above I_0. The lines M_0, M_1, M_2, are iso expenditure lines similar to the budget lines of earlier diagrams. M_0, for example, plots all bundles costing m_0, i.e. satisfying the equation $p_1x_1 + p_2x_2 = m_0$. The problem is to find the point in the feasible set which is on the lowest isoexpenditure line. This will, in the tangency solution shown here be where the indifference curve ℓ_0 is tangent to the isoexpenditure line m_0. The problem confronting the utility maximizing consumer is to move along the budget line until the highest indifference curve is reached. The expenditure minimizing problem is to move along the indifference curve until the lowest isoexpenditure line is reached. The optimal x_1^* in problem (2.1) depend on the prices and the utility level u :

$$x_i^* = H_i\left(p_1.....p_n, u\right) = H_i\left(p, u\right) \qquad i = 1, ...n \quad \text{.................. (2.6)}$$

and $H_i\left(p,u\right)$ is the Hicksian demand function for x_i. Substituting the optimal values of X_i $\sum p_ix_i$ gives.

$$\sum p_ix_i^* = \sum p_iH_i\left(p,u\right) = m\left(p,u\right) \quad \text{...................... (2.7)}$$

$m\left(p,u\right)$ is the expenditure function, showing the minimum level of expenditure necessary to achieve a given utility level as a function of price and the required utility level.

The Hicksion demand function is also called the compensated demand function. In considering the effect of a change in price on demand with utility held constant (the paralal derivative $\partial H_i / \partial P_j \left(i, j-1, n\right)$ we automatically make whatever changes in expenditure are required to compensate for the effects of the price change on real income or utility. This is illustrated in figure 2.1. Assume P_2 remains constant while P_1 falls to give a new family of isoexpenditures lines, with slopes corresponding to that of M_3 in the figure x^1 is he new expenditure minimizing consumption bundle, and the change from x^* to x^1 is the effect of making the relative price change with m varying to keep u constant. The optimal expenditure line slide round the indifference curve from m_0 to m_3 as the optimal bundle changes from x^* to x^1. The minimized total expenditure can be read off from the intercepts of m_0 and m_3 on the x_2 axis. The fall in p_1 lowers m from $p_2 x_2^0$ to $p_2 x_2^3$.

Provided the indifference curves are strictly convex to the origin the optimal x_i (and hence the expenditure function) vary smoothly and continuously with the prices of the goods. Hence the $H_i\left(p, u\right)$ functions have continuous derivatives with respect to the prices. The demand curve we derive from the Hicksian demand function was represented by curve hh. The slope of the Hicksian or compensated demand curve, $\partial H_i / \partial p_i \left(i = 1, ... n\right)$ is the substitution effect of the price change. Since by definition $\partial H_i / \partial p_i$ is taken with u held constant.

The expenditure function gives that smallest expenditure, at a given price vector, that is required to achieve a particular standard of living or utility level, and describes how that expenditure will change as prices or the required utility level change. The assumptions made in chapter. Concerning the nature of the consumer's preference ordering and indifference sets imply certain properties of the expenditure function.

(a) The expenditure function is concave in price choose two price vectors p^1 and p^{11}, and K such that $o \le k \le 1$. Define $p = kp^1 + \left(1-k\right) p^{11}$. We have to prove that C see the definition of concaulity in Appendix B) :

$$M\left(\overline{P}, u\right) \ge km\left(P^1, u\right) + \left(1-k\right) m\left(P^{11}, u\right) \text{ for given u.}$$

Proof

Let x^1 and x^{11} salve the expenditure minimization problem whine the price vector is respectively p^1 and p^{11}. By definition of the expenditure function, $p^1 x = m\left(p^1, u\right)$ and $p^{11}\, x^{11} = m\left(p^{11}, u\right)$

likewise, let $\overline{x}$ solve the problem when the price vector is $\overline{p}$, so that $\overline{p}\,\overline{x} = m\left(p^{11}, u\right)$. Since x^1 *and* x^1 are solutions to there respective expenditure minimization problems we must have

$$P^1\,\overline{x} \geq P^1\,x^1 \text{ and } p^{11}\,\overline{x} \geq P^{11}\,x^{11} \qquad [2.8]$$

Multiplying through the first inequality by k and the second by 1-k and summing gives

$$KP^1\,\overline{x} + (1-k)\,p^{11}\,\overline{x} \geq kp^1x^1 + (1-K)\,p^{11}x^{11} \qquad [2.9]$$

But by definition of $\overline{p}$ this implies $\left(kp^1 + (1-K)\,p^{11}\right)\overline{p}\,\overline{x} \geq kp^1x^1 + (1-k)\,p^1x^{11}$ [2.10]

which is the result we want.

Figure 2.2 illustrates the proof of this important result. It is obvious that, when the is expenditure lines at which x^1 *and* x^{11} are optimal solutions are shifted so as to pass through point $\overline{x}$, they must yield higher expenditure, this giving the key inequalities in (2.8) The rest of the proof then follows by simple algebra.

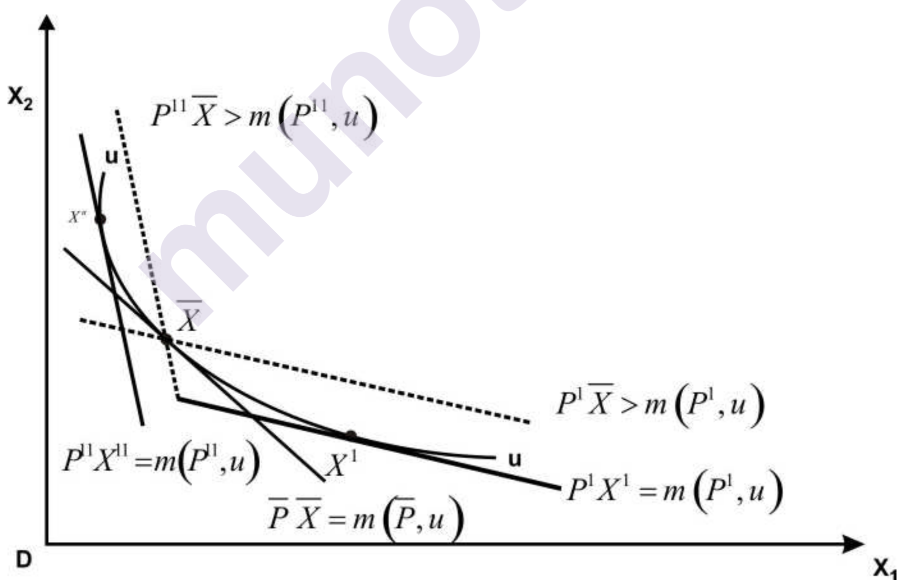


Figure 2.2

The figure could in one sense be misleading. The inequalities (which in this case are strict) appear to follow from the convexity of the indifference curves. Note, however, that the inequalities follow simply from the fact that x^1 (respectively x^{11}) minimizes px at price vector p^1 (respectively p^{11}) while $\overline{x}$ may not 2.8 then follows from the definition of a minimum. Thus the proof of concavity of the expenditure function does not depend on convexity of preference. However the property of uniqueness of solution like $x^1 \; and \; x^{11}$, and the differentiability of Hicksian demands and of the expenditure function, do. Note that strict converity of preference implies strict concavity of the expenditure function at an interior solution to problem [1] Since it implies uniquess of the solution and hence strict inequalities in [2.8].

Figure 2.3 illustrates the strict concauity of the expenditure function when the price vectors $p^1 \; and \; p^{11}$ differ only in respect of one price, p^1. The slope of the expenditure function at a point is equal to the compensated demand for good; at the price p_1 :

(b) Shephard's lemma : $\partial m\left(p_1, u\right) / \partial p := X_i^* = H_i\left(p, u\right)$

The proof is just a version of the Envelope Theorem. Differentiating 2.7 with respect to the 1st price gives.

$$\frac{\partial m}{\partial p_i} = x_i^* + \sum_{i=1}^{n} P_i \frac{\partial x_1^*}{\partial P_i} = x_i^* + \mu \sum_{i=1}^{n} \mu_i \frac{\partial x_i^*}{\partial p_i} = x_i^x \qquad [2.11]$$

The second equality uses the fact that $p_i = \mu u_i$, from the first order condition [2.3] since utility is held constant when p_i varies, differentiating the constraint [2.4] with respect to p_i shows that

$\sum_{i=1}^{n} u_i \, \partial x_j^* / \partial p_i^* = 0$ which gives the third equality in [2.11]

Thus the partial derivative of the expenditure function with respect to the 1st price is the compensated demand for the 1st good. In figure 2.3, the slope of the curve at price $P_i^1 \; is \; x_i^1 = H_i\left(p_1^1, \dots p_i^1 \dots . p_n^1, u\right)$. This can be put intuitively as follows, suppose a consumer buys 12.5 units of a gas a week at a cost of E_1 per unit. The price of gas then rises by p per unit. Shephard's lemma says that, to a first approximation, to maintain the same utility level or standard of living her expenditure must increase by $H_i \Delta P_i = 12.5$: just enough to maintain consumption at the initial price level. The qualification 'to a first approximation' is important. For finite price changes. Fig 2.3 should that $H_i \, \Delta \, p_i$ overstates the required increase in expenditure, since the expenditure function is strictly concave. As a good's price goes up, the consumer substitutes away from the good in question, and this reduces the amount of expenditure otherwise required to keep utility constant. Shephard's lemma tells us that for small enough price changes this distinction can be ignored.

(c) $\partial m / \partial p_i \geq 0$ with strict inequality if $x_i^* > 0$.

This follows immediately from shephard's limma. Since at last one good must be bought, the expenditure function is non-decreasing in the price vector p and strictly increasing in at least one price. Higher prices mean higher expenditure to reach a given utility.

(d) The expenditure function is homogeneous of degree 1 in prices.

Take a given u value and price vector p_1^0 and $Let\ M^o = m(p^o, u) = p^0 x^0$ where x^0 is the expenditure minimizing bundle at p^0, that is $p^0 x^0 \le p^0\ x$ for all bundle x yielding utility of u or more. But this imples that $kp^0 x^0 \le k^0 p^0\ x$ for all bundles yielding at least u and so x^0 is optimal at prices p^0 and kp^0. Then $m(kp^0, u) = km(p^0, u)$. Since relative prices do not change, the optimal bundle is not changed. It has merely become more or less expensive depending on whether k > 1 or k < 1.

(e) The expenditure function is increasing in u Higher utility at given prices requires higher expenditure. Rather than use the envelope theorem again, recall that the lagrange multi plier $\mu > 0$ in 2.2 is equal to the derivative $\partial m / \partial u, \mu$ is the marginal cost of utility since it represents threat of change of minimized expenditure with respect to the required utility level. μ is the reciprocal of the lagrange multiplies λ in the corresponding utility maximization problem, i.e. μ is the inverse of the marginal utility of income note that, although the assumptions underlying ordinal utility theory allow the sign of μ to be estestablished, we cannot say that μ is necessarily increasing or decreasing, with u, because both are possible for different, permissible utility functions.

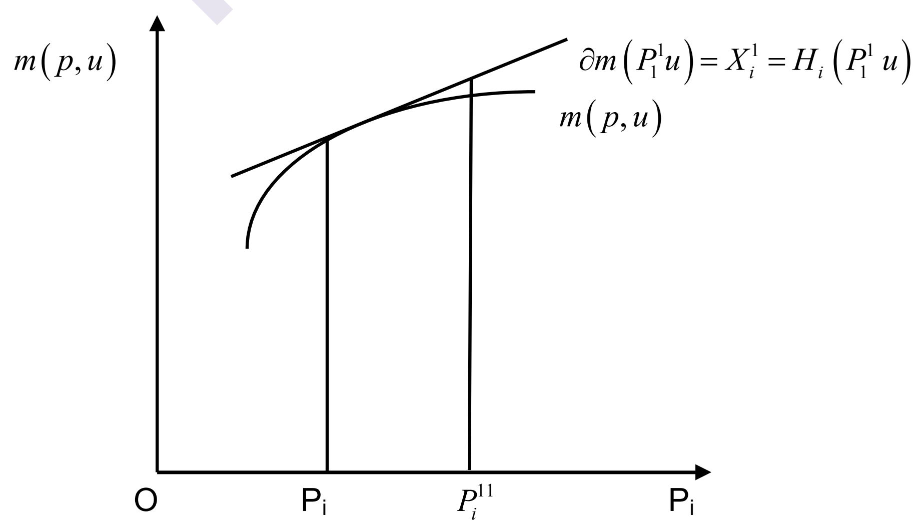


Figure 2.3

It is important to be clear about the relation between expenditure and utility the essential facts about the consumer's preference ordering are contained in the structure of her indifference sets or curves. The minimum expenditure requiredto reacha given indifference set at given prices is unaffected by any number we attach to that indifference set to indicate its place in the ordering on the other hand, once we have chosen a numerical representation of the preference ordering a utility function this will imply a particular relationship between expenditure m and utility u. But the properties we set out above hold for all permissible utility functions, and the only general restriction we can place on the relation between m and u (for a given price vector) is that it is monotonically increasing.

2.3 THE INDIRECT UTILITY FUNCTION, ROY'S IDENTITY AND THE SLUTSKY EQUATION

The indirect utility function is derived from the consumer problem of maximizing $\mu(x_1....,x_n)$ subject to the budget constraint $\sum p_i x_i \leq M$ and non-negativity constraints we saw that the x_i which are optimal for this problem will be functions of the $p_i \text{ and } M : x_i^* = D_i(p_1 ...., p_n, M) = D_i(p, M)$ The maximized value of $u(x_1, ... x_n) = u(x_1^*, ..., x_n^*)$ will therefore also be a function of the $p_i \text{ and } M$

$$u(x_1^* ... x_n^*) = u(D_1(p, m), ... D_n(p, M)) = u^*(p, m) \qquad [2.12]$$

u is known as the indirect utility function since utility depends indirectly on prices and money income via the maximization process, in contrast to the utility function $u(x_1, ..., x_n)$ where u depends directly on the x_i. We can use u^* to investigate the effects of changes in prices and money income on the consumer's utility.

From the interpretation of the Lagrange multiplier, the effect of an increase in money income on the maximized utility is

$$\frac{\partial u^*}{\partial m} = \lambda \qquad [2.13]$$

The effect of a change in p_i on u^* can also be found as a version of the Envelope Theorem, Differentiating u^* with respect to p_i :

$$\frac{\partial u^*}{\partial p_i} = \sum u_k \frac{\partial x_k^*}{\partial p_i} = \lambda \sum P_k \frac{\partial x_k^*}{\partial p_i} \qquad [2.14]$$

The budget constraint must still be satisfied so that

$$\frac{d}{dp_i}\left[\sum P_k x_k^*\right] = \frac{dM}{dP_i} = 0$$

and so

$$\sum p_k \frac{\partial x_k^*}{\partial p_i} + x_i^* = 0$$

or $$-x_i^* = \sum p_k \frac{\partial x_k^*}{\partial p^i}$$

Substitution of this in [2.14] gives Roy's identity :

$$\frac{\partial u^*}{\partial p_i} = -\lambda\, x_i^* = -\frac{\partial u^*}{\partial m} x_i^* \qquad [2.15]$$

The expression on the right hand side of 2.15 has the following intuitive explanation An increase in p_i is a reduction in the purchasing power of the consumer's money income M, and by shephard's lemmat, to the first orders, her purchasing power falls at the rate $-x_i^*$ as p_i varies λ is the marginal utility of money income. The product of λ and $-x_i^*$ is the rate at which utility varies with money income, times the rate at which (the purchasing power of) money income varies with p_i and so this product yields the rate of change of utility with respect to p_i.

Since $\lambda > 0$, Roy's identity should that an increase in the price of good a consumer buys reduces her (maximized) utility or standard of living to a greater extent, the larger the quantity of it she buys.

The indirect utility function tells us that utility depends, via the maximization process, on the price income situation the consumer faces. Note that implies 2.13 that the indirect utility function is monotonically increasing in income, M. thus we can invert the indirect utility function $u = u^*(p, m)$ to obtain the expenditure function M = m (p, u). A given solution point for a given price vector can be viewed equivalently as resulting from minimizing expenditure subject to the given utility level or maximizing utility subject to the given expenditure level. We can choose either to solve the utility maximization problem, obtain the indirect utility function and invert it to obtain the expenditure function, or to obtain the expenditure function and then invert it to obtain the indirect utility function. The two functions are dual to each other, and contain essentially the same information : the forms of the functions and their parameters are completely determined by the form of the original (direct) utility function. But then, since each of these three functions contains the same information, we can choose any one of them as the representation of the consumer's preferences that we wish to work with.

Duality can be used to give a neater derivation of roy's identity. Setting M - m(p,u), rewrite the indirect utility function as

$$u = u^{*}\left(p, m\left(p,u\right)\right) \qquad [2.16]$$

The differentiating through with respect to p_i, allowing m to vary in such a may as to hold u constant, gives

$$o = \frac{\partial u^{*}}{\partial p_i} + \frac{\partial u^{*}}{\partial m}\frac{\partial m}{\partial p_i} \qquad [2.17]$$

which, using shephard's lemma and 2.13, gives Roy's identity 2.15 directly.

Since the indirect utility function is ordinal and not cardinal, we cannot restrict it to be convexs or concave (unlike the expenditure function), bad we can show that is quasi-convex in prices and income, a property that is useful in many application.

Figure 2.4 illustrates quasi-convexity in prices and income. A function is quasi convex if, given any point in its (convex) domain, the worse set of the point, i.e. the set of points giving Fig 2.4.

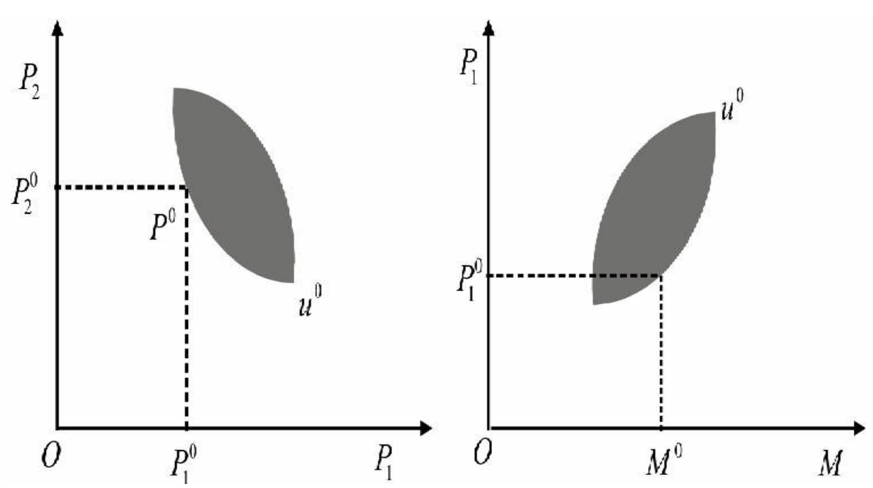


Figure 2.4

the same or lower values of the function, is convex. Take the case of two goods, where the indirect utility function is $u^*(p_1, \ p_2, M)$. In part (a) of the figure, $p^0 = (p_1^0, p_2^0)$ is some arbitrary point, and the indifference curve u^0 or contour of the indirect utility function, through that point is convex to the origin if the function is quasi - convex. The course set $w(p^0) = \{(p_1, p_2); u^*(p_1, p_2, M^0) \leq u^*(p_1^0, p_2^0, M^0)\}$ use to the north east of p^0 chigher prices imply lower utility) and is convex. In (b) of the figure, the contour u^* through the point $(p_1^0, p_2^0 \ M) \leq u^*(p_1^0, p_2^0, M)$ lies to the north west of the point chigher price and lower income implies lower utility) and is convex. (Be sure you can explain the negative and positive slopes of these contours, respectively) Similarly for any point (p_2^0, M).

Prove that the indirect utility function is quais convex in prices and income choose two points in the domain of the function, (p^0, m^0) and (p^1, m^1), such that

$$u^*(p^0, m^0) = u^0 \geq u^1 = (p^1 m^1) \qquad [2.18]$$

So that (p^1, m^1) is in the wares set of (p^o, m^0). We have to show that any convex combination of these two price - income vectors is also in this worse set of $(p^0, m^0): u^*(\overline{p}, \overline{m}) \leq u^0$ [2.19]

where

$$\overline{p} = kp^0 + (1-k)p^1, km^0 + (1-k)m^1 \ k \in [0,1] \qquad [2.20]$$

Now take any goods vector x that satisfies the budget constraint $\overline{p}\, x \leq \overline{m}$ or, given the definitions of $\overline{p} \ and \ \overline{m}$

$$kp^0 x + (1-k) \, p^1 x \leq km^0 + (1-k) M^1 \qquad [2.21]$$

For this to hold either

$$p^0 x \leq M^0 \qquad [2.22]$$

or $\quad p^1 x \leq M^1 \qquad$ [2.23]

or both. Now x satisfying these inequalities cannot yield a higher utility value than the maximized utility at the corresponding budget constraint. Hence 2.22 implies $u^*(\overline{p}, \overline{m}) \leq u^0$, and 2.23 implies

$u^*\left(\overline{p},\overline{m}\right) \leq u^1$ and since one or both of 2.22 and 2.23 must hold and $u^1 \leq u^0$ by assumption, we have establi shed $u^*\left(\overline{p},\overline{m}\right) \leq u^0$ as required.

Figures 2.5 illustrates. Fix the price of good 2 as 1, so that the intercept of the x_2 axis shows total expenditure and the slope of the budget constraint is - $p_1.B^0$ in the figure corresponds to the budget constraint $p_1^0 x_1 + x_2 = m^0 B^1$ corresponds to $p_1^1 x_1 + x_2 = m^1$ and yieldsa lower utility value than $B^0.\overline{B}$ corresponds to $p_1, x_1 + x_2 = \overline{M}$ where $\overline{p}_1 = kp^0 + (1-k)\,p_1^1, \overline{m} = KM^0 + (1-k)\,M^1.\overline{B}$ also yields a labour value of utility than B^0 we have.

$$M^1 < \overline{M} < M^0 \qquad [2.24]$$

$$p_1^1 < \overline{p}_1 < p_1^0 \qquad [2.25]$$

That $\overline{B}$ passes through the intersection point $\left(x_1^0, x_2^0\right) of\ B^0\ and\ B^1$ follows by nothing that if we sum.

$$k\left(p_1^0\ x_1^0 + x_2^0\right) = KM^0 \qquad [2.26]$$

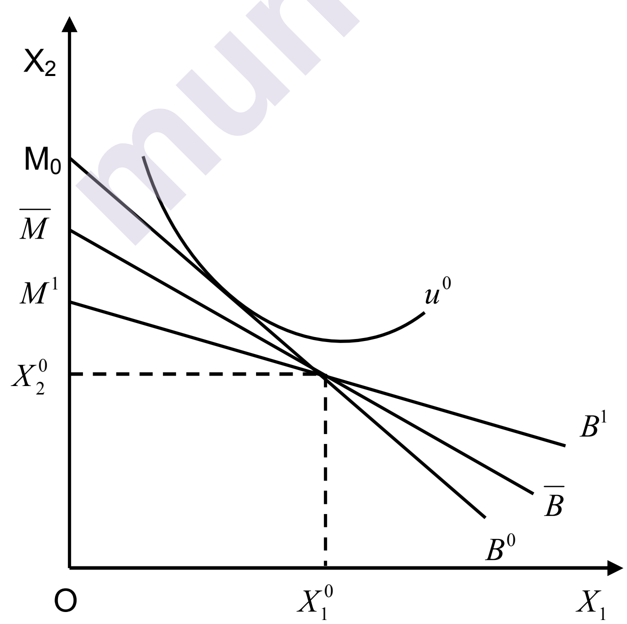


Figure 2.5

and

$$(1-k)\left(p_1^1\ x_1^0 + x_2^0\right) = M^1 \qquad [2.27]$$

We obtain

$$\overline{p_1}\, x_1^0 + x_2^0 = \overline{m} \qquad [2.28]$$

Thus $\left(p^1, m^1\right)$ and $\left(\overline{p}, \overline{m}\right)$ are both in the worse set of $\left(p^0, m^0\right)$ and $\left(\overline{p}, \overline{m}\right)$ is a convex combination of $\left(p^0, m^0\right)$ and $\left(p^1, m^1\right)$.

2.3.1 The Slutsky Equation :-

The slutsky equation plays a central role in analyzing the properties of demand functions. It is derived as follows. If we take as the constraint in the utility maxi mization problem the level of expenditure resulting from solution of the explanditure minimization problem core equivalently take as the constraint in the latter problem the level of utility resulting from the solution to the formers) then the solutions x_i^* to the two problems, the values of the Marshallian and Hicksian demand functions, will be identical setting $m = m\left(p, u\right),$ we can write for the 1st goods

$$H_i\left(p,u\right) = D_i\left(p, m\left(p,u\right)\right) \qquad [2.29]$$

since 2.29 is an identity we can differentiable through with respect to the 1st price, allowing expenditure to change in whatever way is required to keep utility constant, to obtain

$$\frac{\partial H_i}{\partial P_i} = \frac{\partial D_i}{\partial P_i} + \frac{\partial D_i}{\partial M}\frac{\partial m}{\partial P_i} \qquad [2.30]$$

Using shepherds lemma and rearranging gives the slutsky equation.

$$\frac{\partial D_i}{\partial P_j} = \frac{\partial H_i}{\partial P_i} - x_j \frac{\partial D_i}{\partial M} \qquad [2.31[$$

Taking I = j, so that we consider the effect of a price change on its own demand, we see from 2.31 that the slope of the marshalion demand function is the sum of two effect : the substitution effect, $\partial H_i / \partial P_i$, which is the slope of the Hicksian or compensated demand curve, and the income effect, $-x_i\, \partial D_i / \partial m$. Thus the slutsky equation gives a precise statement of the conclusions of the diagrammatic analysis of chapter 1. We show in a moment that $\partial H_i / \partial P_i \leq 0$. Then 2.31 again with i = j establishes that if the good is normal, so that $\partial D_i / \partial m > 0$, the slope of its marshallian demand curve is negative. If the good is inferior, so that

$\partial D_i / \partial M$ so, the slope is negative, positive or zero depending on the relative sizes of the absolute values.

$$\left| \partial H_i \middle| \partial P_i \right| and \left| x_i \, \partial D_i \middle| \partial M \right|$$

It is useful to express the slutsky equation in elasticity form. Again taking i = j, multiplying through 2.31 by p_i / x_i, and the income term by M / M, gives

$$\in_{ii} = \sigma_{ii} - S_i n_i \qquad [2.32]$$

where $\in_{ii}$ is the marshallian demand elasticity, σ_{ii} is the Hicksian or compensated demand elasticity, n_i is the income elasticity of demand, and $s_i = p_i x_i / m$ is the share of good in total expenditure. Thus the difference between Hicksian and Marshallian elasticity for a good will be smaller, the smaller its income elasticity and the less significant it is in the consumer's budget with $i \neq j$, 2.31 becomes

$$\in_{ii} = \sigma_{ij} - \sin i \qquad [2.33]$$

which emphasizes that cross price marshallian demand elasticities and on income elasticities weighted by expenditure shares. Equality of the Marshalliah cross-price elasticities therefore requires strong restrictions on preferences.

We define the slutsky matrix as the n x n matrix $\left[\partial Hi / \partial p_i \right]$ of Hicksian demand derivatives. It is a straight forward extension of shephard's lemma and the properties of the expenditure function to show that this matrix is a symmetric, negative semi definite matrix. From shephard's lemma

$$\frac{\partial m\left(p,u\right)}{\partial p_i} = H_i\left(p,u\right) \quad i = 1 \ldots.., n \text{ we have}$$

$$\frac{\partial^2 m\left(p,u\right)}{\partial p_i \partial p_i} = \frac{\partial H_i}{\partial P_i} i, j = 1, \ldots. n \qquad [2.34]$$

Then, from young's Theorem we have immediately that $\partial H_i / \partial P_i = \partial H_i / \partial p_i$, and so the slutsky matrix is symmetric. The slutsky matrix $\left[\partial H_i / \partial P_i \right]$ is the matrix of second order partials of the expenditure function and the concauity of the expenditure function implies that matrix is negative semi-definite since $\partial^2 m / \partial p_1^2 = \partial H_i / \partial p_i \leq 0$ by the definition of negative semi definiteness, the Hicksian demand curve cannot have a positive slope. We have seen earlier that strict convexity of preference and $x_i > 0$ at the optimum establish the stronger result that $\partial^2 m / \partial p_i^2 = \partial H_i / \partial p_i < 0$.

The Hicksian demand derivative $\partial H_i / \partial P_i$ is often used to define complements and substitutes. Two goods i and j are called Hicksian complement if $\partial H_i / \partial P_i < 0$ and Hicksian substitutes if $\partial H_i / \partial P_i > 0$. The advantage of this definition is that symmetry implies that the nature of the complementarily or substitutability between the goods cannot change if we take $\partial H_j / \partial P_i$ rather than $\partial H_i / \partial P_j$. This would not be true if we defined complements and substitutes in terms of the Marshallian demand derivatives.

2.4 PROPERTIES OF DEMAND FUNCTIONS

We have seen that it is possible to draw definite conclusions about the effects of price changes on the Hicksian demands. The Hicksian demand functions are not however, directly observable since they depend on the consumers, utility level as well as prices on the other hand, the marshallian demand functions can be estimated from information on purchases, prices and money income. The slutsky equation enables us to reformulate the predictions about the properties and Hicksian demand functions in terms of the observable Marshallian demand functions and thus to rider the set of testable predictions from consumer theory.

We can summarize the testable implication derived in this and the previous chapter:

(a) Marshallian demand functions are homeneous of degree zero in prices and money income:

(b) The Marshallian demand functions satisfy the 'adding up' property : $\sum P_i x_i^* = M$;

(c) The Hicksian demand derivatives (cross substitution effects) are symmetric :

$\partial H_i / \partial P_i = \partial H_i / \partial P_i$ or using the slutsky equation,

$$\partial D_i / \partial P_i + x_j \, \partial D_i / \partial M = \partial D_i / \partial P_i + x_i \partial D_i / \partial M;$$

(d) The slutsky matrix $\left[\partial H_i / \partial P_j \right] = \left[\partial D_i / \partial P_j + x_i \, \partial D_i / \partial M \right]$ is negative semi definite.

These are all the predictions about the Marshallian demand functions which can be made on the basis of the consumer preference axioms. The converse question of whether a system of demand functions with these properties implies the existence of a utility function from which the demand functions could have been derived is known as the integrability problem. In next section we will show that this is in fact so by considering the equivalent problem of retrieving an expenditure function from a set of Marshallian demand functions which satisfy the above properties.

2.5 CHOICE UNDER UNCERTAINTY

Introduction :-

The analysis in the preceding chapters has assumed that all decisions are taken in conditions of certainty. Any decision would result in one and only one outcome. When a firm chooses a set of input quantities, there is only one level of output which will result, and it knows the profit which it will receive from the sale of each output, no matter how far in the future production and sale will take place. Like wise, in planning their purchases of goods and services, and borrowing or lending decisions, households are assumed to know with certainty the expenditure and utility associated with each consumption vector.

But uncertainty is pervasive. There is technological uncertainty, when the firm is not able to predict for sure the output level which would result from a given set of input quantities Machines may break down; crops may be affected by the weather. There is market uncertainty when a single household or firm is not able to predict for sure the prices at which it will buy or sell. Market uncertainty is associated with disequilibrium and change : If an economy were permanently in long run static equilibrium, then firms and house holds would expect to trade at equilibrium prices, which, by experience, become known. If, however, changes are taking place through time which change equilibrium positions, the individual agents in the markets cannot know the new equilibrium in advance, and can only form expectations of prices which they know may be wrong.

Extension of the theory to take account of uncertainty has two main aims. It should first tell us something about the usefulness and validity of the concepts and propositions already derived. What becomes of the conclusions about the working of a decentralized price mechanism, for example? Can we still establish existence and optimality of competitive equilibrium? Are the predictions about household's and firms responses to changes in parameters affected qualitatively? The answers are important positively and normatively. Second, many important aspects of economic activity can not be adequately analyses without explicit recognition of uncertainty. For examples, the joint stock limited liability company, the basic institutional form of the firm in capitalist economies, has no real rationale in a world of certainty, and neither has the stock market. Insurance furthers markets and speculation cannot be understood expect in the context of uncertainty Relaxation of the certainty assumption gives new insights into many other areas, for example investment decisions.

As with models of an economy with certainty, we begin with the optimization problem of a single decision-taker. The optimization problem under uncertainty has the some basic structure as under certainty objects of choice; objective function, and constraints defining a feasible set of choice objects. The main interest centers on the first two of these, and, in particular, the construction of a set of axioms which allows us to define a preference ordering, represent able by a utility function, over the objects of choice.

A formualization of 'uncertainty'

Uncertainty arises because the consequence of a decision is not a single sure outcome but rather a number of possible outcomes. our first task in developing a theory of choice under uncertainty is to set out a precise formulization of the decision taking situation we can begin by distinguishing three kinds of variables which play a part in an economic system these are:

(a) The choice variables of the decision taker which are directly under his control such variables are not only endogenous to the mode of the economic system, but are also endogenous to the model of the individual economic agent. Example in earlier chapters include firms output levels and consumers purchases.

(b) Variables whose values are determined by the operation of the economic system, i.e. by the interaction of the choices of individual economic agents, and which are regarded as parameters by them. Prices are an example in a competitive economy. Such determined variables are endogenous to the model of the economic system, but exogenous to the model of the individual economic agent.

(c) Environmental variables, whose values are determined by some mechanism outside the economic system and which can be regarded as parameters of the economic system. They influence its outcome, but are not in turn. affected by it. The weather is an example, at least for some problems, though, in the light of such events as global warming, even this could be seen as endogenous in some models.

Suppose that the economy operates over only two periods, period 1 (the present) and period 2 (the future). In period 1 the environmental variables take on specific values which are known to all economic agents. We assume that the economy produces a resource allocation and a set of relative prices. If there were complete independence between the decisions made in period 1 and those to be made in period 2, then the state of knowledge at period 1 about the environmental variables at period 2.5 irrelevant. In this case, decisions for period 2 can be left until period 2, and do not affect decision taking at period 1. We assume that this kind of temporal separability of decision-taking does not exist. At period 1, economic agents will have to choose values of variables such as investment (purchase of durable good) and financial assets (bonds and shares), which effect what they will be able to do in period 2. Agents plans for the values of variables they will choose at period 2 influenced by their expectation about the value of variable outside their control at period 2-determined variables such as prices, and environmental variables like the wealthier will condition their choices at period 1. We therefore need a theoretical framework to analyse the formation of plans and expectation, and their influence on current choices.

We proceed as follows. Suppose there exists a vector of environmental variables $(e_1, e_2 ... e_n)$ where each environmental variable is capable of taking on a finite number of values in period 2. Let E_j denote the set of values which can be taken by environmental variable $e_j \left(i = 1.2, ... n\right)$. For example e_1 could be the average temperature over period 2, measured to the nearest degree centigrade, and E_1 could be the set $\left\{e_1 / 50^0 C \geq e_1 \geq -80^0 C\right\}$, which has a finite number of elements (since the temperature is measured in unites of $1^0 C$) Define a state of the world as a specific combination of the values of the environmental variables, i.e. as a specific value of the vector $(e_1, e_2 .... e_n)$ since each element of the vector can take only a finite number of values the number 5 of states of the world is also finite, though possibly very large. We index the states of the world by a number s=1,2 …5 and use the index to label the value of the choice variable or determined variables in each state of the world. Thus, for example, we can use y, to denote the level of income the individual gets in state 5.

There fundamental properties of the set of states of the world should be clear :

(a) The set is exhaustive, in that it contains all the states of the world which could possibly obtain at period 2.

(b) Members of the set are mutually exclusive in that the occurrence of any one rules out the occurrence of any other.

(c) The states of the world are outside the control of any decision-taken, so that the occurrence of any one of them cannot be influenced by the choice of any economic agent, or indeed by any coalition of agents.

The definition properties of states of the world are basic to all subsequent analysis. They can be regarded as an attempt to eliminate the elements of doubts, apprehension, and muddle which are part of the every day meaning of the word uncertainty, and to give the situation a precise formulization, for purposes of the theory. Three further assumptions which can be made are:

(a) All decision takers have in their minds the same sets of states of the world. They classify the possible combination of environmental variables in the same way.

(b) When period 2 arrives, all decision acres will be able to recognize which state of the world exists, and will all agree on it.

(c) At period 1, each decision taker is able to assign a probability to the event that a particular state of the world will accurse at period 2. The probabilities may differ for different decision-takers, but all probability assignments satisfy the basic probability laws. The probability associated with the 5th state by decision taker i, denote Π's, lies on the interval $1 \geq \Pi^{1}s \geq 0$, with $\Pi's = 1$ implying that I he regards state 5 as certain not to occur. The probability of one or another of several states occurring is the sum of their probabilities of their simultaneous occurrence being zero, and, in particular one of the 5 states must occur, i.e.

$$\sum_{5=1}^{s} \Pi_5^1 = 1$$

Each of these assumptions is quite strong and plays an important part in what follows. The first is necessary if we are to portray decision - takers as making agreements in state contingent terms : in order for one to agree with another that if state 1 occurs I will do A,A in return for your doing B if state 2 occurs: it is necessary that they should understand each other's references to states.

The second assumption is also required for the formation and discharge of agreements framed in state-contingent terms. If parties to an agreement would differ about which state of the world exists ex post, they are unlikely to agree ex ante on some exchange which is contingent on states of the world. The assumption also rules out problems which might arise from differences in the information which different decision takers may possess. Suppose, for example, that individual I cannot tell whether it is state 1 or state 2 which actually prevails at period 2, while individual J does know. Then I is unlikely to conclude an agreement with J under which say, I gains and I loses if state 1 occurs, while J gains and I loses if state 2 occurs, because of course. I could be exploited by J.

Choice under uncertainty.

We now consider the question of optimal choice under uncertainty. First, we need to define the objects of choice, and then we can consider the question of the decision taker's preference ordering over these choice objects. We present what is usually called the van-Neumann-Morgenstern theory of Expected utility.

Initially, we assume that there is a single good, which is measured in units of account, and which can be thought of a 'income'. Let $y_s\left(5=1,2,....5\right)$ denote an amount of income which the decision-taker will have if and only if state s occurs C in this section we shall be concerned only with a single decision-taker and so do not need to burden ourselves with a notation which distinguishes among decision-takers) Assume that the invidual assigns a probability Π_s to state of the world 5, and denote the vector of probabilities by $\Pi=[\Pi_1,\Pi_2...\Pi_3]$, while $y=\left[y_1,y_2\ ...\ y,\right]$ is the corresponding vector of state-contingent incomes. Define a prospect, P, as a given income vector with an associated probability vector,

$$P=\left(\Pi,y\right)=\left(\Pi_1.....\Pi_5,y_1\ ....\ y_5\right)$$

changing the probability vector Π, or the income vector y (or both) produces a different prospect. Another term for a prospect would be a probability distribution of incomes.

The choice objects of our theory are prospects such as P Any decision has as its only and entire consequence some prospect P, and so choice between alternative actions or decisions is equivalent to choice between alternative prospects. A preference ordering over decisions can only be derived from a preference ordering over their associated prospects.

For example, consider the decision of a market gardener to insure or not against loss of income through sickness or poor weather such as sever frost. Decision A is the decision not to insure, decision B is to insure Associated with A is a prospect,

$P^A = \left(\Pi_2 y^A\right)$ where y^A is on income vector, the components of which vary across states of the world. In the subset of states in which he is sick, income will take on one value; in the subset of states in which there is frost, income takes on another value; in the subset in which he is sick and there is frost, there will be a third value; and when he is not sick and there is no frost, there will be a fourth can presumably the highest) value Associated with B is a certain prospect (assuming that compensation for loss of income through sickness or frost is complete) $p^B = \left(\Pi, y^b\right)$, where each element of y^B is equal to what income would be in the absence of sickness and frost, minus the insurance premium, which must be paid in all states of the world. The choice between A and B, i.e. the decision whether or not to insure, depends on whether P^A is or is not preferred to P^B, To analyses choice under uncertainty there fore requires us to construct a theory of the preference ordering over prospects.

If certain assumption (axioms) concerning a decision-taker's preferences are satisfied, then we are able to represent those preferences the criterion by which he takes his choices in a simple and appealing way. A test of the appropriateness of the assumptions would be to show that we can correctly predict choices not yet observed, on the basis of observation of choice already made. It should be emphasized that our theory is a device for permitting such predictions, rather than for describing whatever thought process a decision taker goes through when making choices. The objects of choice consist of a set of prospects, which we can denote by $\left\{p^1, p^2, ... p^n\right\}$. The five axioms are described next.

Axiom 1 : ordering of prospects

Given any two prospects, the decision taker prefers one to the other, or is indifferent between them, and these relations of preference and indifference are transitive. In the notation of chaper for any two prospects p^i, p^s, exactly one of the statements.

$p^i > p^k, p^j < p^k, p^j \sim p^k$, is true, while

$p^i > p^k \text{ and } p^k > p^1 \Rightarrow p^i > p^1$

and similarly for the indifference relation $\sim$. This oxiom means that the preference ordering over prospects has the same desirable properties of completeness and consistency which were attributed to the preferences ordering over bundles of goods.

Before stating the second axiom, we need to introduce the concept of a standard prospect. Given the set of prospects under consideration we can take all the income values which appear in them, regardless of the state and the prospect to which they belong, as defining a set of values of the variable, income. Since there is a finite number of states and prospects, there is a finite number of such income values (at most, ns of them) There will be a greatest and a smallest income value Denote these values by $y_u \text{ and } y_I$ respectively. It follows that all income values lie on the interval $[y_{I,}y_u]$ and we can construct the theory so as to apply to this entire interval on the real line Define a standard prospect, p as a prospect involleing only the two out comes $y_u \text{ and } y_2$ with probabilities v and 1-v respectively, where $1 \geq v \geq 0$. A specific standard prospect, p_0^1, can be written as

$$p_0^1 = \left(v^1, y_u, y_1\right)$$

(where for convenience, we do not bother to write the second probability $1-v^1$) we obtain a second standard prospect, p_0^{11} by changing v^i, the probability of getting the better outcome, to v^{11} so that

$$p_0^{11} = \left(v^{11}, y_u, y_1\right)$$

we can then state the second axiom.

2.6 SUMMARY

The expenditure function is derived from the problem of minimizing the total expenditure necessary for the consumer to achieve a specified level of utility. Since utility depends indirectly on the prices and money income hence it is refomed to as indirect utility function.

Slutsky equation plays a central role in analyzing the properties of demand function.

2.7 QUESTIONS FOR REVIEW

1. Examine the expenditure function.
2. Explain the concept of indirect utility function.
3. Elaborate the concept of Roy's identity.
4. What are the properties of demand function.

❖❖❖❖

Unit-3

TECHNOLOGY OF PRODUCTION AND PRODUCTION FUNCTION

Unit Structure :

3.0 OBJECTIVES

After going through this module you will come to know the concepts, like –

- Technology of production,
- Specification of technology,

- Input Requirement Set and production function,
- Convex Technology
- Leontief – Technology
- Technical Rate of substitution (TRS)
- Elasticity of Substitution
- Returns to Scale (Long-Run Production Function)
- Efficient Production
- Homogeneous Production Function
- Homothetic production Function
- The CES Production Function

3.1 INTRODUCTION

The simplest and the most common way to describe the technology of a firm is the production function, which is generally studied in intermediate courses. However, there are other ways to describe firm technologies that are both more general and more useful. We will discuss several of these ways to represent firm production possibilities in this unit, along with ways to describe economicaly relevant aspects of a firm's technology.

3.2 TECHNOLOGY OF PRODUCTION

A firm produces outputs from various combinations of inputs. In order to study firm choices we need a convenient way to summarise the production possibilities of the firm, i.e., which combinations of inputs and outputs are technologically feasible.

A certain amount of inputs are used to produce certain amount of outputs per unit time period. We may also want to distinguish inputs and outputs by the calendar time in which they are available, the location in which they are available, and even the circumstances under which they become available. By defining the inputs and outputs with regard to when and where they are available, we can capture certain aspects of the temporal or spatial nature of production.

The level of detail that we will use in specifying inputs and outputs will depend on the problem at hand, but we should remain aware of the fact that a particular input or output good can be specified in arbitrarily fine detail.

3.2.1 SPECIFICATION OF TECHNOLOGY:

Suppose the firm has 'n' possible goods to serve as inputs and /or outputs. If a firm uses y_j^j units of a good j as an input and produces y_j^o of the good as an output, then the net output of good j is given by $y_j = y_j^o - y_j^i$. If the net output of good j is positive, then the firm is producing more of good j than it uses as inputs; if the net output is negative, then the firm is using more of good j than it produces.

A production plan is simply a list of net outputs of various goods. We can represent a production plan by a vector y in R^n where y_j is negative if the j^{th} good serve as a net input and positive if the j^{th} good serve as a net output. The set of all technologically feasible production plans is called the firm's production- possibilities set and will be denoted by Y, a subset of R^n. The set Y gives us a complete description of the technological possibilities facing the firm.

When we study the behaviour of a firm in certain economic environments, we may want to distinguish between production plans that are "immediately feasible" and those that are "eventually feasible". For example, in the short run, some inputs of the firms are fixed so that only production plans compatible with these fixed factors are possible. In the log run, such factors may be variable so that the firm's technological possibilities may well change.

We will generally assume that such restrictions can be described by some vector z in R^n. For example, z could be a list of maximum amount of the various inputs and outputs that can be produced in the time period under consideration. The restricted or short-run production possibilities set will be denoted by Y(z); this consists of all feasible net output bundles consistent with the constraint level z.

3.2.2 INPUT REQUIREMENT SET:-

Suppose we are considering a firm that produces only one output. In this case we write the net output bundle as (y,-x) where x is vector of inputs that can produce y units of output. We can then define a special case of a restricted production possibilities set, i.e., the input requirement set, as-

$$v(y) = \{x \text{ in } R^n : (y, -x) \text{ is in } Y\}$$

The input requirement set is the set of all input bundles that produce at least y nits of outputs.

Here the input requirement set measures inputs as positive numbers rather than negative.

3.2.2 (i) ISOQUANT

The isoquant gives all input bundles that produce exactly y units of output. In other words, an isoquant is the combination of all inputs that produce same level of output i.e., y.

An isoquant can also be defined as:

$$Q(y) = \{x \text{ in } R^n : x \text{ is in } V(y) \text{ and } x \text{ is not in - } V(y') \text{ for } y' > y\}$$

3.2.2 (ii) SHORT-RUN PRODUCTION POSSIBILITY SET

Suppose a firm produces some output from labour and capital. Production plans then look like $(y, -\ell, -k)$ where y is the level of output, ℓ the amounts of labour input, and k the amount of capital input. We know that the labour can be varied immediately in the short run but the capital remains fixed at the level $\bar{k}$. Then the short-run production possibility set can be expressed as –

$$Y(\bar{k}) = \{(y, -\ell, -k) \text{ in } Y : k = \bar{k}\}$$

3.2.2 (iii) PRODUCTION FUNCTION

The production function for a firm which has only one output can be defined as –

$f(x) = \{y \text{ in } R : y$ is the maximum output associated with x in y}

3.2.2 (iv) TRANSFORMATION FUNCTION

A production plan y in Y is technologically efficient if there is no y' in Y such that $y' \geq y$ and $y' \neq y$; in other words, a production plan is efficient if there is no other way to produce more output with the same inputs or to produce the same output with less inputs.

The set of technologically efficient production plans can be described by a transformation function:

$$T : R^n \rightarrow R$$

Where T(y)=0 if and only if y is efficient. The transformation function gives the maximal vectors of net outputs.

3.2.3 COBB-DOUGLAS TECHNOLOGY

Let ‘a’ be a parameter such that 0 < a < 1. Then the Cobb-Douglas technology can be defined as –

1. Production possibility set -

$$Y = \{(y, -x_1, -x_2) \text{ in } R^3 : y \leq x_1^a x_2^{1-a}\}$$

2. Input requirement set –

$$V(y) = \{(x_1, x_2) \text{ in } R^2 : y \leq x_1^a x_2^{1-a}\}$$

3. Isoquant

$$Q(y)=\left\{(x_1, x_2) \text{ in } R^2 \ : y = x_1^a \ x_2^{1-a}\right\}$$

4. Short-run production possibility set –

$$Y(z)=\left\{(y, -x_1, -x_2) \text{ in } R^3 : y \le x_1^a \ x_2^{1-a}, x_2 = z\right\}$$

5. Transformation function –

$$T(y, x_1, x_2) = y - x_1^a \ x_2^{1-a}$$

6. Production function –

$$f(x_1, x_2) = x_1^a \ x_2^{1-a}$$

3.2.4 LEONTIEF TECHNOLOGY

Let a > o and b > o be parameters. Then the Leontief Technology can be defined as –

1. Production possibility set -

$$Y=\left\{(y, -x_1, -x_2) \text{ in } R^3 : y \le \min(ax_1, bx_2)\right\}$$

2. Input requirement set –

$$V(y)=\left\{(x_1, x_2) \text{ in } R^2 : y \le \min(ax_1, bx_2)\right\}$$

3. Isoquant

$$Q(y)=\left\{(x_1, x_2) \text{ in } R^2 : y = \min(ax_1, bx_2)\right\}$$

4. Transformation function –

$$T(y, x_1, x_2) = y - \min(ax_1, bx_2)$$

5. Production function –

$$f(x_1, x_2) = \min(ax_1, bx_2)$$

The general shape of Cobb-Douglas and Leontief technology can be depicted diagrammatically as in the figures (a) and (b) respectively.

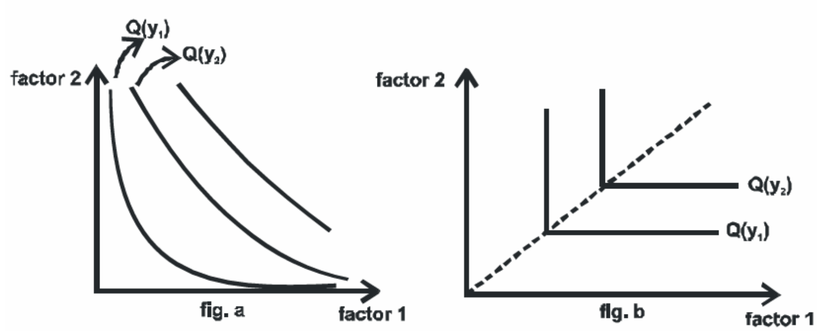


Figure 3.1

3.3 ACTIVITY ANALYSIS

The most straightforward way of describing production sets or input requirement sets is simply to list the feasible production plans. For example, suppose that we can produce an output good using factor inputs 1 and 2. There are two different activities or technologies by which this production can take place.

Technique A: One unit of factor 1 and two units of factor 2 produces one unit of output.

Technique B: Two units of factor 1 and one unit of factor 2 produces one unit of output.

Let the output be good 1; and factors be goods 2 and 3. Then we can represent the production possibilities implied by these two activities by the production set –

$$Y = \{(1,-1,-2),(1,-2,-1)\}$$

or the input requirement set –

$$V(1) = \{(1,2),(2,1)\}$$

This input requirement set is depicted in the figure 3.2(A).

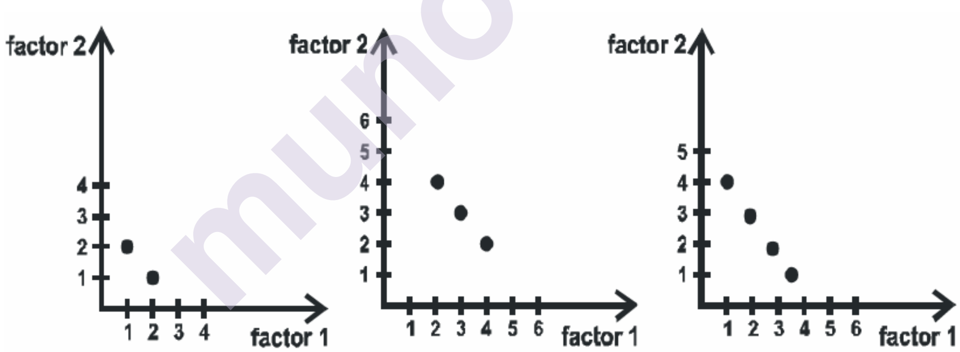


Figure 3.2(A)

It may be the case that to produce y units of output we could just use y times as much of each input for y=1,2, …. In this case one might think that the set of feasible way to produce y units of output would be given by

$$V(y) = \{(y,2y),(2y,y)\}$$

However, this set does not include all the relevant possibilities. It is true that (y, 2y) will produce y units of output if we use technique A and that (2y, y) will produce y units of output if we use technique B- But what if we use a mixture of technique A & B.

In this case we have to let y_A be the amount of output produced using technique A and y_{AB} be the amount produced using technique B. The V(y) will be given by the set –

$$V(y) = \{(y_A + 2y_B, y_B + 2y_A) : y = y_A + y_B\}$$

So, for example, V(2) = {(2,4),(4,2),(3,3)} . Both V(y) & V(2) are depicted in the above figures.

3.4 MONOTONIC TECHNOLOGY

Suppose that we had an input vector (3, 2). Is this sufficient to produce one unit of output? We may argue that since we could dispose of 2 units of factor 1 and be left with (1,2), it would indeed be possible to produce 1 unit of output from the inputs (3,2). Thus, if such free disposal is allowed, it is reasonable to argue that if x is feasible way to produce y units of output and x' is an input vector with at least as much of each input, then x' should be a feasible way to produce y. Thus, the input requirement set should be monotonic in the following sense.

Monotonicity : x is in V(y) and x' is in V(y).

If we assume monotonicity, then the input requirement sets depicted in figure 4.2 become the sets depicted in figure 3.3.

Figure 3.3

This assumption of monotonicity is often an appropriate assumption for production sets as well. In this context we generally want to assume that if y is in Y and $y' \leq y$ then y' must also be in Y. That is to say that, if y in Y is feasible then y' in Y is also feasible.

3.5 CONVEX TECHNOLOGY

Let us now consider what the input requirement set looks like if we want to produce 100 units of output. As a first step we might argue that if we multiply the vectors (1,2) and (2,1) by 100, we should be able just to replicate what we were doing before and thereby produce 100 times as much. It is clear that not all production processes will necessarily allow for this kind of replication, but it seems to be plausible in many circumstances.

If such replication is possible, then we can conclude that (100, 200) and (200, 100) are in V(100). Are there any other possible ways to produce 100 units of output? Well we could operate 50 processes of technique I and 50 process of activity II. This would use 150 units of good 1 and 150 units of good 2 to produce 100 units of output; hence (150 ,150) should be in the input requirement set. Similarly, we could operate 25 process of activity I and 75 processes of activity II. This implies that

25(100,200) 75(200,100) (175,125) should be in V(100). More generally, t(100,200) (1 t)(200,100) (100t 200 (1 t), 200t (1 t)100) Should be V(100) for t = 0, .01, .02 ….

We might as well make the obvious approximation here and let t take on any fractional value between 0 and 1. This leads to a production set of the form depicted in figure 2.4 A. Thus,

Convexity: If x and x' are in V(y), then $tx+(1-t)x'$ is in V(y), for all $0 \le t \le 1$. That is, V(y) is a Convex set.

We applied the arguments given above to the input requirement sets, but similar arguments apply to the production sets. It is common to assume that if y and y' are both in Y, then $ty+(1-t)y'$ is also in Y for $0 \le t \le 1$ in other words Y is a convex set.

Now we will describe a few of the relationships between the convexity of V(y) and the convexity of Y.

Convex production set implies convex input requirement set. i.e., if the production set Y is a convex set, then the associated input requirement set, V(y), is a convex set.

Convex input requirement set is equivalent to quasiconcave production function. V(y) is a convex set if and only if the production function f(x) is a quasiconcave function.

3.6 REGULAR TECHNOLOGY

Finally, we will consider a weak regularity condition concerning V(y)

V(y) is a closed, nonempty set for all $y \geq o$

The assumption that V(y) is nonempty requires that there is some conceivable way to produce any given level of output. This is simply to avoid qualifying statements by phrases like "assuming that y can be produced".

The assumption that V(y) is closed is made for technical reasons and is innocuous in most contexts. Roughly speaking, the input requirement set must include its own boundary.

3.7 THE TECHNICAL RATE OF SUBSTITUTION

Assume that we have some technology summarized by a smooth production function and that we are producing at a particular point $y^* = f\left(x_1^*, x_2^*\right)$. Suppose that we want to increase the amount of input 1 and decrease the amount of input 2 so as to maintain a constant level of output. How can we determine this technical rate of substitution between these two factors?

In the two dimensional case, the technical rate of substitution is just the slope of an isoquant; how one has to adjust x_2 to keep output constant when x_1 changes by a small amount, as depicted in figure 3.4

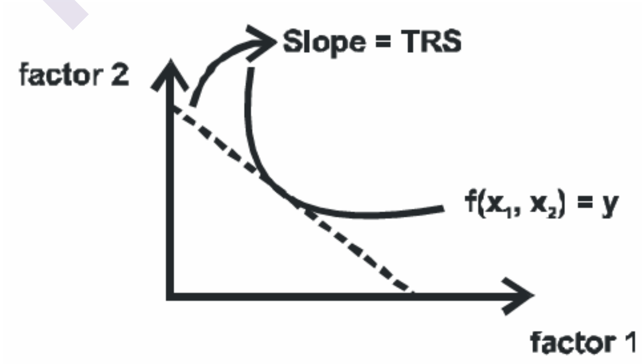


Figure 3.4

In the 'n' –dimensional case, the technical rate of substitution is the slope of an isoquant surface, measured in a particular direction.

Let $x_2(x_1)$ be the (implicit) function that tells us how much of x_2 it takes to produce y if we are taking x_1 units of the other input. Then by definition, the function $x_2(x_1)$ has to satisfy the following identity - $f\left(x_2, x_1(x_1)\right) = y$.

Actually, we require an expression for - $\partial x_2\left(x_1^*\right)/\partial x_1$

Then, differentiating the above identity, we get –

$$\frac{\partial f(x^*)}{\partial x_1} + \frac{\partial f(x^*)}{\partial x_2} \cdot \frac{\partial x_2(x_1^*)}{\partial x_1} = 0$$

$$\frac{\partial x_2(x_1^*)}{\partial x_1} = -\frac{\partial f(x^*)/\partial x_1}{\partial f(x^*)/\partial x_2}$$

This gives us an explicit expression for the technical rate of substitution.

Here is the another way to derive the technical rate of substitution. Think of a vector of small changes in the input levels which we write as $dx = (dx_1, dx_2)$ The associated changes in the output is approximated by $dy = \frac{\partial f}{\partial x_1}dx_1 + \frac{\partial f}{\partial x_2}dx_2$ this expression is known as the total differential of the function f(x). Consider a particular change in which only factor 1 and factor 2 changes, and the change is such that output remains constant. That is dx_1 and dx_2 adjust "along an isoquant".

Since output remains constant, we have

$$o = \frac{\partial f}{\partial x_1}dx_1 + \frac{\partial f}{\partial x_2}dx_2,$$

which can be solved for -

$$\frac{dx_2}{dx_1} = -\frac{\partial f/\partial x_1}{\partial f/\partial x_2}$$

Either the implicit function method or the total differential method may be used to calculate the technical rate of substitution.

3.8 TRS FOR A COBB-DOUGLAS TECHNOLOGY

Given that $f(x_1, x_2) = x_1^a x_2^{1-a}$ we can take the derivatives to find -

$$\frac{\partial f(x)}{\partial x_1} = ax_1^{a-1}\ x_2^{1-a} = a\left[\frac{x_2}{x_1}\right]^{1-a}$$

$$\frac{\partial f(x)}{\partial x_2} = (1-a)x_1^{a}\ x_2^{-a} = (1-a)\left[\frac{x_1}{x_2}\right]^{a}$$

It follows that,

$$\frac{\partial x_2(x_1)}{\partial x_1} = \frac{\partial f/\partial x_1}{\partial f/\partial x_2} = -\frac{a}{1-a}\frac{x_2}{x_1}$$

3.9 THE ELASTICITY OF SUBSTITUTION

The technical rate of substitution measures the slope of an isoquant. The elasticity of substitution measures the curvature of an isoquant. More specifically, the elasticity of substitution measures the percentage change in the factor ratio divided by the percentage change in the TRS, with output being held fixed.

If we let $\Delta(x_2/x_1)$ be the change in the factor ratio and ΔTRS be the change in the technical rate of substitution, then the elasticity of substitution denoted by 'σ' can be given as –

$$\sigma = \frac{\dfrac{\Delta(x_2/x_1)}{x_2/x_1}}{\dfrac{\Delta TRS}{TRS}}$$

The elasticity of substitution, which is a relatively natural measure of curvature, asks how the ratio of factor inputs changes as the slope of the isoquant changes. If a small change in slope gives us large change in factor input ratio, then the isoquant is relatively flat which means that the elasticity of substitution is large.

In practice we think of the percentage change as being very small and take the limit of this expression as Δ goes to zero. Then, the expression for σ becomes –

$$\sigma = \frac{TRS}{(x_2/x_1)}\frac{d(x_2/x_1)}{d\,TRS}$$

It is often convenient to calculate σ using the logarithmic derivative. In general, if y=g(x), the elasticity of y with respect to x refers to the percentage change in y induced by a small percentage change in x.

That is, $\in = \frac{\frac{dy}{y}}{\frac{dx}{x}} = \frac{dy}{dx}\frac{x}{y}$

Provided that x and y are positive, this derivative can be written as $\in = \frac{dIny}{dInx}$

To prove this, note that by the chain rule $\frac{dIny}{dInx}\frac{d\ In\ x}{dx} = \frac{dIny}{dx}$

Carrying out the calculations on the left-hand and right-hand side of the equals sign, we have –

$$\frac{d\ln y}{d\ln x}\frac{1}{x} = \frac{1}{y}\frac{dy}{dx}$$

$$\frac{d\ln y}{d\ln x} = \frac{x}{y}\frac{dy}{dx}$$

Alternatively we can use total differential to write –

$$d\ln y = \frac{1}{y}dy$$

$$d\ln x = \frac{1}{x}dx,$$

So that,

$$\in = \frac{d\ln y}{d\ln x} = \frac{dy}{dx}\frac{x}{y}$$

Applying this to the elasticity of substitution, we can write –

$$\sigma = \frac{d\ln(x_2 / x_1)}{d\ln|TRS|}$$

Here, it should be noted that the absolute value sign in the denominator is to convert the TRS to a positive number so that the logarithm makes sense.

The Elasticity of Substitution for the Cobb-Douglas Production Function:

We have seen above that –

$$TRS = -\frac{a}{1-a}\frac{x_2}{x_1}$$

or

$$\frac{x_2}{x_1} = -\frac{1-a}{a} TRS$$

It follows that,

$$\ln\frac{x_2}{x_1} = \ln\frac{1-a}{a} + \ln|TRS|$$

This in turn implies –

$$\sigma = \frac{d\ln(x_2/x_2)}{d\ln|TRS|} = 1$$

Hence, it is clear from the above expression that the elasticity of substitution for the Cobb-Douglas production function is equal to one.

3.10 RETURNS TO SCALE AND EFFICIENT PRODUCTION

Suppose that we are using some vector of inputs x to produce some output y and we decide to scale all inputs up or down by some amount $t \geq 0$. What will happen to the level of output?

In the case we described earlier, where we wanted only to scale output up by some amount, we typically assumed that we could simply replicate what we were doing before and thereby produce 't' times as much output as before. If this sort of scaling is always possible, we will say that the technology exhibits constant returns to scale. More formally, a technology is said to exhibit constant returns to scale if any of the following are satisfied.

(1) y in Y implies ty is in Y, for all $t \geq 0$;
(2) x in V(y) implies tx is in V(ty), for all $t \geq 0$;
(3) f(tx) = tf(x) for all $t \geq 0$; i.e., the production function f(x) is homogeneous of degree 1.

The replication argument given above indicates that constant returns to scale is often a reasonable assumption to make about technologies. However, there are situations where it is not a plausible assumption.

One circumstance where constant returns to scale may be violated is when we try to "subdivide" a production process. Even if it is always possible to scale operations up by integer amounts, it may not be possible to scale operations down in the same way.

Another circumstance where the constant returns to scale may be violated is when we want to scale operations up by noninteger amounts. Certainly, replicating, what we did before is simply enough, but how do we do one and one half times what we were doing before.

A third circumstance where constant returns to scale is inappropriate is when doubling all inputs allows for a more efficient means of production to be used. Replication says that doubling our output by doubling our inputs is feasible, but there might be a better way to produce output. Consider, for example, a firm that builds an oil pipeline between two points and uses labour, machines and steel as inputs to construct the pipeline. He may take the relevant measure of output for this firm to be the capacity of resulting pipeline. Then it is clear that if we double all inputs to the production process, the output may more than double since increasing the surface area of a pipe by 2 will increase the volume by a factor of 4. in this case when output increases by more than the scale of the inputs, we say the technology exhibits increasing returns to scale.

A technology exhibits increasing returns to scale if,

$$f(tx) > t\,f(x) \text{ for all } t > 1.$$

A fourth situation where constant returns to scale may be violated is by being unable to replicate some inputs.

Consider for example, a 100 acre farm. If we wanted to produce twice as much output, then we could use twice as much of each input. But this would imply using twice as much land as well. It may be that this is impossible to do since more land may not be available. Even though the technology exhibits constant returns to scale if we increase all inputs, it may be convenient to think of it as exhibiting decreasing returns to scale with respect to the inputs under our control.

More precisely, we have a technology that can be said to exhibit decreasing returns to scale if,

$$f(tx) < t\,f(x) \text{ for all } t > 1.$$

The most natural case of decreasing returns to scale is the case where we are unable to replicate some inputs. Thus, we should expect that the restricted production possibility sets would typically exhibit decreasing returns to scale. It turns out that it can always be assumed that decreasing returns to scale are due to the presence of some fixed factor input.

Finally, it should be noted that the various kinds of returns to scale explained above are global in nature. It may well happen that a technology exhibits increasing returns to scale for some values of x and decreasing returns to scale for other values.

3.10.1 THE ELASTICITY OF SCALE

The elasticity of scale measures the percent increase in output due to a one percent increase in all inputs – that is, due to an increase in the scale of operations.

Let y = f (x), be the production function. Let t be a positive scalar, and consider the function y(t)= f (tx). If t = 1, we have the current scale of operations; if t >1, we are scaling all inputs up by t; and if t < 1, we are scaling all inputs down by t.

The elasticity of scale is then given by –

$$e(x) = \frac{\frac{dy(t)}{y(t)}}{\frac{dt}{t}}$$

evaluated at t=1

Rearranging this expression, we have -

$$e(x) = \frac{dy(t)}{dt}\frac{t}{y}\bigg|_{t=1} = \frac{df(tx)}{dt}\frac{t}{f(tx)}\bigg|_{t=1}$$

from the above expression, we may say that the technology exhibits – locally;

(1) Increasing returns to scale, if e(x) >1;
(2) Constant returns to scale, if e(x) =1; and
(3) Decreasing returns to scale, if e(x)<1.

3.10.2 RETURNS TO SCALE AND COBB-DOUGLAS TECHNOLOGY

Suppose that $y = x_1^a x_2^b$

Then,

$$\begin{aligned} f(tx_1, tx_2) &= (tx_1)^a\,(tx_2)^b \\ &= t^{a+b}\,x_1^a\,x_2^b \\ &= t^{a+b}\,f(x_1, x_2) \end{aligned}$$

$$\therefore f(tx_1, tx_2) = t^{a+b}\,f(x_1, x_2)$$

Hence, $f(tx_1, tx_2) = tf(x_1, x_2)$ if and only if a + b = 1. It, therefore, implies that the,

(1) Technology exhibits constant returns to scale, if a+b =1;
(2) Increasing returns to scale, if a+b >1; and

(3) Decreasing returns to scale if a+b < 1.

In fact, the elasticity of scale for the Cobb-Douglas technology turns out to be precisely a+b. To see this consider the definition of elasticity of substitution –

$$\frac{d(tx_1)^a(tx_2)^b}{dt} = \frac{dt^{a+b}x_1^a\, x_2^b}{dt}$$

$$= (a+b)t^{a+b-1}\, x_1^a\, x_2^b$$

Evaluating this derivative at t=1 and dividing by

$$f(x_1, x_2) = x_1^a\, x_2^b =$$

$$= \frac{(a+b)1^{a+b-1}x_1^a\, x_2^b}{x_1^a\, x_2^b}$$

$$= a+b$$

3.11 HOMOGENEOUS AND HOMOTHETIC TECHNOLOGY

A function f(x) is homogeneous of degree k if $f(tx) = t^k f(x)$ for all $t > o$. The two most important "degrees" in economics are the zeroth and first degree. A zero degree homogeneous function is one for which f(tx) = f(x), and first degree homogeneous function is one for which f(tx)= t f(x).

Comparing this definition to the definition of constant returns to scale we see that a technology has constant returns to scale if and only if its production function is homogeneous of degree one.

A function $g: R \to R$ is said to be a positive monotonic transformation if g is strictly increasing function; that is, a function for which x > y implies that g(x) > g(y).

A homothetic function is a monotonic transformation of a function that is homogeneous of degree one. In other words, f(x) is homothetic if and only if it can be written as f(x)= g(h(x)), where $h(\bullet)$ is monotonic function. Both, homogeneous and homothetic functions are depicted in the figure 3.5.

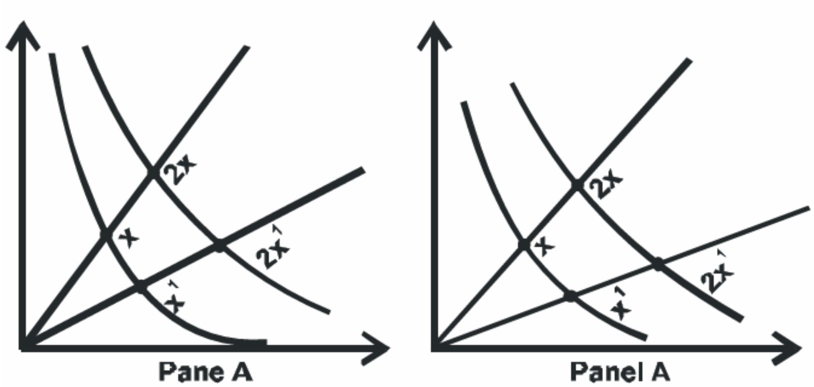


Figure 3.5

Panel A of the figure 3.5 depicts the function that is homogeneous of degree one. That is, if x and x^1 can both produce y units of output, then 2x and $2x^1$ can both produce 2y units of output.

Panel B of the figure 3.5 depicts a homothetic function. That is, if x and x^1 produce the same level of output, y, then 2x and $2x^1$ can produce the same level of output, but not necessarily 2y.

Homogeneous and homothetic functions are of interest due to the simple ways that their isoquants vary as the level of outputs varies. In the case of a homogeneous function the isoquants are all just "blown up" versions of a single isoquant. If f(x) is homogeneous of degree one, then if x and x^1 produce y units of output, it follows that tx and tx^1 can produce ty units of output, as depicted in figure 3.5A.

A homothetic function has almost the same property: if x and x^1 produce the same level of output, then tx and tx^1 can also produce the same level of output – but it wont necessarily be t times as much as the original output. The isoquants for a homothetic technology look just like the isoquants for homogeneous technology, only the output levels associated with the isoquants are different.

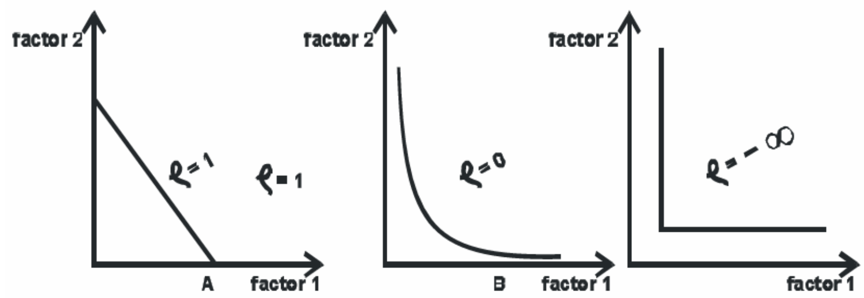
Homogeneous and homothetic technologies are of interest since they put specific restrictions on how the technical rate of substitution changes as the scale of production changes. In particular, for either of these functions the technical rate of substitution is independent of the scale of production.

3.11.1 THE CES PRODUCTION FUNCTION

The constant elasticity of substitution or CES production function has the following form;

$$y = [a_1 x_1^{\rho} + a_2 x_2^{\rho}]^{1/\rho}$$

It is quite easy to verify that CES function exhibits constant returns to scale. The CES function contains several other well-known production functions as special cases, depending on the value of the parameter ρ. These are illustrated in figure 3.6.



In figure 3.6 above, panel A depicts the case where $\rho = 1$, panel B the case where $\rho = 0$ and the panel C the case where $\rho = -\infty$.

The production function contained in the CES function can be described as –

1) The linear production function ($\rho = 1$).

Simple substitution yields – $y = x_1 + x_2$

2) The Cobb-Douglas production function ($\rho = 0$). When $\rho = 0$ the CES production function is not defined, due to division by zero. However, we will show that as ρ approaches zero, the isoquants of the CES production function looks very much like the isoquants of the Cobb-Douglas production function.

This is easiest to see using the technical rate of substitution. By direct calculation –

A ρ approaches zero, this tends to a limit of $TRS \frac{x_2}{x_1}$

Which is simply the TRS for the Cobb-Douglas production function.

3) The Leontief production function $\rho = -\infty$. We have just seen that the TRS of CES production function is given by equation (1) above, A P approaches $-\infty$, this expression approaches -

$$TRS = -\left(\frac{x_1}{x_2}\right)^{-\infty} = -\left(\frac{x_2}{x_1}\right)^{\infty}$$

If $x_2 > x_1$ the TRS is negative infinity; if $x_2 > x_1$ the TRS is zero. This means that Q as ρ approaches $-\infty$, a CES isoquant looks like an isoquant associated with the Leontief technology.

The CES production function has a constant elasticity of substitution. In order to verify this, remember that the technical rate of substitution is given by –

$$TRS = \left(\frac{x_1}{x_2}\right)^{\rho-1}$$

So that,

$$\frac{x_2}{x_1} = |TRS|^{\frac{1}{1-\rho}} = |TRS|^{\frac{1}{1-\rho}}$$

Taking logs we see that,

$$\ln\frac{x_2}{x_1} = \frac{1}{1-\rho}\ln|TRS|$$

3.12 SUMMARY

In short production is the creation of utility by transforming physical units of inputs into physical units of output. Production function is the technology of combining physical units of inputs to produce the given level of output.

3.13 QUESTIONS

Q.1 Explain the concept of technology of production.
Q.2 Elaborate the concept of input requirement set.
Q.3 Define and explain the concepts of Cobb-Douglas and Leontief Technology.
Q.4 Discuss the concept of monotonic, convex and Regular technology.
Q.5 What is technical rate of substitution? Explain
Q.6 Explain returns to scale and the concept of efficient production.
Q.7 Explain the concept of CES production function.

❖❖❖❖

Unit-4

COST FUNCTION

Unit Structure :

4.0 OBJECTIVES

After going through this unit you will able to explain the concepts, like –

- Cost Function,
- Average and marginal costs,
- Long-run and Short-run costs,
- Properties of the cost function,
- Shephard's Lemma,
- The Envelope Theorem for Constrained Optimisation,
- Duality of cost and Production function,
- Geometry of Duality

4.1 INTRODUCTION :-

People without a background in economics usually make a mistake between cost and price. Price is the amount paid by the consumer and received by the producer. Cost is the amount spent by the producer in manufacturing the commodity or the service.

Cost can be understood in a variety of ways. The opportunity cost is the returns from the next best alternative. There are implicit costs which may not be seen in the accounts statements and explicit costs which could be clearly understood.

An important division of costs is between Fixed and Variable Costs. Fixed costs are those which do not depend on the quantity of output produced, they include costs like rent, payment of loan installments, permits, etc. Variable costs depend upon the quantity of output produced and increase with output (for total variable costs).

Another concept of classifying costs is total, average and marginal costs. Total cost is divided into total fixed and total variable costs. The total cost refers to the cost incurred in producing the given quantity of output. The usual total cost function is of a cubic form. Average cost is the per unit cost of producing the commodity which can be obtained by dividing the total cost with quantity of output. Marginal cost is the rate of change in total cost with respect to output and so there can not be any marginal fixed cost by definition.

4.2 COST FUNCTION

The cost function measures the minimum cost of producing a given level of output for some fixed factor prices. As such it summarizes information about the technological choices available to the firms. The behaviour of the cost function can tell us a lot about the nature of the firm's technology.

Just as the production function was our primary means of describing the technological possibilities of production, the cost function will be our primary means of describing the economic possibilities of a firm. Here we will investigate the behaviour of the cost function $c(w, y)$ with respect to its price and quantity arguments.

4.2.1 AVERAGE AND MARGINAL COST

Let us consider the structure of the cost function. In general, the function can always be expressed simply as the value of the conditional factor demands.

$$c(w, y) \equiv wx(w, y)$$

This just says that the minimum cost of producing y units of output is the cost of the cheapest way to produce y.

In the short run some of the factors of production are fixed at predetermined levels. Let x_f be the vector of fixed factors, x_v be the vector of variable factors, and break up 'w' into $w = (w_v, w_f)$, the vectors of prices of the variable and fixed factors. The short-run conditional factor demand functions will generally depend on x_f, so we write them as $x_v(w, y, x_f)$. Then the short-run cost function can be written as –

$$c(w, y, x_f) = w_v x_v(w, y, x_f) + w_f x_f$$

The term $w_v x_v(w, y, x_f)$ is called short-run variable cost (SVC), and the term $w_f x_f$ is the fixed cost (FC).

From these basic units, we can define various derived cost concepts, as follows –

Short run total cost (STC)

$$STC = w_v x_v(w, y, x_f) + w_f x_f$$

Short run average cost (SAC)

$$SAC = \frac{c(w, y, x_f)}{y}$$

Short run average variable cost (SAVC)

$$SAVC = \frac{w_v x_v(w, y, x_f)}{y}$$

Short run average fixed cost (SAFC)

$$SAFC = \frac{w_f x_f}{y}$$

Short run marginal cost (SMC)

$$SMC = \frac{\partial c(w, y, x_f)}{\partial y}$$

When all factors are variable, the firm will optimize in the choice of x_f. Hence, the long-run cost function only depends on the factor prices and level of output as indicated earlier.

We can express this long-run function in terms of the short-run cost function in the following way. Let $x_f(w, y)$ be the optimal choice of the fixed factors, and let $x_v(w, y) = x_v(w, y, x_f(w, y))$ be the long-run optimal choice of the variable factors. Then the long-run cost function can be written as –

$$c(w,y) = w_v x_v(w,y) + w_f x_f(w,y) = c(w,y,x_f(w,y))$$

The long-run cost function can be used to define cost concepts similar to those defined above:

Long run average cost $= LAC = \dfrac{c(w,y)}{y}$

Long run marginal cost $= LMC = \dfrac{\partial c(w,y)}{\partial y}$

It should be noted here, that the "long-run average cost" equals "long-run average variable cost" since all costs are variable in the long-run; and the "long-run fixed costs" are zero.

4.2.2 THE SHORT-RUN COBB-DOUGLAS COST FUNCTION :

Suppose the second factor in a Cobb-Douglas technology is restricted to operate at a level 'k'. Then the cost minimizing problem is –

min $w_1 x_1 + w_2 k$

Such that $y = x_1^a k^{1-a}$

Solving the constraint for x_1 as a function of y and k gives,

$$x_1 = \left(y k^{a-1}\right)^{\frac{1}{a}}$$

Thus,

$$c(w_1, w_2, y, k) = w_1 \left(y k^{a-1}\right)^{\frac{1}{a}} + w_2 k$$

The following variations can also be calculated –

Short-run average cost $= w_1 \left(\dfrac{y}{k}\right)^{\frac{1-a}{a}} + \dfrac{w_2 k}{y}$

Short-run average variable cost $= w_1 \left(\frac{y}{k}\right)^{\frac{1-a}{a}}$

Short-run average fixed cost $= \frac{w_2 k}{y}$

Short-run marginal cost $= \frac{w_1}{a}\left(\frac{y}{k}\right)^{\frac{1-a}{a}}$

4.2.3 THE GEOMETRY OF COSTS

The cost function is the single most useful tool for studying the economic behaviour of the firm. In a sense, the cost function summarizes all economically relevant information about the technology of the firm.

Since, we have taken factor prices to be fixed, costs depend only on the level of output of a firm. The total cost curve is always assumed to be monotonic in output : the more you produce the more it costs. The average cost curve, however, can increase or decrease with output, depending on whether total cost rise more than or less then linearly. It is often thought that the most realistic case, at least in the short-run, is the case where the average cost curve first decreases and then increases. The reason for this is as follows –

In the short-run the cost function has two components : fixed costs and variable costs. We can therefore write short-run cost as–

$$SAC = \frac{c(w, y, x_f)}{y} = \frac{w_f x_f}{y} + \frac{w_v x_v(w, y, x_f)}{y}$$
$$= SAFC + SAVC$$

In most applications, the short-run fixed factors will be such things as machines buildings, and other types of capital equipments while the variable factors will be labour and raw material. Let us consider how the costs attributable to these factors will change as output changes.

As we increase output, average variable costs may initially decrease, if there is some initial region of economies of scale. However, it seems reasonable to suppose that the variable factors required will increase more or less linearly until we approach some capacity level of output determined by the amounts of the fixed factors. When we are near to capacity, we need to use more than a proportional amount of the variable inputs to increase output. Thus, the average variable cost function should eventually increase as output increases, as depicted in figure 4.1A. Average fixed costs must of course decrease with output, as indicated in figure 4.1B. Adding together the average variable cost curve and the average fixed cost curve gives us the U shaped average cost curve as is depicted in figure 4.1C.

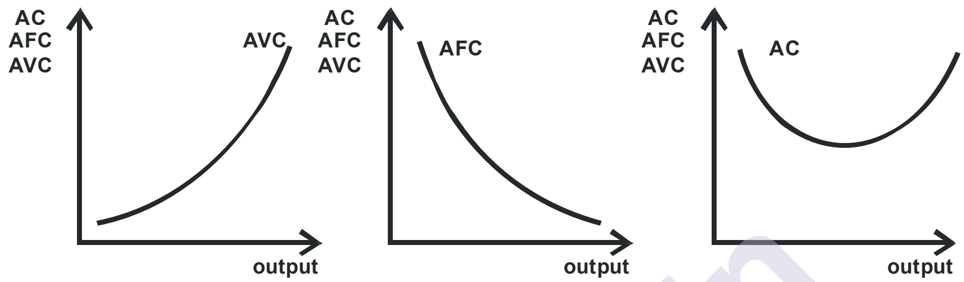


Fig: 4.1A **Fig: 4.1B** **Fig: 4.1C**

The initial decrease in the average cost is due to the decrease in average fixed costs; the eventual increase in the average cost is due to the increase in average variable costs. The level of output at which the average cost of production is minimized is sometimes known as the minimal efficient scale.

In the long-run all costs are variable costs; in such circumstances increasing average costs seems unreasonable since a firm could always replicate its production process. Hence, the reasonable, long-run possibilities should be either constant or decreasing average costs. On the other hand, certain kinds of firms may not exhibit a long-run constant-returns-to-scale technology because of long-run fixed factors. If some factors do remain fixed even in the long-run, the appropriate long-run average cost curve should presumably be U-shaped.

Let us now consider the marginal cost curve. What is its relationship with the average cost curve? Let $y*$ denote the point of minimum average cost; then to the left of $y*$ average costs are declining so that for $y \le y*$

$$\frac{d}{dy}\left(\frac{c(y)}{y}\right) \le 0$$

Taking the derivatives, it gives,

$$\frac{yc'(y) - c(y)}{y^2} \le 0 \text{ for } y \le y*$$

This inequality says that marginal cost is less than average cost to the left of the minimum average cost point. A similar analysis shows that,

$$c'(y) \geq \frac{c(y)}{y} \text{ for } y \leq y*$$

Since both inequalities must hold at $y*$, we have

$$c'(y*) = \frac{c(y*)}{y*} \;;$$

That is marginal cost equal average cost at the point of minimum average cost.

The Cobb-Douglas Cost Curves

The generalized Cobb-Douglas technology has a cost function of the firm,

$$c(y) = Ky^{\frac{1}{a+b}} \qquad a+b \leq 1$$

Where, k is a function of factor prices and parameters. Thus,

$$AC(y) = \frac{c(y)}{y} = Ky^{\frac{1-a-b}{a+b}}$$

$$MC(y) = c'(y) = \frac{K}{a+b} y^{\frac{1-a-b}{a+b}}$$

If $a+b<1$, the cost curves exhibit increasing average costs; if $a+b=1$, the cost curves exhibits constant average costs.

4.2.4 LONG-RUN AND SHORT-RUN COST CURVES

Let us now consider the relationship between long-run cost curves and the short-run cost curves. It is clear that the long-run cost curves should never lie above any short-run cost curves, since the short-run cost minimization problem is just a constrained version of the long-run cost minimization problem.

Let us write the long-run cost function as $c(y) = c(y, z(y))$. Here we have omitted the factor prices since they are assumed fixed and we let $z(y)$ be the cost minimizing demand for a single fixed factor. Let $y*$ be some given level of output, and let $z* = z(y)$ be the associated long run demand for the fixed factor. The short run cost, $c(y, z*)$, must be at least as great as the long run cost,

$c(y, z(y))$, for all levels of output, and the short-run cost will equal the long-run cost at output $y*$ so $c(y*, z*) = c(y*, z(y*))$. Hence, the long-run and the short-run cost curves must be tangent at $y*$. This is just the geometric restatement of the envelope theorem. The slope of the long-run cost curve at $y*$ is –

$$\frac{dc(y*, z(y*))}{dy} = \frac{\partial c(y*, z*)}{\partial y} - \frac{\partial c(y*, z*)}{\partial z} \frac{\partial z(y*)}{\partial y}$$

But since $z*$ is the optimal choice of the fixed factors at he output level $y*$, we must have –

$$\frac{\partial c(y*, z*)}{\partial z} = 0$$

Thus, long-run marginal costs at $y*$ equal short-run marginal costs at $(y*, z*)$.

Finally, we note that if the long-run and short run cost curves are tangent then the long-run and short-run average cost curves must also be tangent. A typical configuration is illustrated in figure 4.2

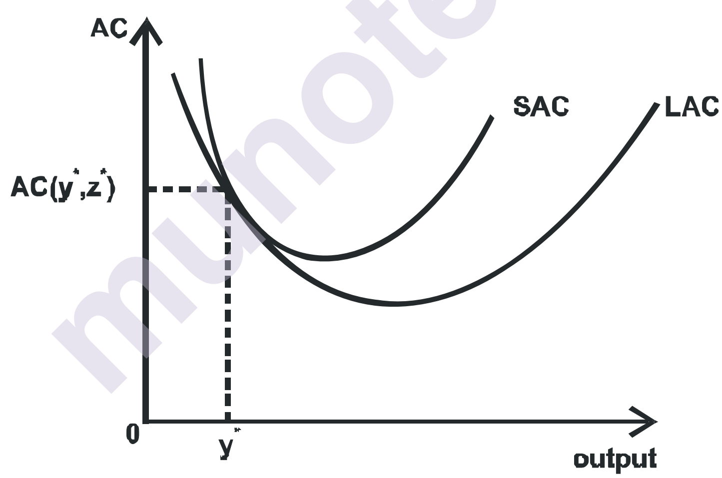


Figure 4.2

4.3 FACTOR PRICES AND COST FUNCTIONS

We turn now to the study of the price behaviour of cost functions. Several interesting properties follow directly from the definition of the functions. These properties of the cost functions are summarized as below –

Properties of the Cost Functions –

1) Non-decreasing in w :

If $w' \geq w$, then $c(w^1, y) \geq c(w, y)$

2) Homogeneous of degree 1 in w :

$c(tw,y) = tc(w,y)$ for $t > 0$

3) Concave in w :

$c(tw + (1-t)w^{'}, y \geq tc(w,y) + (1-t)c(w^{'}, y)$ for $0 \leq t \leq 1$

4) Continuous in w :

$c(w,y)$ is continuous as a function of w, for $w \geq 0$

Proof :

1) Cost function is non-decreasing in w :

Let x and $x^{'}$ be cost minimizing bundles associated with w and w^{1}. Then $wx \leq wx^{'}$ by minimization and $wx^{'} \leq w^{'}x^{'}$. Since, $w \leq w^{'}$. Putting these inequalities together gives $wx \leq w^{'}x^{'}$ as required.

2) Cost function is homogeneous of degree 1 in w :

We show that if x is the cost minimizing bundle at price w, then x also minimizes costs at prices tw. Suppose this is not so, and let x' be a cost minimizing bundle at tw so that $twx' < twx$. But this inequality implies $wx' < wx$, which contradicts the definition of x. Hence, multiplying factor prices by a positive scalar t does not change the composition of a cost minimizing bundle, and thus, costs must rise by exactly a factor of t :

$$c(tw,y) = twx = tc(w,y)$$

3) Let (w,x) and (w',x') be two cost-minimizing price factor combinations and let $w" = tw + (1-t)w'$ for any $0 \leq t \leq 1$. Now,

$$c(w",y) = w"x" = twx" + (1-t)w'x"$$

Since $x"$ is not necessarily the cheapest way to produce y at price w' or w, we have $wx" \geq c(w,y)$ and $w'.x" \geq c(w',y)$. Thus,

$$c(w",y) \geq tc(w,y) + (1-t)\,c\,(w',y)$$

4.4 SHEPHARD'S LEMMA

Let $x_i(w,y)$ be the firms conditional factor demand for input $'i'$. Then if the cost function is differentiable at (w,y), and $w_i > 0,$ for $i = 1, \ldots\ldots\ldots\ldots, n$ then

$$x_i = (w,y) = \frac{\partial c(w,y)}{\partial w_i}$$

$$i = 1, \ldots\ldots\ldots, n$$

Proof :

Let $x*$ be a cost – minimizing bundle that produce y at prices $w*$. Then define the function,

$$g(w) = c(w, y) - wx*$$

Since, $c(w, y)$ is the cheapest way to produce y, this function is always non-positive, at $w = w^*,\ g(w^*) = 0.$

Since, this is the maximum value of $g(w)$, its derivative must vanish:

$$\frac{\partial g(w^*)}{\partial w_i} = \frac{\partial c(w^*, y)}{\partial w_i} - x_i = 0$$

$$i = 1, \ldots\ldots\ldots, n$$

Hence, the cost minimizing input vector is just given by the vector of derivatives of the cost function with respect to the prices.

4.5 THE ENVELOPE THEOREM

Shephard's Lemma is another example of the envelope theorem. However, in this case we must apply a version of the envelope theorem that is appropriate for constrained optimization problems.

Consider a general parameterized constrained maximization problem of the form –

$$M(a) = \max_{x_1, x_2} g(x_1, x_2, a)$$

Such that $h(x_1, x_2, a) = 0$

In the case of the cost function –

$$g(x_1, x_2, a) = w_1 x_1 + w_2 x_2, h(x_1, x_2, a) =$$

$= f(x_1, x_2) - y,$ and 'a' could be one of the prices.

The Langrangian of this problem is

$$L = g(x_1, x_2, a) - \lambda\, h(x_1, x_2, a)$$

and the first order conditions are –

$$\frac{\partial g}{\partial x_1} - \lambda \frac{\partial h}{\partial x_1} = 0$$

$$\frac{\partial g}{\partial x_2} - \lambda \frac{\partial h}{\partial x_2} = 0 \text{ ------------------------------------ (1)}$$

$$h(x_1, x_2, a) = 0$$

These conditions determine the optimal choice functions $(x_1(a), x_2(a))$, which in turn determine the maximum value function

$$M(a) \equiv g(x_1(a), x_2(a), a) \text{ ------------------------ (2)}$$

The envelope theorem gives us the formula for derivative of the value function with respect to a parameter in the maximization problem. Specifically, the formula is –

$$\frac{dM(a)}{da} = \left.\frac{\partial L(x,a)}{\partial a}\right|_{x=x(a)}$$

$$= \left.\frac{\partial g(x_1, x_2, a)}{\partial a}\right|_{x_i = x_i(a)} - \lambda \left.\frac{\partial h(x_1, x_2, a)}{\partial a}\right|_{x_i = x_i(a)}$$

These partial derivatives are the derivatives of g and h with respect to a holding x_1 and x_2 fixed at their optimal values.

Application of the Envelope Theorem to the Cost Minimization Problem :

In this problem the parameter $'a'$ can be chosen to be one of the factor prices, w_i. The optimal value function $M(a)$ is the cost function $c(w, y)$.

The envelope theorem asserts that –

$$\frac{\partial c(w,y)}{\partial w_i} = \frac{\partial L}{\partial w_i} = x_i\big|_{x_i = x_i(w,y)} = x_i(w,y)$$

Envelope Theorem: Marginal Cost Revised:

It is another application of the envelope theorem, consider the derivative of the cost function with respect to y. According to the envelope theorem, this is given by the derivative of the

Langrangian with respect to y. The Lagrangian for the cost minimization problem is

$$L = w_1 x_1 + w_2 x_2 - \lambda [f(x_1, x_2) - y]$$

Hence,

$$\frac{\partial c(w_1, w_2, y)}{\partial y} = \lambda$$

In other words, the Lagrange multiplier in the cost minimization problem is simply marginal cost.

4.6 DUALITY

Suppose, set $VO(y)$ is an "outer bound" to the true input requirement set $V(y)$. Given data (w^t, x^t, y^t) $VO(y)$ is defined to be

$$VO(y) = \{x : w^t x \geq w^t x^t \text{ for all t such that } y^t \leq y\}$$

It is straightforward to verify that $VO(y)$ is a closed, monotonic and convex technology. Furthermore, it contains any technology that could have generated the data (w^t, x^t, y^t) for t = 1,………,T

If we observe choices for many different factor prices, it seems that $VO(y)$ should "approach" the true input requirement set in some sense. To make this precise, let the factor prices vary over all possible price vectors $w \geq 0$. Then the natural generation of VO becomes –

$$V^*(y) = \{x : wx \geq wx(w, y) = c(w, y) \text{ for all } w \geq 0\}$$

Relationship between $V^*(y)$ will contain $V(y)$ and the true input requirement set $V(y)$:

Of course $V^*(y)$ will contain $V(y)$. In general, $V^*(y)$ will strictly contain $V(y)$. For example, in figure 4.3A we see that the shaded area can not be ruled out of $V^*(y)$ since the points in this area satisfy the condition that $wx \geq c(w, y)$.

The same is true for figure 4.3B.

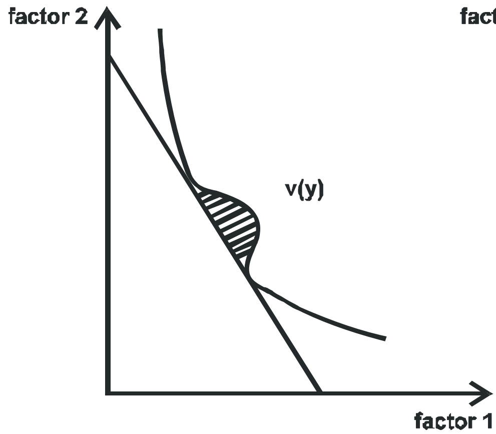


Fig : 4.3A

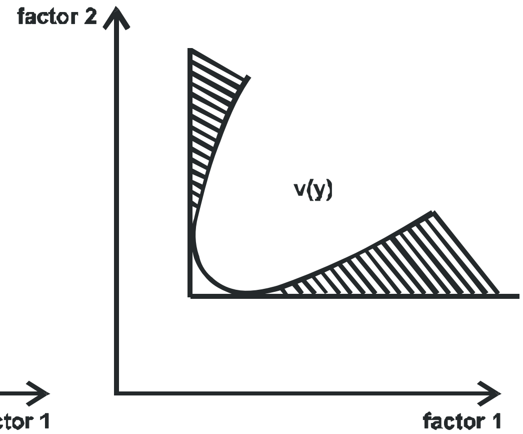


Fig : 4.3B

The cost function can only contain information about the economically relevant sections of $V(y)$, namely, those factor bundles that could actually be the solution to a cost minimization problem, i.e. that could actually be conditional factor demands.

However, suppose that our original technology is convex and monotonic. In this case $V*(y)$ will equal $V(y)$. This is because, in the convex monotonic case, each point on the boundry of $V(y)$ is a cost minimizing factor demand for some price vector $w > 0$. Thus, the set of points where $wx \geq c(w, y)$ for all $w \geq 0$ will precisely describe the input requirement set more formally –

When $V(y)$ equals $V*(y)$. Suppose $V(y)$ is regular, convex, monotonic technology.

Then $V*(y) = V(y)$

Proof: We already know that $V*(y)$ contains $V(y)$, so we only have to show that if x is in $V*(y)$ then x must be in $V(y)$.

Suppose that x is not an element of $V(y)$. Then since $V(y)$ is a closed convex set satisfying the monotonicity hypothesis, we can apply a version of separating hyperplane theorem to find a vector $w* \geq 0$ such that $w*x < w*z$ for all z in $V(y)$. Let $z*$ be a point in $V(y)$ that minimizes cost at the prices $w*$. Then in particular we have $w*x < w*z* = c(w*, y)$. But then x can not be in $V*(y)$, according to the definition of $V*(y)$.

This proposition shows that if the original technology is convex and monotonic then the cost function associated with the technology can be used to completely reconstruct the original technology. If we know the minimal cost of operation for every possible price vector w, then we know the entire set of technological choices open to the firm.

This is a reasonably satisfactory result in the case of convex and monotonic technologies but what about less well-behaved cases? – Suppose we start with some technology $V(y)$, possibly non-convex. We find its cost function $c(w, y)$ and then generate $V^*(y)$. We know from the above results that $V^*(y)$ will not necessarily be equal to $V(y)$, unless $V(y)$ happens to have the convexity and monotonicity properties. However, suppose we define –

$c^*(w, y) = \min wx$
Such that x is in $V^*(y)$

What is the relationship between $c^*(w, y)$ and $c(w, y)$?

When $c(w, y)$ equals $c^*(w, y)$. It follows from the definition of the functions that $c^*(w, y) = c(w, y)$

Proof: It is easy to see that $c^*(w, y) \le c(w, y)$; since $v^*(y)$ always contains $v(y)$, the minimal cost bundle in $v^*(y)$ must be at least as small as the minimal cost bundle in $v(y)$. Suppose that for some prices w', the cost minimizing bundle x' in $v^*(y)$ has the property that $w'x' = c^*(w', y) \le c(w', y)$. But that can not happen, since by definition of $v^*(y) - w'x' \ge c(w', y)$.

This proposition shows that the cost function for the technology $v(y)$ is the same as the cost function for its convexification $V^*(y)$. In this sense, the assumption of convex input requirement sets is not very restrictive from an economic point of view.

In short, it can be stated that –

(1) Given a cost function we can define an input requirement set $V^*(y)$

(2) If the original technology is convex and monotonic, the constructed technology will be identical with the original technology.

(3) If the original technology is non-convex or non-monotonic, the constructed input requirement will be convexified, monotonized version of the original set, and most importantly, the constructed

technology will have the same cost function as the original technology.

The above three points can be summarized succinctly with the fundamental principle of duality in production : the cost function of a firm summarizes all the economically relevant aspects of its technology.

4.7 SUFFICIENT CONDITIONS FOR COST FUNCTIONS

We know that the cost function summarizes all the economically relevant information about a technology. We also know that all cost functions are non-decreasing, homogeneous, concave, continuous functions of prices. The question arises : suppose that you are given a non-decreasing, homogeneous, concave continuous function of prices – is it necessarily the cost function of some technology?

The answer is yes, and the following proposition shows how to construct such a technology.

When $\phi(w, y)$ is a cost function. Let $\phi(w, y)$ be a differentiable function satisfying –

1) $\phi(tw, y) = t\phi(w, y)$ for all $t \geq 0$;
2) $\phi(w, y) \geq 0$ for $w \geq 0$ and $y \geq 0$;
3) $\phi(w', y) \geq \phi(w, y)$ for $w' \geq w$;
4) $\phi(w, y)$ is concave in w.

Then $\phi(w, y)$ is the cost function for the technology defined by $V^*(y) = \{x \geq 0 : wx \geq \phi(w, y),$ for all $w \geq 0\}$

Proof: Given $w \geq 0$ we define

$$x(w, y) = \left(\frac{\partial \phi(w, y)}{\partial w_1}, ..., \frac{\partial \phi(w, y)}{\partial w_n} \right)$$

And note that since $\phi(w, y)$ is homogeneous of degree 1 in w, Euler's law implies that $\phi(w, y)$ can be written as

$$\phi(w, y) = \sum_{i=1}^{n} w_i \frac{\partial \phi(w, y)}{\partial w_i} = wx(w, y)$$

Here it should be noted that the monotonicity of $\phi(w, y)$ implies $x(w, y) \geq 0$

Her we need to show that for any given $w' \geq 0,\ x(w', y)$ actually minimizes $w'x$ over all x in $V*(y)$:

$$\phi(w', y) = w'x(w', y) \leq w'x \text{ for all } x \text{ in } V*(y):$$

First, we show that $x(w', y)$ is feasible; that is, $x(w', y)$ is in $V^*(y)$. By the concavity of $\phi(w, y)$ in w we have –

$$\phi(w', y) \leq \phi(w, y) + D\phi(w, y)(w' - w)$$
$$\text{- for all } w \geq 0$$

Using Euler's law as above it reduces to

$$\phi(w', y) \leq w'x(w, y) \text{ for all } w \geq 0$$

It follows from the definition of $V^*(y)$, that $x(w', y)$ is in $V^*(y)$.

Next we show that $x(w, y)$ actually minimizes wx over all x is in $V^*(y)$, then by definition it must satisfy.

$$wx \geq \phi(w, y)$$

But by Euler's law,

$$\phi(w, y) = wx(w, y)$$

The above two expressions imply –

$$wx \geq wx(w, y)$$

for all x in $V^*(y)$ as required.

4.8 SUMMARY

Concept of cost plays a vital role in determining the performance of a firm. One requires to known the cost of production together with the revenue to find the total amount of profits or losses if any. Per unit cost of production i.e. average cost and average revenue has a greater role in determining the profits or losses. Marginal cost of production is necessary in knowing the equilibrium level of output.

4.9 QUESTIONS

1) What is cost function?
2) Discuss the concept of average and marginal costs.
3) What is geometry of costs?
4) Explain long-run and short-run cost curves.
5) Explain the Shephard's Lemma.
6) Explain the Envelope Theorem.
7) Discuss the duality of costs.

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(Module 3)

Unit-5

PRICE AND OUTPUT DETERMINATION UNDER PERFECT COMPETITION

Unit Structure

5.0 OBJECTIVES

This unit will enable you to understand.

- The features of perfect competition.
- The short-run and long-run equilibrium of a perfectly competitive firm.
- The stability of equilibrium in the short run and long-run.
- The Tatonmement Process and
- The Marshall's Process.

5.1 INTRODUCTION

A market in which we find perfect competition between a large number of buyers and a large number of seller of a homogeneous product and uniform price is called perfect competition market or perfectly competitive market. In other words in a perfect competition market all the potential sellers and buyers are fully aware of the prices at which transactions take place and all the affers made by them and any buyer can purchase any commodity from any of the sellers at the prices quested by them.

5.2 FEATURES OF PERFECT COMPETITION

The main features of a perfect competition market are discussed below.

1) **A large Number of Bayers and sellers :-** There are a large number of buyers and sellers of the commodity in this market. Each one of them is too small relative to the market and it cannot exert any perceptible influence on price.

2) **Homogenous Product :-** The output of each firm in the market is homogenous, identical or perfectly standardized. As a result, the buyer cannot distinguish between the output of one firm and that of another and is therefore, indifferent to the particular firm from which he buys.

3) Freedom of Entry or Exit. Entry (or exit) of the firms into (or from) the market is free in the perfect competition market. This means that any new firm is free to start production if it so whishes, and that any existing firm is free to cease production and leave the industry if it so whishes. Existing firms cannot bar the entry of new firms and there are no legal prohibition on entry or exit.

4) **Perfect Mobility :-** There is perfect mobility of factors of production geographically (i.e., from one place to the other) as well as occupationally (i.e., from one job to the other).

5) **Perfect knowledge :-** there is perfect and complete knowledge on the part of all buyers and sellers about the conditions in the market. For a market to be perfect it is essential that all buyers and sellers should be aware of what is happening in any part of the market.

5.3 INTRODUCTION TO THE PROCESS OF EQUILIBRIUM

In preceding chapters we considered models of the optimal choices of consumers and firms. In these models, prices were always taken as parameters outside the control of the individual decision-taker. We now examine how these prices are determined by the interaction of the decisions of such price, taking individuals. Since the interaction takes place through markets, we examine theories of markets whose participants act as price-takers, that is competitive markets.

Here we drew a distinction between production and supply in the short run and in the long run we maintain that distinction in market analysis, since supply conditions are an important determinant of the market outcome we again think of demand and supply as rates of flow per unit time. The short run is the period over which firms have fixed capacity. In the long run all inputs are variable. For example, if it takes a year to plan and implement capacity changes then the short run is this year and the long run is next year. Since decisions for the long run are necessarily planning decisions, expectations must come into the picture. So should uncertainty.

The chapter adopts a partial equilibrium approach a single market is considered in isolation. This is not entirely satisfactory, since there may be interaction between markets. For example, we shall see that in aggregating firms' supply curves to obtain a market supply curve we may wish to take account of the effect of expansion of aggregate market output on the prices of inputs used by the firms. The justification for a partial equilibrium analysis is that is is simple and can give useful insights. Moreover, the key issues concerning. The existence and stability of equilibrium can be introduced in a particularly simple context.

5.4 SHORT-RUN EQUILIBRIUM

Let $x_i = D_i(p)$ be the 1st consumer's demand for the commodity at price p and

$$X = \sum_i X_i = \sum_i D_i\left(p\right) = D(P) \qquad [5.1]$$

be the market demand function the short run supply function of firm I is

$$y_i = S_i\left(p, w\right) \qquad [5.2]$$

where y_i is the output of firm j and w is the price of the variable input.

It might appear that we could proceed to abstain a market supply function by aggregating the firm supply functions as we did the consumers demand function in 5.1, but this is not in general the case. In deriving the firms supply function we assumed input prices constant this was a natural assumption to make, since any one firm is a competitive industry (defined as the set of all producers of a given commodity) could be expected to be faced with perfectly classic input supply curves. Then, as its output price is raised, the firm could expand its desired production and input levels without raising input price. The assumption may not be appropriate for the industry as a whole, however: as the price at which they can sell their outputs rises for all firms, expansion in production and input demands may raise input prices because the increase in demand for inputs is no longer insignificant, and input supply functions have positive slopes to the industry as a whole.

Denote the total amount of the variable input used by the industry by $z(y)\left(z^1(y)>0\right)$ If

$$w=w\left(z(y)\right) \qquad [5.3]$$

with $w^1(z)>0,$ there are pecuniary external diseconomies: on increase in the total output of firms in the industry increases the price of an input.

The consequences for the firms actual supply are shown in fig 5.1. In the figure, price is assumed to rise from p to p. The firms initial supply $(\equiv SMC)$ curve is in each case s, $\left(p, w^0\right)$. If simultaneous expansion by all firms raises input prices from w^0 *to* w^1 the marginal cost curves and short run supply curves of each firm must rise. Figure 5.1(a) shows one possible Fig 5.1.

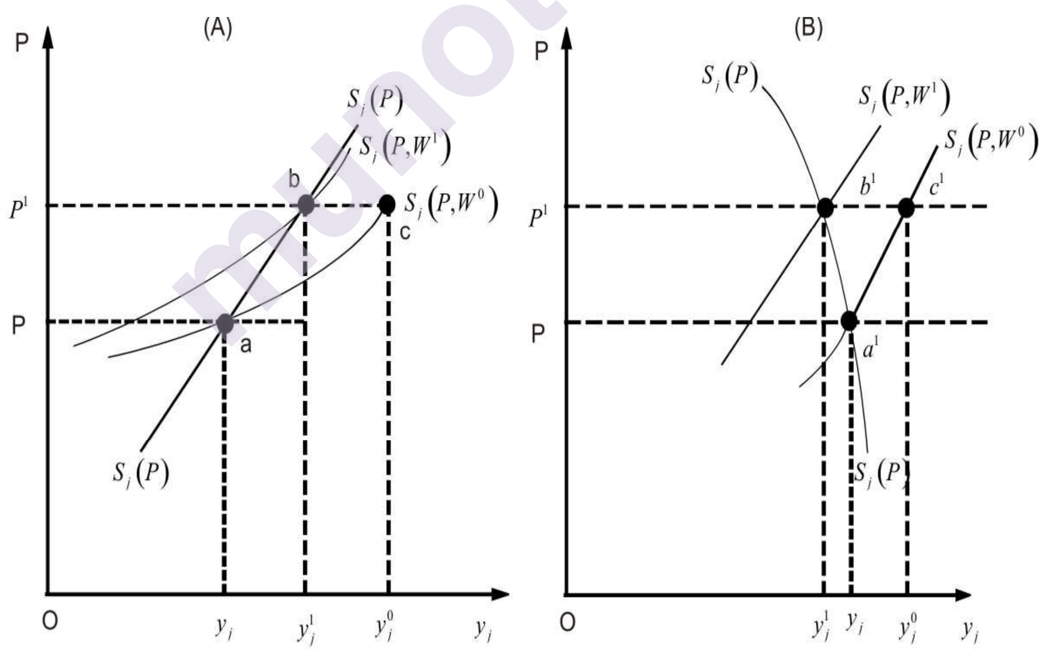

Figure 5.1.

result of the expansion of firms in response to the higher price. The short run supply curve has risen to $s_i\left(p,w^1\right)$ and so at prices p^1 the firm will want to supply y_i^1 and not y_i^0. Hence the points on the firms supply curve corresponding to p and p when all firms explain, are a and b respectively and $s_i\left(p\right)$ is the locus of all such price. Supply pairs. clearly, the firms effective market supply curve $s_i\left(p\right)$ will be less elastic than its ceteris paribus supply curve $s_i\left(p,w\right)$, They would concise if input prices were not bid up by simultaneous explain sion of output by all firms (and there were no technological externalities.

In (b) of the figure a more extreme case is shown. The increase in input prices causes a sufficient shift in the firm's SMC curve to make the post. adjustment output y_i^1 actually less than y_i and so its effective market supply curve s_j has a negative slope. Thus, although the law of diminishing returns ensures that each firms ceteris paribus supply curve has a positive slope this is not sufficient to ensure that the firms effective supply curve has a positive slope, if input prices increase with the expansion of outputs of all firms.

Denoting the effective industry supply function by $y\left(p\right)$ and substituting 6.3 in 6.2 gives the effective supply function of firm :

$$y_i = s_j\left(p, w\left(z\left(y\right)p\right)\right) = s_j\left(p\right) \qquad [5.4]$$

and summing gives the effective industry supply function

$$y = \sum_j y_j = \sum_j s_j\left(p\right) = s\left(p\right) \qquad [5.5]$$

Differentiating 6.4 with respect to the market price gives the effective supply response of firm j (after allowing for the effect of the increase in induced by the change in output of all firms) as

$$\frac{dy_i}{dp} = s_{jp} + 5_{jw}W^1\left(z\right)z^1\left(y\right)\frac{dy}{dp} = s_j^1\left(p\right) \underset{<}{\geq} 0 \qquad [5.6]$$

Since $s_{jp} = \partial s_j\left(p,w\right)/\partial p > 0$ and $s_{jw} = \partial s_i / \partial w < 0$ we see that the firms effective supply could be increasing or decreasing in P.

The change in industry supply as a result of the increases in p is the sum of the effective changes in the firms supplied and so from 6.5 and 6.6.

$$\frac{dy}{dp} = \sum_j \frac{dy_i}{dp} = \sum_j s_{jp} + w^1 z^1 \frac{dy}{dp} \sum_j s_{jw}$$

Since $s_{jp} > 0$ and $w^1 > 0, z^1 > 0, s_{je} < 0,$ soluing for dy / dp gives

$$\frac{dy}{dp} = \frac{\sum_j s_{ip}}{1 - w^1 z^1 \sum_j s_{jw}} > 0 \qquad [5.8]$$

Thus the effective industry supply curve is positively sloped despite the fact that some of the firms may have negatively sloped effective supply curves. The slope of the market supply function depends on the extent to which increases in input demands increases input prices and the consequent increases in marginal costs at all output levels. Not that at a market supply $s = s(p)$ i.e. a point on this supply function, each firms marginal cost is exactly equal to p, given that all output adjustments have been complete. We define p as the supply price of the corresponding rates of output y_i since it is the price at which each firm would be content to supply and to go on supplying - the output y_i. At any greater price firms would find it profitable to expand production; at any lower price, they would wish to contract.

Figure 5.2 shows a number of possible situations which might arise when we put the market supply function together with the demand function. In (a) we show a well behaved case. The price p^*, with demand x^* equal to supply y^*, is obviously on equilibrium, since sellers are receiving the price they require for the output they are producing, and this output is being taken off the market by buyers at that price. There is no reason either for sellers to change their output (since each $y_i = s_i(p^*)$ maximizes is profit at price p^*) or for buyers to change the amount they buy.

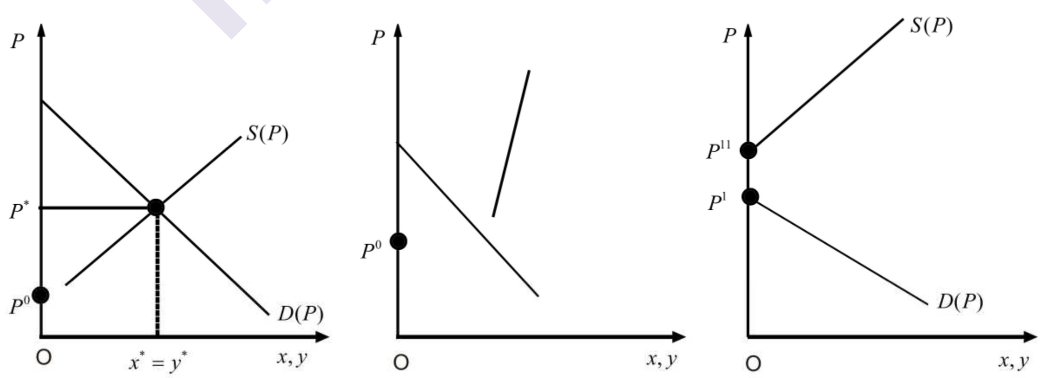

Figure 5.2

In (C) we show a third possibility, Suppose that firms do not all have the some AVC, but instead are evenly distributed over a range of AVC, with the minimum point of the lowest AVC curve being equal to p^{11}. If there are many sellers, and each seller is an insignificant part of the market we can then take the s(p) curve as continuous, with intercept at p^{11}. However, at price $p^{1} < p^{11}$ demand is zero-no one would be prepared to pay p^{1} or more for this good. It follows that equilibrium in this market implies a zero output and a price in the interval $\left[p^{1}, p^{11}\right]$ the highest price any buyer would pay is insufficient to cover the AVC of the firm with the lowest minimum AVC. We have a non produced good which firms would supply if the price were high enough, but which nobody wants to buy at such a price. The reader will find it instructive at this point to construct the excess demand functions

$$z(p) = D(p) - s(p) \qquad [6.9]$$

in these there cases, and illustrate them in a price-excess demand graph of the type shown in Figure 5.3

Figure 5.3 (b) suggests that a discontinuity in a supply or demand function and thus in the excess demand function. may imply that there is no equilibrium. This is a matter of some concern, since our theory of the market predicts the market outcome to be the equilibrium outcome, and raises the question what do we have to assume to ensure that the market has an equilibrium? To take the case of one market is to give only a provisional answer to the question since we ignore the interdependence among markets Nevertheless, it is instructive to consider the existence question in the simple context of one market.

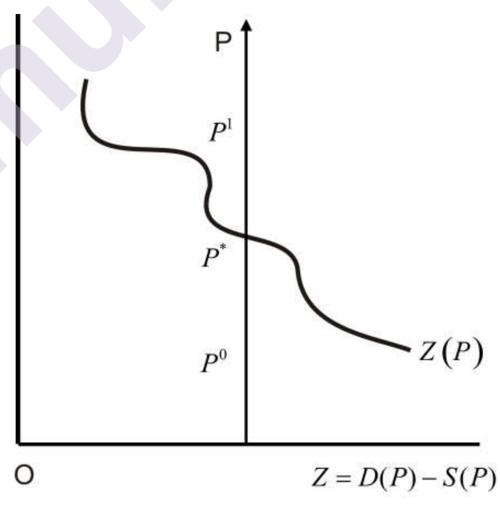

Figure 5.3

Figure 5.3 shows that discontinuity is a problem. Is it then enough to assume that z(p) is a continuous function of p? clearly not An equilibrium is a price $p^{*} > 0$ such that $z\left(p^{0}\right) = 0$. If $z(p) < 0, or\ z(p) > o$ for all $p > o$, $then\ z(p)$ may be continous but we will not have an equilibrium. This suggests the following existence theorem for a single market. If.

a) the excess demand function $z(p)$ is continuous for $p \geq 0$.

b) there exists a price $p^0 > 0$ such that $z(p^0) > 0,$ and

c) there exists a price $p^1 < 0$ such that $z(p^1) < 0,$

then there exists exists an equilibrium price $p^* > 0$ such that $z(p^*) = 0$

The intuition is clear from fig 5.3. If the excess demand curve is continues us and passes from a point at which excess demand is positive to a point at which excess demand is negative, it must cross the price axis, giving an equilibrium price.

The significance of the equilibrium price is that it induces buyers to demand exactly the output that results from individual sellers' profit-maximizing decision at that price. Plans are all mutually consistent and can be realized. We now turn to the equally important question of the stability of a market in the short-run.

5.5 STABILITY OF EQUILIBRIUM

Stability is an important characteristic of a market since predictions of the effects of changes in supply or demand conditions typically take the form of comparisons of the equilibrium before and after the change stability, like the question of existence considered in section A, is also relevant for analyses of welfare, which typically focus on properties of equilibria. Such analyses would have less point if one could not be sure that the market had an equilibrium to which it would tend.

A market is stable if, when ever the market price is not an equilibrium price, the price converges over time to on equilibrium price. The market is locally stable if it tends to an equilibrium when it starts off in a small neighborhood of that equilibrium and globally stable when it tends to some equilibrium price whatever its initial disequilibrium price.

In general we are more interested in global stability and whether the market will eventually end up in some equilibrium local stability does not imply global stability but, if there is only one equilibrium, global stability implies local stability. If a market has multiple equilibria it may be locally stable in the neighbourhood of some equilibria and unstable in the neighbourhood of other. Global stability then implies that at least one of the multiple equilibria is locally stable, though other may be unstable. Even if all the equilibria were locally stable this would not imply that the market was globally stable.

Formally a market is stable if $\lim_{t \to \infty} p(t) = p^*$ where p^* is an equilibrium price, $t \geq o$ is time, p(t) is the time path of price and the initial price $p(o) \neq p^*$.

The analysis of stability is concerned with a market's disequilibrium behaviour and requires a theory of how markets operate out of equilibrium. Any such theory rests on answers to three fundamental questions.

1] How do the market price or prices respond to non-zero excess demand?
2] How do buyers and sellers obtain information on the price or prices being offered and asked in the market?
3] At what point does trading actually take place, i.e. when do buyers and sellers enter into binding contracts?

There questions are important because answers to them may differ and differences in the answer lead to significant differences in the models of disequilibrium. In questions 1 and 2 we use the phrase price or prices because at this stage theories may provide for a single price to prevail through out the market even out of equilibrium, where as other allow there to be differences in prices offered by buyers and asked by sellers throughout the market. Whether or not a unique price will always prevail depends on the answers to questions 2 and 3.

To begin with we consider two contiruious time models of market adjustment. The first, known as the tatonnement process (tatonnement can be interpreted as 'groping') was proposed by walras. The second, which it can be argued is better suited to markets with production, was suggested by marshall.

5.6 THE TATONNEMENT PROCESS (TP)

The TP is an idealized model of how a market may operate out of equilibrium, in the sense that it may not describe the may a market works, but under certain conditions a market may operates as if its adjustment process were a TP there is a central individual, who can be called the market 'umpire', and who has the rate of a market coordinator. He announces to all decision takers a single market price (the answer to question 2), which they take as a parameter in choosing their planned supplies or demands. They each inform the umpire of their choices and he aggregates them to find the excess demand at the announced price. He then revises the announced price by the following rule (the answer to question 1).

$$\frac{dp}{dt} = \lambda z\left(p\left(t\right)\right)\lambda > 0 \qquad [6, 10]$$

that is, he changes the price at a rate proportionate to the excess demand. No trading takes place unless and until equilibrium is reached (the answer to question 3) at which time sellers deliver their planned supply and buyers take their planned demand. Notice that in this process there is no contact between buyers and sellers out of equilibrium every thing is mediated through the umpire.

Figure 5.4 shows three possible market excess demand functions. In (a), the excess demand acure has a negative slope. If, initially, the umpire announces the price $p^0 < p^*$, excess demand will be positive and he will revise the announced price upwards towards p^* if the announced price where above p^* it would be revised down words. Since these movements are always in the equilibrating direction, from wherever the process sarts, equilibrium will be global stable.

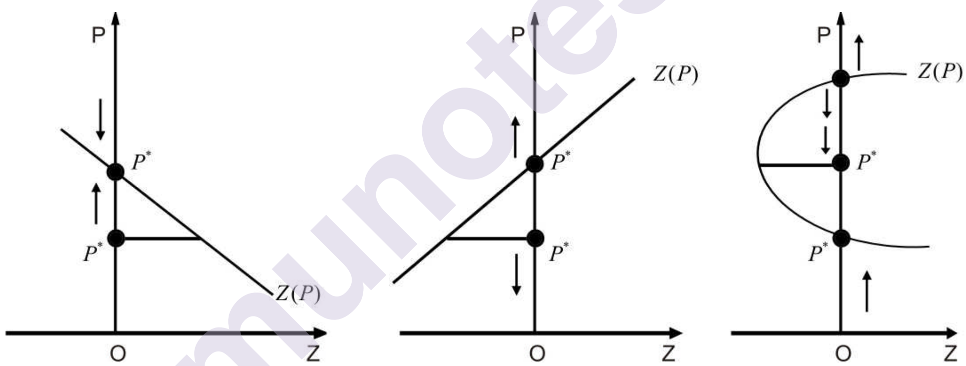

Figure 5.4

In (b), the excess demand curve has a positive slope. If the announced price is initially at p^0 the umpire will now reduce price, since $z < 0$, and hence the TP leads away from equilibrium. A similar result used occur if the initial price were above p^*. Hence in this market the equilibrium is globally unstable.

In (c) we have a somewhat more complex case. The excess demand curve is backward bending, having a negative slope over one range of prices and a positive slope over another. In this case, if the initial price were anywhere in the interval $o \leq p < p^1$, the TP would converge to the equilibrium p^*. If, however, the initial price was $p^{11} > p^1$ the market would move away from equilibeium, since excess demand is positive for $p > p^1$ and so price would be increased. Therefore the market is not globally stable, since on initial point sufficiently far from the equilibrium. p^* qould lead away from market equilibrium. The market has two equilibrium positions, one at p^* and one at p^1; the former is locally but not globally stable, the latter is locally (and therefore globally) unstable.

From this discussion we can deduce the following stability conditions, i.e. sufficient conditions for the TP to be stable: (a) equilibrium is globally stable if excess demand is positive whenever price is less than its equilibrium value and negative when price is above its equilibrium volue;

(b) equilibrium is locally stable if the condition holds for prices in all small neighbouxhood of an equilibrium.

For a more formal analysis of stability we can use a distance function, which measures the distance between two point. Thus define

$$\delta\left(p(t), p^*\right) = \left(p(t) - p^*\right)^2 \qquad [5.11]$$

which measures the distance between an equilibrium price p^* and some other price p(t) (note that $\delta\left(p(t), p^*\right) > 0 \Leftrightarrow p(t) \neq p^*.$) A necessary condition for the time path of price $p(t)$ to converge to p^* is that $d\delta / dt < 0,$ i.e. the distance between the price path and p^* is falling through time. Differentiating we have

$$\frac{d\delta}{dt} = 2\left(p(t) - p*\right)\frac{dp}{dt} = \left(p(t) - p^*\right)\lambda z\left(p(t)\right) \qquad [5.12]$$

from 5.10 then clearly $d\delta / dt < 0$ if any only if $p(t) - p^*)$ *and* $z\left(p(t)\right)$ have opposite signs, as in the stability condition Note that this is true regardless of the value of λ : the 'speed of adjustment' parameter determines only now fast, and no whether, the TP converges to equilibrium.

Is the condition also sufficient for convergence, however? It may seem 'intuitively obvious' that it is, but consider the example of the function $y = a + 1/t.$ Here we have $dy / dt < 0, but \lim t \to \infty\, y = 0$. So we have to provide a further argument to justify the claim that $\delta\left(p(t), p^*\right)$ is not bounded away from zero under the TP.

We do this by establishing a contradiction suppose, without loss of generality, that $p(o) > p*$ and suppose that

$\ell im \infty\ p(t) = \overline{p}\ where\ \overline{p} > p^*$. The interval $\left[p(o), \overline{p}\right]$ is non-empty, closed and bounded and the function $d\delta / dt$ is continous, so at some t we must heave that $d\delta / dt$ takes an a maximum, by were strass. Theorem, call this maximum s^* Note that, since for $p(t) \neq p^*$ we must have $d\delta / dt < 0,$ then s < 0 also for only arbitrary $t = \bar{t}$ integrate to obtain.

$$\int_0^{\bar{t}} \frac{d\delta}{dt} dt = \delta\left(p\left(\bar{t}\right), p^*\right) - \delta\left(p(o), p^*\right) \qquad [5.13]$$

and

$$\int_0^{\bar{t}} s^*\ dt = s^*\ \bar{t} \qquad [5.14]$$

Then by definition of s^* we must have

$$\delta\left(p\left(\bar{t}\right), p^*\right) - \delta\left(p(o), p^*\right) \leq s^*\ \bar{t} \qquad [5.15]$$

or

$$\delta\left(p\left(\bar{t}\right), p^*\right) \leq s^*\ \bar{t} + \delta\left(p(0), p^*\right) \qquad [5.16]$$

By choosing $\bar{t}$ large enough, we can make the right hand side of 5.16 negative, implying we must have on the left-hand side a negative value of the distance function, which is impossible. Thus we have the contradiction.

This proof makes precise the intuition that, if $p(t)$ is always moving closes to p^* whenever $p(t) \neq p^*$, it cannot tend to anything other than p^*.

5.7 MARSHALL'S PROCESS

Marshall suggested the following alternative to Walra's TP suppose that when sellers bring their output to market they sell it for whatever it will fetch. Refer to fig 5.5. If supply is less than the equilibrium supply y^* then the price buyers will be prepared to pay if it is auctioned off to the highest bidders, the demand price, p_D^0, exceeds the supply price, p_s^0 conversely, if supply exceeds equilibrium supply auctioning off the available supply causes demand price to fall below supply price. Marshall argued that when demand price p_0 exceeds supply price p_s sellers will expand supply, and conversely when p_o is less then p_s. This is because p_s equals each seller's marginal cost, and so $p_0 > p_s$ implies output explansion increases profits, while when $p_o < p_s$ profits are increased by an output contraction. This suggests the adjustment rule.

$$\frac{dy}{dt} = \lambda\left(p_0\left(y\right) - p_s\left(y\right)\right) \qquad [5.17]$$

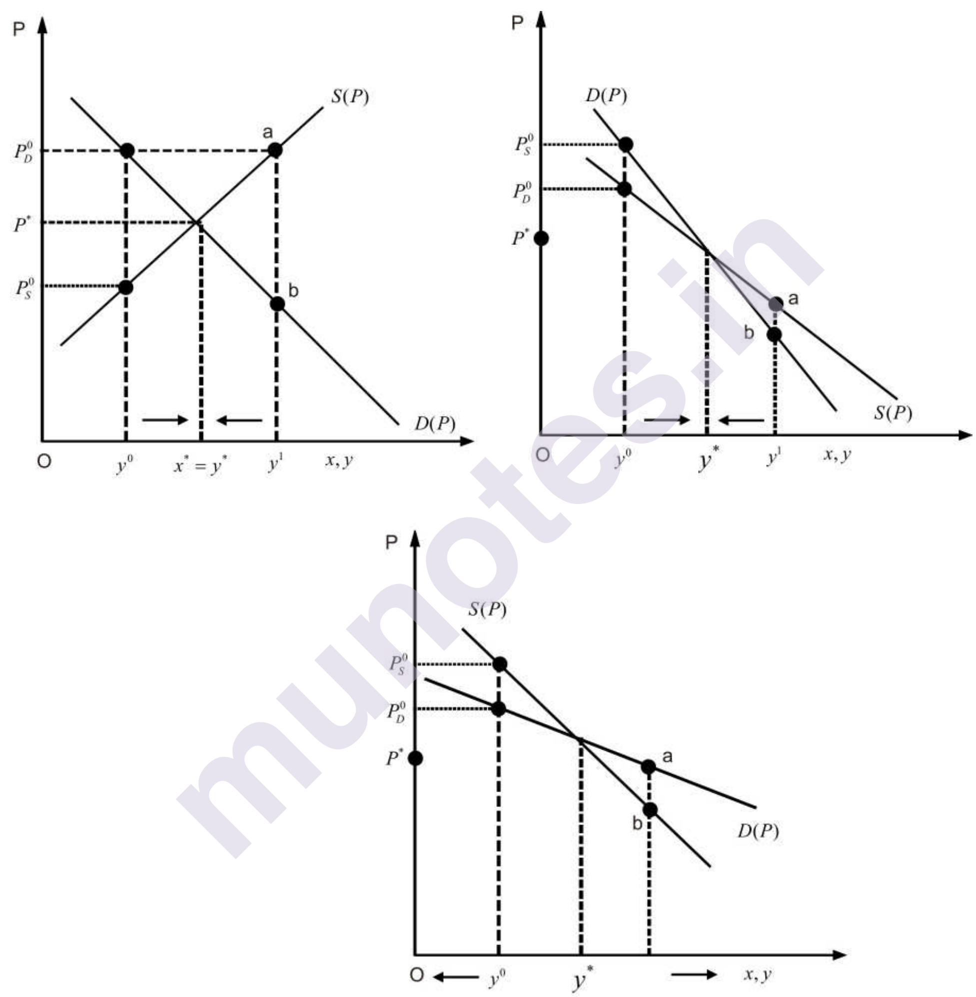

Figure 5.5

$$P_S^0 abS(P)bD(P)y^0x^* = y^*y^1x, y$$

where $p_0\left(y\right)$ is inverse demand function, giving demand price as a function of quantity supplied (= quantity traded at any t) and similarly, $p_s\left(y\right)$ is the inverse supply function (derived from the firms marginal cost functions as before). Note that, at equilibrium quantity $y^*, p_D = p_s = p^*$

Under what conditions is Marshall's process stable? If output expands when $p_D > p_s$ and contracts when $p_0 < p_s$ then fig 5.5 (a). Suggests that, when the supply and demand curves have the usual slopes, the market is stable Figures 6.5(b) and (c) show that, when the supply curve has a negative slope, the process is stable if the demand curve cuts the supply curve from above but unstable in the converse case. This is interesting, not only because back ward bending supply curves are possible but also because the walrasion TP has precisely the opposite outcomes in these cases. In figure 6.5 (b), the corresponding excess demand function $z(p) = D(p) - s(p)$ increases with price with price and so the walrasion TP would be unstable. In figure 5.5 (c) z(p) has a negative slope and so the Walrasion TP is stable. Thus although the two adjustment processes have then some outcomes in the 'standard case', it matiers which we adopt in a 'non-standard' case.

To make the stability conditions for Marshall's process more precise, we again adopt a distance function approach. Define the distance function

$$\delta\left(y(t), y^*\right) = \left(y(t) - y^*\right)^2 \qquad [5.18]$$

Then

$$\frac{d\delta}{dt} = 2\left(y(t) - y^*\right)\frac{dy}{dt} = 2\left(y(t) - y^*\right)\lambda\left[p_D(t) - p_s(t)\right] \qquad [5.19]$$

using 5.17. Then, for $d\delta / dt < 0$, we require $\left(y(t) - y^*\right) and \left(p_D(t) - p_s(t)\right)$ have opposite signs, confirming the diagrammatic analysls. We can establish the sufficiency of this condition along similar lines to those used in the case of the TP process.

We have already noted that in non-standard cases the walrasian TP and Marshall's process have opposite implication for market stability-it matters whether we take price as adjusting to a difference in quantities, or quantity as adjusting to a difference in demand and supply prices. We can also compare the processes in terms of the answers to the three questions at the beginning of this section:

1) Responsiveness of price to non-zero excess demand. In the standard case of negatively sloped demand and positively sloped supply, both processes result in market price rising (falling) when there is positive (negative) excess demand. In the Walrasian case this happens directly through the TP; in the Marshallian case, it happens via the auction mechanism which establishes the demand price.

2) Infromation on price (s). In the TP, this is transmitted simultaneously to all buyers and sellers by the umpire; in Marshalls

process, at each instant the auction mechanism rations off available output and the demand price is immediately made known. Buyers never need to know the supply price-sellers know their own marginal costs and so once the demand price is known an output change can result.

3) When does trade take place? In the TP, only at equilibrium, under Marshall's process, at every instant as available supply is auctioned off marshals process has trading out of equilibrium, with an efficient rationing rule, so that available supply is auctioned off to the highest bidders. Alternatively, think of Marshall's process as consisting of a sequence of 'very short-run' or instantaneous equilibria, with a vertical supply curve at each of these equilibria, and the analysis then establishes conditions under which this sequence of instantaneous equilibria converges to a full equilibrium of supply and demand.

Which model is 'better' depends on which process captures more closely the way a particular market works. Walras' TP may seem unrealistic in its reliance on a central 'umpire' collecting buying and selling intentions and announcing an equilibrium price, but some markets, for example markets in stocks and shares, and minerals such as gold and silver, are highly organized with brokers who may function much as a walrasian umpire.

There are two features of both models which are unsatisfactory in the light of observations of how many markets work. First, both process are centralized : some device - the umpire or the auction mechanism ensures that all buyers and sellers simultaneously face the some price. However, in many real markets, price formation is decentralized. Individual buyers meat, haggle and deal with individual sellers, and pressures of excess demand or supply exert their influence by causing sellers and buyers to bid price up or down. If information on all the prices being offered and asked is fully and costlessly available throughout the market then this would be equivalent to a centralized adjustment process. But this is often not the case. Buyers and sellers have to seek each other out to find the prices at which they are prepard to trade, and this search process is costly.

Second, in neither model do buyers and sellers form expectations and act upon them In the TP this possibility is simply excluded. In Marshalls process, sellers must make some forecast of further price in order to market decisions which determine their future supply, but this is not module explicitly, being subsumed in the adjustment rule 6.17. In the rest of this section therefore we consider the explicit modeling of expectations in market adjustment processes.

5.8 LONG RUN EQUILIBRIUM

We saw that the firm's long-run supply curve is that part of its long-run marginal cost curve above its long-run overage cost curve. There are several reasons why the market supply curve cannot be obtained simply by summing these supply curies:

(a) External preliminary effects. As all firms vary output, input prices may change, causing each firm's cost caves to shift.

(b) External technological effects. Individual firms' cost curves shift as a result of expansion of scale by all firms leading to congestion or improvement in common facilities such as transport or communications.

(c) Changes in the number of firms in the market. As price rises firms which previously found it unprofitable to produce the commodity now find it profitable, and so invest in capacity and add to output. In a competitive market there are no barriers such as patents, legal restrications, ownership of raw material sources. Which impede the entry of new firms. A firm which at the going price just breaks even, with total revenue equal to long-run total cost (induding the opportunity cost of capital and effort supplied by its owners (s)) is called a marginal firm at that price one which makes on 'excess profit' (total revenue > total long-run opportunity costs) is called an intro-marginal firm, and one which would market a loss, but breaks even at a higher price, is called an extrmarginal firm. As rpice rises, marginal firms become intro-marginal and some extra marginal firms enter.

It is therefore by no means assured that the long run market supply curve will be positively sloped (see questions 1,2). However, in figure 5.6 (c) we assume this to be the case. S(p) shows how the rate of output varies with price when capacity is adjusted and the number of sellers may change. It should be noted that underlying this curve is a possibly complex set of adjustments, and the transition from one point on the curve to another is not so smooth and effortless as the curve suggest. It should be interpreted as showing the aggregate output which will be forthcoming at each price after all these adjustments have been made. Or, alternatively, it shows the price at which a given number of firms would remain in the industry, maintain their capacity and supply in aggregate a given rate of output. The P-Coordinate of any point y is then the long-run supply price of that rate of output.

The long-run equilibrium is shown in Figure 5.6 (c) as the point $\left(y^{*}, p^{*}\right)$ At this point firms are prepared to maintain the rate of supply y^{*} and consumers are prepared to buy this output at price

p^*. If therefore, the short run supply curve s(p) was as shown in the figure the short-run equilibrium we have earlier been examining would also be a long run equilibrium. It would be maintained indefinitely in the absence of any change in demand, input prices or technology.

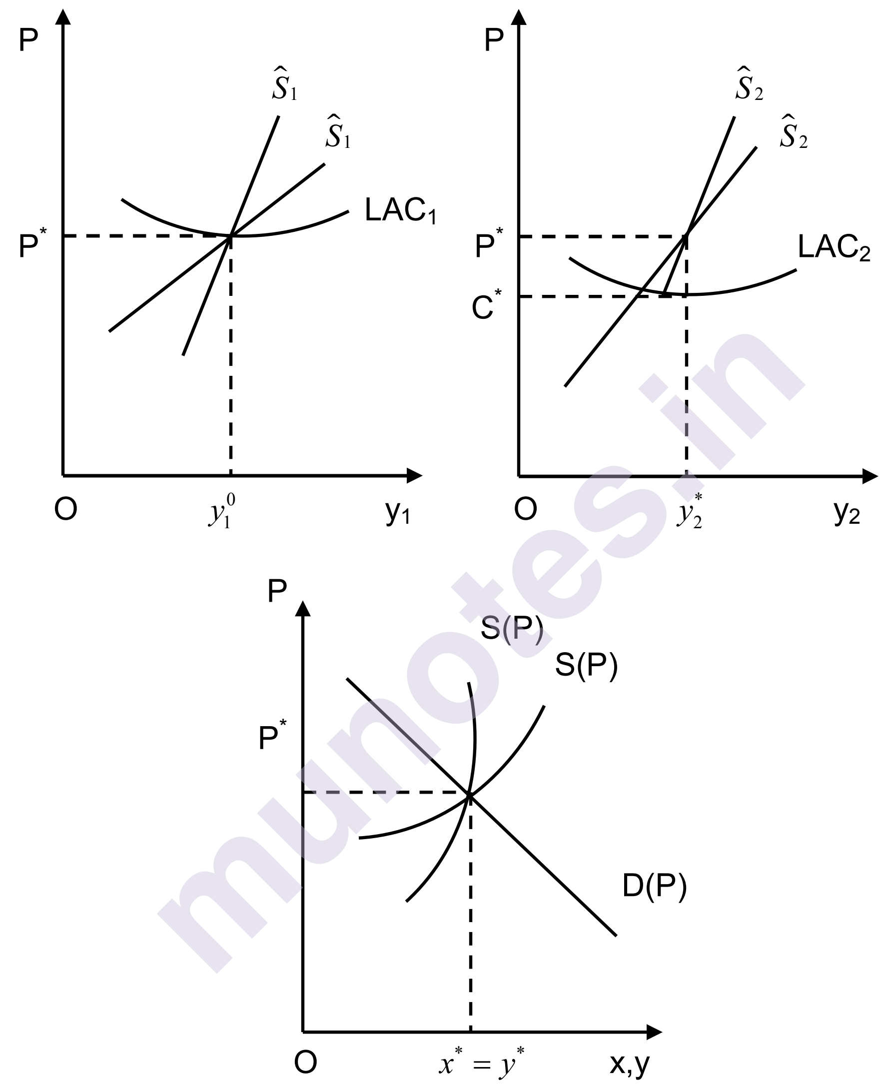

Figure 5.6

The other parts of the figure show the implications of the long-run equilibrium for two 'representative firms' In (a), firm 1 is a marginal firm. At market price p^* it chooses a long run profit maximizing scale of output y_1^* and at that output p^* is equal to its minimum long-run average cost. Firm 2, on the other hand, shown in (b) of the figure, is on intra-marginal firm; at its profit-maximizing scale of output y_2^*, its long run average cost $c^* < p^*$, and it makes an excess profit equal to $\left(p^* - c^*\right)y_2^*$. However, such 'excess profits' which may be earned temporarily, will not persist indefinitely, but rather should be regarded as true opportunity costs to the firm in long run.

The argument goes as follows the fact that the intra marginal firms average costs are lower than these of a marginal firm must reflect the possession of some particularly efficient input, for example especially fertile soil or exceptionally skilful management. Since these generate excess profits, we expect other firms to compete for them, so that after a period long enough for contracts to lapse, the firms which currently enjoys the seruices of these super productive inputs will have to pay them what they ask or lose them. The maximum these inputs can extract is the whole of the excess profits $\left(p^* - c^*\right) y_2^*$ and so what was a profit during the period when the contract was in force becomes a free opportunity cost to the firm after that time. Such excess profits are therefore called quasi-rents, to emphasize that they are not true long-run excess profits, but merely rents accruing to the contractual property rights in certain efficient input serucies, which become transformed into costs in the long run once this transformation has taken place, the 'intro-marginal' firms LAC curve will rise until its minimum point is equal to p^*. Hence in the long run all firms in the market will be marginal firms in the sense that they just break even.

Figure 5.6 industries the three conditions which must hld in long-run equilibrium 1) Each firm in the market equates its long-run marginal cost to price, so that output maximizes profit.

2) For each firm price must equal long run average cost (if necessary after quotients have been transformed into opportunity costs) so that profits are zero and no entry or exit takes place.

3) Demand must equal supply.

Condition (1) and (2) then imply that each firm produces at the minimum point of its long-run average cost curve, as fig 5.6(a) illustrates. This is a strong result on the efficiency of the competitive market equilibrium, since it implies that total market output is being produced at the lowest possible cost.

As with the short run supply curves in fig 5.6 (b) discontinuities in the long-run supply curve may imply that equilibrium does not exist suppose that (a) all firms, whether currently in the market or not, have identical . u-shaped LAC curves as shown in figure 5.6 (a) (b) input prices do not vary with industry output.

Then, there could be a discontinuity in the long-run market supply curve at price p^* in figure 5.6. At any price below p^*, all firms would have the market, and market supply will fall to zero, while at price p^* planned market supply is y_1^* multiplied by the number of firms which are capable of producing the good with the given LAC curve. This discontinuity could be avoided if there is some mechanism which selects potential suppliers in such a may as to ensure that any given market demand at price p^* is just met by the appropriate number of firms each producing at minimum long-run average cost then, the long-run market supply curve would be a horizontal line at price p^* expansion of market output is brought about entirely by new entry rather than through output expansion by existing firms. Long-run equilibrium price can not differ from p^*, and so is entirely cost determined. The level of demand determines only aggregate output and the equilibrium number of firms. Note that for a long-run market supply curve which is a continuous horizontal line we need the least cost output of a firm $\left(y_1^*\right)$ in fig 5.6 (a) to be 'very small' relative to market demand, and the number of firms to be 'very large'.

More simply, if the technology of production is such that there is no range of outputs over which there are increasing returns to scale, then there is no discontinuity in market supply. For example, if all firms experience decreasing returns to scale at all outputs then long-run average and marginal cost curves will be every where upward sloping and their horizontal sum (taking into account any input price effects) will have an intercept on the price axis.

Alternatively, if we assume all firms have identical production functions with constant returns to scale, and face identical (constant) input prices, then the long-run market supply curve is again a horizontal straight line. Each firm's long-run marginal cost curve is a horizontal line and coin cides with its long-run average cost curve, and these are at the some level for all firms. Then, the only possible equilibrium price is given by this common maxginal average cost so that price is again completely cost determined Demand again determines only the aggregate equilibrium market output. Note that, in such a market model, the equilibrium output of each firm, as well as the equilibrium number of firms producing in the market, are indeterminate.

5.9 STABILITY IN THE LONG-RUN

The analysis of the stability of long-run equilibrium in a competitive market must take into account the interaction between short-and long-run decisions of firms, the effects of new entry and the role of price expectations. We carry out the analysis for the case in which input prices increases with aggregate market output, and all firms have U-shaped cost curves. As shown in figure 5.7, the long run market output, and all firms have U-shaped cost curves. As shown in figure 5.7, the long run market supply curve is upward sloping. It should be thought of as the locus of price-quantity points at which the long run equilibrium conditions are satisfied at each paint, price-long run marginal cost for each firm in the market, and no further entry or exit will take place at a given price because firms are just breaking even at that price given that the quast rents of intra marginal firms have been transformed into opportunity costs). Thus corresponding to each paint on the curve is a particular set of firms, each with a profit maximizing capacity and output level As price rises, output increases along the curves as a result of both output expansion by existing firms and entry of new firms. However, the actual timl path of price and output may not lie one the supply curve. For that to happen, we again need the assumption of rational expectation, as we shall now see.

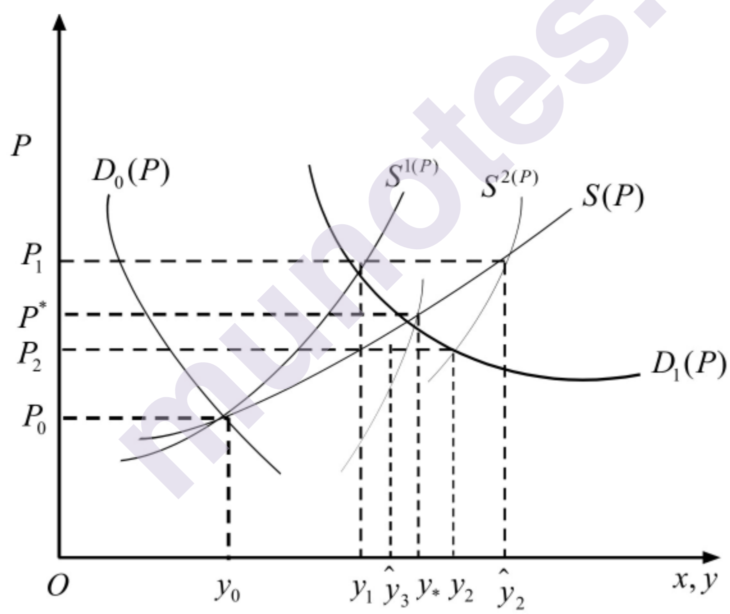

Figure 5.7

Suppose at year O the market is initially in long-run equilibrium at the price and output pair $(p_0\ y_0)$ in figure 5.7 In year 1 demand shift to $D_1(p)$. In the short run year 1 output can only expand along the short run supply functions $s^1(p)$, determined by the shrot run marginal cost functions of the firms already in the market (together with only effects of increasing input prices as analysed in section A). Thus price in year 1 is established as p_1. Since p_0 corresponded to zero profit of the existing firms, p_1 must imply positive profits. The market is clearly not in long run equilibrium. What happens next depends upon the assumption we make about price expectations formation.

Begin, as in the cobweb model of section B, with the assumption of naïve expectations all firms, whether currently in the market or contemplating entry, expect price p_1 to prevail next year, in year 2. The existing firms expand capacity and new firms enter and install capacity to the extent that planned market output expands to y_2, since this is the aggregate output corresponding to jong-run profit maximization at price p_1. But of course, when period 2 arrives, $\left(p_1, y_2^1\right)$ is not an equilibrium price will have to fall to p_2, whether demand equals short-run supply as indicated by the sort-run supply curves $s^2\left(p\right)$. This is determined by the short run marginal cost curves of all firms in the market initial incumbents and new entrants in year 2. If all firms again assume, naively, that p_2 will prevail in year 3, then capacity will be contracted and some firms will have the market unitl y_3 will be the aggregate market supply that will be planned for year 3. And so on under naïve expectations, price fluctuates around the equilibrium value p^* and in the case illustrated in figure 6.7 eventually converges to it (in the absence of further demand change). The fact that capacity can only be adjusted in the long run introduces the some kind of supply lag that we assumed for an agricultural market. The main difference is that here the short-run supply curve is positively sloped whereas in the Gobweb model it was in effect vertical. The role of the long-run supply curve in the present analysis is to show how future price. Although the ultimate effect of the demand shift is to move the market from one point on the long run supply curve to another, the actual time path of price and output through the adjustment process lies along the demand curve and describes a diminishing sequence of jumps from one side of equilibrium point to the other.

However, our previous criticisms of the naïve expectations assumption apply equally here. It is irrational for a profit maximizing firm to form its expectations in this may because then it is consistently sacrificing potential profits suppose instead that all firms have rational expectations, that is, they know the market model and use its predication as their price expectation. Then, if the change in demand between periods 0 and 1 is unanticipated, the year 1 short-run equilibrium is at $\left(p_1, y_1\right)$ as before, but now firms can predict the now long-run equilibrium price p^*. This is the only price with the property that the planned output which maximize profits at that price can actually be realized, i.e sold, on the market next period Hence existing firms will expand capacity and new firms will enter so as to expand market output to y^*, and the market moves to its long run equilibrium in years 2. If the change in demand had been fully anticipated at year 0, then the same argument leads to the conclusion that the market would move to its new long-run equilibrium in year 1. In that case, the market adjusts smoothly along its long-run supply curve to change in demand.

5.10 SUMMARY

This unit explains the features of perfect expedition and elaborates the process of price and output determination of a perfectly competitive firm in the short-run and long run. It also explains the concept of Tatonnement process and Marshall's process.

5.11 QUESTIONS

1. Explain the short-run equilibrium of perfectly competitive firm.
2. Examine the stability of equilibrium of perfectly completive firm in the short-run.
3. Describe the concept of the Tatonnement Process.
4. Examine the concept of Marshall's Process.
5. Explain the equilibrium of a perfectly7 competitive firm in the long-run.

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Unit-6

PRICE & OUTPUT DETERMINATION UNDER PERFECT COMPETITION

UNIT STRUCTURE:

6.0 OBJECTIVES

After going through this unit you will be able to explain the concepts of -

- Existence & Stablility of General Equilibrium
- First & Second Fundamental Theorems of Welfare Economics
- The Market Failure
- The consumer surplus
- Theory of second best

6.1 INTRODUCTION:

The general equilibrium phenomenon is the interdependent and interrelated. General equilibrium indicated equilibrium of consumer in the market by two forces these are demand and supply where price of commodity determines at such a point, this point is called equilibrium point of market. In other words, general equilibrium concept is related to price determination in the market by various forces. How the general equilibrium concept is interdependent? Consumer demand for a particular commodity affects by various factors such as taste of consumer, preferences, price of substitute commodity, climate, income and many more and his demand affected by these factors. Therefore, general equilibrium of market affects because of these factors. So, the concept of general equilibrium becomes interdependent.

There are two important views these are Marshallian general equilibrium and Walarasian general equilibrium. In Marshallian equilibrium analysis, Marshall explains partial equilibrium by taking only two variables to determine prices. He assumed that other factors being constant. The second view of Walarasian was first scientific view, because he considered all variable or relevant variables which plays important role in price determination in the market or market equilibrium. That is why Walatasian analysis is called general equilibrium analysis.

6.2 EXISTENCE OF GENERAL EQUILIBRIUM:

General equilibrium is the concept which is complex in the nature, because it is interdependent on various variables. So, existence of general equilibrium in the market is difficult task. There are various problems are arisen. Among of them three problems arised by Koutsoyiannis which are as follows-

1. Existence of equilibrium. Does a general equilibrium solution exist?
2. Uniqueness of equilibrium. If an equilibrium solution exists, is it unique?
3. Stability of equilibrium. If an equilibrium solution exists, is it stable?

In short, existence of equilibrium in the market is the condition or situation where neither excess demand exist nor excess supply exist. It means whatever supply in market at particular price is demanded, there is no issue of excess stock. So, this scenario called market equilibrium. The existence of equilibrium is shown by below diagram.

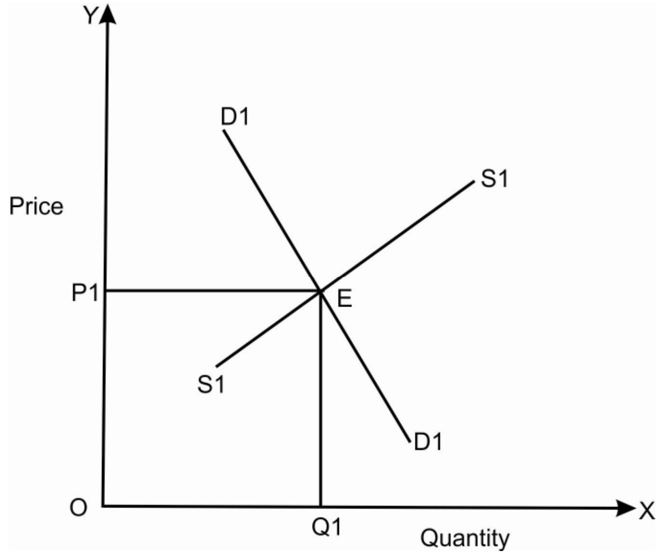

Figure 6.1

In the above figure, price has been shown on Y axis and quantity on X axis. S1S1 is supply curve and D1D1 is demand curve. D1D1 and S1S1 curves intersect to each other at E point. E point is the equilibrium point at which price and quantity determines P1 and Q1 respectively. At E point there is existence of equilibrium.

6.3 STABILITY OF EQUILIBRIUM:

If equilibrium exists in market, is it stable? It means when the various forces of market disturbs the equilibrium, is the tendency of market towards the equilibrium? If answer is yes, so we can call, this equilibrium is stable equilibrium.

The stability of equilibrium in the market is depends on shape and slope of demand and supply curve. These two factors determine whether the equilibrium is stable? For the stable equilibrium demand curve should be downward sloping and supply curve should be upward sloping.

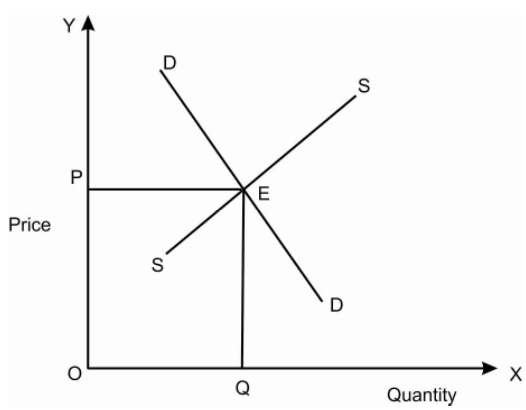

Figure 6.2

Above figure shows stable equilibrium. In this figure, DD demand curve is downward sloping and SS supply curve is upward sloping which intersects to each other at E point. And at the E point there is stable equilibrium. If any situation, the market equilibrium disturbs, the price mechanism will make it again stable.

6.4 FIRST & SECOND FUNDAMENTAL THEOREMS OF WELFARE ECONOMICS:

6.4.1 FIRST FUNDAMENTAL THEOREM OF WELFARE ECONOMICS:

The first fundamental theorem of welfare economics is related to the concepts of Pareto optimality and the perfect competition. What is Pareto optimality? Pareto optimality states, "without making someone worse off, no one will better off." It means there is no further improvement possible in social welfare.

The social welfare equilibrium happens in only perfectly competitive market, this is known as first fundamental theorem of welfare economics where all marginal conditions of Pareto will be fulfilled. In other words, first fundamental theorem of welfare economics explains Pareto efficiency in economy or social welfare equilibrium in the perfect competitive market or first fundamental theorem of welfare economics assumes that general competitive equilibrium is the Pareto optimal.

Marginal Conditions of Pareto Efficiency or Conditions of Pareto Optimality:-

1. **Efficiency in Exchange:** The marginal rate of substitution (MRS) between any two products or commodity must be the same for every individual who consumes both.
2. **Efficiency in Production:** The marginal rate of technical stubstitution (MRTS) between any two factors must be the same for any two firms using these factors to produce the same product.
3. **Efficiency in Product Mix:** The marginal rate of substitution between any pair of products for any person consuming both must be the same as the marginal rate of transformation between them."
4. Efficiency in Consumption or Exchange
5. Pareto Optimality in Consumption or Exchange and Perfect Competition
6. Pareto Optimality Conditions when the External Effects are Present
7. Efficiency in the Allocation of Factors among Commodities or Efficiency in Product-Mix or Composition of Output.

Critics on First Fundamental Theorem of Welfare Economics:

1) This theorem ensures only about Pareto efficiency not the social justice.
2) Externalities found in consumption and production.
3) Second order condition of the equilibrium must be fulfilled.
4) Economic efficiencies are quite restrictive.
5) The concept of perfect competition is hypothetical in the practical life imperfect completion prevails.

6.4.2 SECOND FUNDAMENTAL THEOREM OF WELFARE ECONOMICS:

There is also second fundamental theorem of welfare economics. This theorem of welfare economics states that "Every Pareto optimal situation, there is competitive equilibrium." Given the initial income distribution in the economy or factor endowment in the economy. In the other word, Pareto efficiency or Pareto optimality situation found in competitive market equilibrium.

6.5 WELFARE EFFECTS OF PRICE CHANGES:

Welfare of the society is depends on various variable factors like as level of production in an economy, government fiscal policies, variation in tax rate and changes in prices of commodity etc.

Welfare of a society or individual have main constraints and these are income and price of commodity and services, because these two factors have ability to enhance social choices as said by Amartya Sen. Therefore, change in the prices of commodities and services effect consumer's welfare.

In the mid nineteen century, Engineer Jules Dupuit who first propounded the concept of economic surplus. Then Alfred Marshall gave fame to this concept and developed the two major concepts as consumer surplus and producer surplus with the help of demand curve and cost curve (supply curve).

6.6 CONSUMER SURPLUS:

The concept of consumer surplus is very useful to understand that what the welfare effects of price change are?

"Consumer surplus is the difference between the price which consumers are willing and able to pay for a good or service and actually do pay." In other words, consumer surplus is the difference between potential price and actual price.

Consumer surplus is the area under the demand curve and market price.

Consumer Surplus = Potential Price – Actual Price

Consumer Welfare/Consumer Surplus at Initial Level:

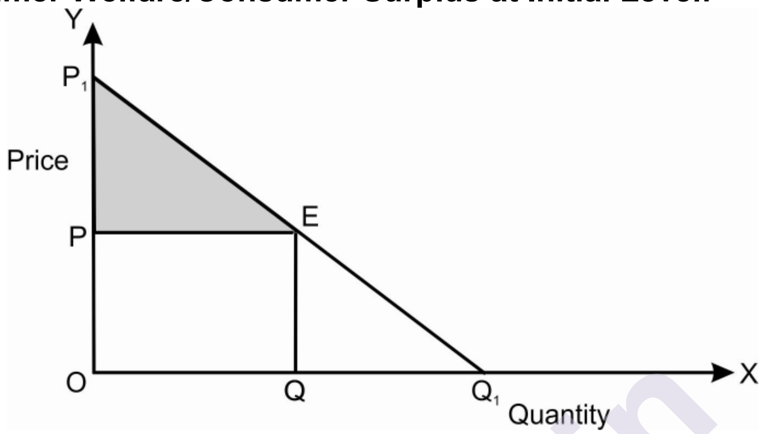

Figure 6.3.1

Consumer Welfare/Consumer Surplus when Price increases:

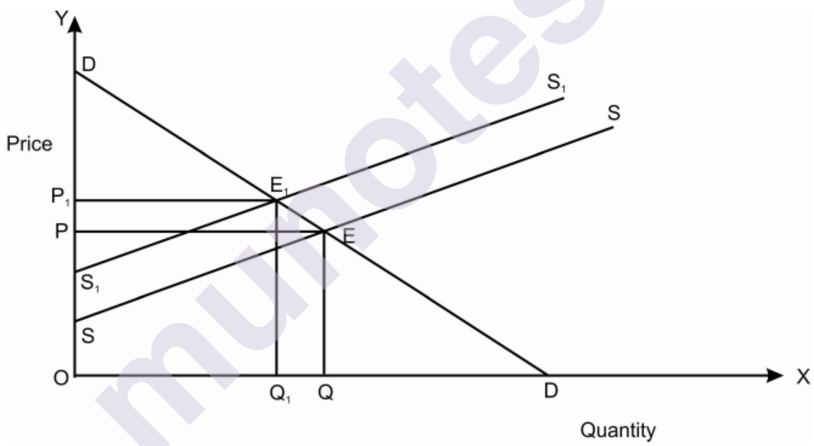

Figure 6.3.2

Consumer Welfare/Consumer Surplus when Price decreases:

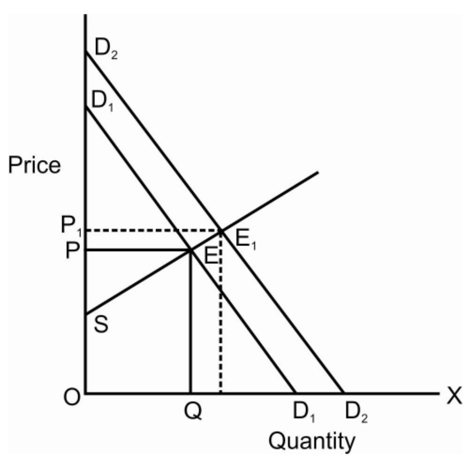

Figure 6.3.3

Above figures 1, 2 and 3 explored the effect of price change (increase/decrease) on consumer welfare/ social welfare/ social advantage.

Figure no. 1 indicates that DD and SS curve which are demand curve and supply curve intersect to each other at E point which is the equilibrium point where consumer is ready for willing to pay at OP1 price for OQ quantity, but actually he pays at OP price. So, EPP1 is the area of consumer surplus.

Figure no. 2 indicates when the price of commodity rise in the economy, consumer surplus or welfare of a consumer will decrease. In this diagram, when SS and DD curves exist in the economy, the area EPD is the area of consumer Surplus. If the market price increases, supply curve shifts to S1S1 which intersect to DD demand curve at E1 point and OP1 market price determines in the economy. Because of price increase, consumer surplus decrease from EPD to E1P1D. It means inflation in economy adversely affects the social welfare of society. Therefore, government of the country and central monetary authority always tries to control inflation in the economy to save public welfare and public interest.

In the case of price decreases (in Figure 3), consumer surplus is increased. In the short, when price decreases in the economy, real income of people and society will increase due to this able to expand their social choices which leads to increase in social welfare and in the case of producer surplus adverse situation will be find.

6.7 MARKET FAILURE:

Market failure is the concept related to Pareto Optimality criteria or perfect competition. In the condition of perfect competition, equilibrium of market is Pareto optimum. It means, there is no further improvement possible or it means market is successful to attain the Pareto optimum equilibrium. So, this is the case of market success. Then question is that what is market failure? Before understanding the concept of market failure, there is need to study the function of market.

Function of market:

The main function of market is that price determination of commodity in the market where two factors of market determined it and these are supply of commodity and demand of a commodity. According to the condition of supply and demand for a commodity, price of a commodity will determine.

Therefore, if market is working very well at the level of Pareto to decide price of commodity in the market, it is called market success and if market is unable to decide prices of commodities, it is called market failure. It happens due to various reasons like as externalities, public good, imperfect competition, asymmetric information etc. In the case of externalities there is very difficult to decide the prices, because demand is no explicitly given. So, one factor of the market is partially missing and price of commodity will not be decided at Pareto level. Samething happens with public goods where people wants various commodity and services from the government, but they are directly not ready to pay for it. It means that wants are not converting in the demand, that is why it is known as demand is missing in market. So, if demand is missing in the market, prices of commodities will not decide in the market. It is the concept of market failure.

6.8 THEORY OF THE SECOND BEST:

The Pareto criteria and marginal conditions of Pareto optimality lead to maximum social welfare or economic efficiency. If the conditions of Pareto optimality fulfilled, maximum social welfare will achieve, but if the Pareto optimality not achieved, then what is the solution for it? Or what is second best solution?

Prof. Lipsey and Lancaster raised the same question and developed the theory of second best.According to them, if the condition of Pareto efficiency not possible to achieve means maximum social welfare or maximum social advantage is unattainable, whether or not efforts should be made to achieve the second best position by satisfying the remaining marginal conditions of Pareto optimum.

In the theory of second best, they assert that the theory of second best or second best solution will not to lead in increase the social welfare. According to second best theory, social welfare will not be increased, if any condition of Pareto optimality is not fulfilled. That is theory of second best solution which is not desirable.

6.9 SUMMARY:

In this unit we have studied the existence and stability of equilibrium, the first and second theorem of welfare economics, the concept of market failure, the functions of market, consumer's surplus (at initial level, when price increases and prices decreases), the theory of second best etc.

6.10 QUESTIONS:

Q1. Explain the first & second fundamental theorems of welfare economics.

Q2. Write notes on following.

- Functions of market
- The theory of Second Best

Q3. Explain the concept of consumer surplus with the help of diagram.

Q4. Explain the existence and stability of equilibrium.

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Unit-7

MONOPOLY

Unit Structure :

7.0 OBJECTIVES

- Market power may exist in both the buyer's and seller's market. Major objective of this unit is to understand the concept of market power.
- We will also analyse the benefits and social cost of monopoly power in the seller's market.
- Monopsony or the market power in the buyer's market will also be analysed in this unit.

7.1 INTRODUCTION

The neo-classical economics criticized monopoly market and monopoly power on the ground that misutilises the resources and leads to inefficiencies in the market. As a result, optimization of social welfare is not possible. The neo-classical ideology regarding monopoly power is reflected in many anti-monopolistic legislations passed by the governments of the countries, like UK, USA etc. The major objective of such anti-monopoly laws is to identify the presence of monopoly power, and then regulate or eliminate any such monopoly power. The problem in this respect, is that the monopoly power is not easily identifiable. There is an absence of unanimity on the factors that lead to monopoly power and hence it is difficult to quantify monopoly power so that an appropriate action can be taken to regulate it.

In this unit, we will try to understand the factors that lead to monopoly power. We will also understand welfare impact of monopoly power. We will also discuss how the monopoly power can be measured to enable the government authorities to take appropriate policy measures.

7.2 MARKET POWER AND MONOPOLY MARKET

7.2.1 Meaning of Monoploy Power :

According to neo-classical economic thoughts, monopoly power of a seller is determined by two factors.

1. The degree of freedom the seller has in deciding his price.
2. The extent to which price exceeds marginal cost or the seller enjoys long-run abnormal profit.

Since the seller under perfect competition does not have freedom either to determine price or to charge price higher than marginal cost, there is no monopoly power present under such market. Greater or lower degrees of monopoly power may be present with monopoly or oligopoly markets. Under the monopoly market, even if there is a single seller, (who does not have any competitor at the moment) the monopolist may not be able to enjoy excess profit out of the fear of potential new competitors. Hence, existence of monopoly with single seller or oligopoly with a few sellers does not necessarily imply an existence of monopoly power.

7.2.2 Benefits of Monopoly Power

It is important, at this stage, to understand whether the cost conditions are likely to remain the same when a number of firms are combined to become a monopoly. In other words, it is necessary to analyse whether a monopoly firm (which produces on a large scale) faces similar cost conditions as compared to a number of competitive firms producing the same product (on a smaller scale). Two views need to be considered in this case:-

1. The monopolist can enjoy various economies of scale such as grater specialization, larger markets, cheaper finance, buying raw materials in bulk, spending more money on research and development, applying modern techniques of production and management, etc. All these will result in the fall in cost of production. If these economics of scale (leading to fall in cost production) are large enough leading to substantial fall in cost, the monopoly price may be smaller than that under perfect competition.

2. The monopolists can charge different price, different buyers, as he is the sole producer in the market. In other words, there is a possibility of price discrimination under monopoly and not so under perfect competition. The monopolist can maximize his profit by charging higher price from the market where his product faces inelastic demand and less price from the market where the demand for his product is highly elastic. The possibility of charging different

price also may promote social welfare as explained in the unit "Price Discrimination".

7.3 SOCIAL COST OF MONOPOLY POWER (WELFARE EFFECTS OF MONOPOLY POWER)

7.3.1 MONOPOLIES WITH THE COSTS HIGHER THAN COMPETITIVE MARKET.

As you are aware, under monopoly, the consumers will have to pay the price which is higher than the marginal cost and the monopolist enjoys supernormal profit at the expense of consumers. This result in two things.

a. Consumers face welfare loss as their consumer surplus declines. (because they have to pay higher price)
b. Producer gains (as he enjoys super normal profit)

In order to understand the welfare loss or gain under monopoly, the concept of consumer surplus can be sued. (It may be recalled that the consumer surplus is the difference between price the consumer is willing to pay and the price which he actually pays). Following diagram explains welfare gains or losses arising out of existence of monopoly power (When the costs under monopoly are higher than the costs under perfect competition).

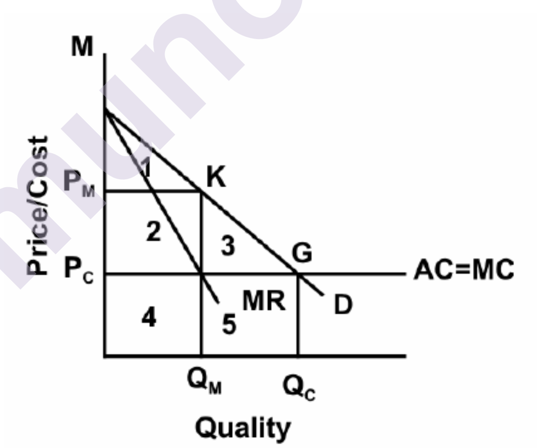

Figure 7.1 Welfare gains / losses from monopoly power

In the diagram 7.1

MD – demand curve for the product.

MR – Marginal Revenue Curve

AC – Average Cost

MC – Marginal Cost

Pc – Price under perfect competition. (The students may recall that the price under perfect competition is equal to marginal cost.

QCc - Output under perfect competition

MGPc - Consumer surplus (which is equal to the area under demand curve a006Ed above the price) under perfect competition.

The consumer surplus perfect competition. The consumer surplus arises because the consumers are willing to pay higher price (maximum OM in the case) but they are actually paying less (OPc)

P_M - Price under monopoly.
OP_M – Output produced by the monopolist (The monopolist equates marginal revenue and marginal cost to determine equilibrium price & quantity.)

It may be noted that for the monopoly market, the equilibrium price is higher and the equilibrium quantity is lower than that under perfect competition.

MKP_M – Consumer surplus under monopoly (which is less than that under perfect competition)

P_MK E P_C (Area 2) – excess profit earned by the monopolist.
At this point it may be noted that there is a fall in consumer surplus and a redistribution of income from consumers to producers (in the form of excess profit which was not so under the perfect competition).

P_M K E P_C – Gains to the producer (Area 2)
An important point to be noted here is that the loss to the consumer in the form of reduction in consumer surplus is more than the gains to the producers in the form of excess profit. Only a part of the loss to the consumers, is redistributed to the producer. Out of total loss to the consumer (Area 2 + 3), only a part (area 2) is the gain to the monopolist. The rest, shown by triangle KGEC (area 3) is called as the dead –weight loss. It arises due to inefficiency of resource allocation under monopoly. It is considered as the social cost of monopoly. In short, monopoly leads to misallocation of resources and hence there is a social cost of monopoly in the form of dead weight loss. The extent of welfare reduction depends upon the price- elasticity of demand for the product and the difference between monopoly and competitive prices.

Table 7.1 summaries the welfare implications of monopoly power.

Area in fig 1.1	Competitive market	monopoly
1	Consumer surplus	Consumer surplus
2	Consumer surplus	Excess profit
3	Consumer surplus	Dead-weight loss
4	Input costs	Input cost resources used some where else in the economy
5	Input costs	

The analysis in this section deals with the situation where the costs under monopoly are higher than those under the prefect competition. But this may not always be the case. Next section deals with the situation where the monopolist produces at the cost lower than that under perfect competition.

7.3.2 MONOPOLIES WITH COST LOWER THAN THE COMPETITIVE MARKETS

Sometimes, the monopoly firms may be able to produce at the cost lower than that under competitive market. This may be so because of the economic of scale that may be enjoyed by the big size monopoly firm or due to an easier access to superior technology as compared to the competitive firm, etc. Under such a situation, it is possible that the welfare gains associated with more efficiency (Production at a lower cost) may compensate for the dead-weight loss as shown in the following figure.

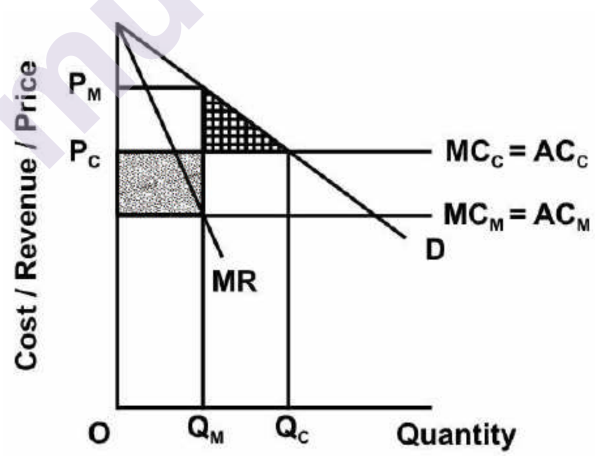

Figure 7.2 :
Gains and losses for a monopoly firm with lower costs.

Figure 7.2 depicts a situation where costs are lower (MCM) under monopoly as compared to the costs under perfect competition (MCPC). Price under perfect competition (OPc) is lower than that under monopoly (OPM). it implies that the monopoly firms enjoy super-normal profits. As explained in the earlier diagram

(7.1), monopoly market faces a deadweight loss equal to the shaded triangle in the figure. The monopolist makes abnormal profits, but in this case, the profits are due to lower costs than the higher price. The cost reduction arised out of various factors mentioned earlier, economies on the use of resources which can be allocated to some other lines of production. These production gains shown by the shaded area, more than compensate the dead-weight loss and hence lead to overall improvement in welfare. Thus, in spite of existence of monopoly power, in spite of existence of monopoly power, in spite of market concentration, the welfare improvement will take place. To conclude, the monopolies may or may not reduce welfare. It would depend on whether and to the extent to which their costs are higher or lower than that in the competitive industry.

Check Your Progress :

1. Define following terms
   a) Monopoly power
   b) Economies of scale
   c) Price discrimination
   d) Social cost of monopoly power
   e) Consumer surplus
   f) Dead-weight loss

7.4 MEASUREMENT OF MONOPOLY POWER

The degree of monopoly power is measured by taking perfect competition as a base, professor A. P. Learner has regarded perfect competition as the market providing socially optimum (maximum) welfare. Any deviation from perfect competition implies an existence of monopoly power, according to him.

Under perfect competition, price is equal to marginal cost at the equilibrium level. The level of output associated with equilibrium price implies optimum allocation of resources. When the degree of competition is less than perfect, i.e. under the imperfect market, the demand curve is downward sloping and price is not equal to marginal cost. The divergence between price and marginal cost is an indicator of the existence of monopoly power, according to Prof. Lerner. Greater. The divergence between the price and marginal cost, higher is the monopoly power enjoyed by the seller, symbolically.

Degree of monopoly $= \frac{P - MC}{P}$

Where P – is equilibrium price.

MC - Marginal cost at the equilibrium level of output.

Under perfect competition, difference between marginal cost and price is zero so

Degree of monopoly $= \frac{P - MC}{P} = \frac{O}{P} = O$

There is an absence of monopoly power under perfect competition. Greater the value of the index $\left[\frac{P - MC}{P}\right]$, the greater is the degree of monopoly power possessed by the seller.

Lerner's Measure of monopoly power is criticized on the following grounds-

1. This measure is not useful in the market where there is non-price competition or product differentiation. Such as under the monopolistic competition. In other words, when the products compete with each other, not in terms of price, but in terms of product variation, advertising, or any other sales promotion practices, the above-mentioned formula can not be used to measure the degree of monopoly power.

2. Another important point of criticism against Lerner's measure of monopoly power is that, this measure is based on only one aspect of monopoly and that is the control over prices. The degree of control over prices depends on the availability of existing substitutes. But the monopoly power may also be threatened by potential substitute which is not considered by this measure.

7.5 MONOPSONY AND BACKWARD INTEGRATION

The term monopsony is opposite of the term monopoly. Whereas, monopoly refers to a condition or activity in the seller's market, monopsony is a seller's market, monopsony is a condition or activity in the buyer's market. In the recent times, the issues arising out of monopsony are gaining prominence in the developed markets the buying power of the supermarkets and other retail chains has been increasing. Hence it is necessary to examine consequences of growing monopsony power through consolidation mergers and the buyer's groups. The monopsony may also bring about wholesale price changer.

According to K. Lancater, monopsony is the economic term, used to describe a market involving a buyer with sufficient market power to exclude competitors and affect the price paid for its products. Monopsony in the buyer's market is the counter part of monopoly in the seller's market. Monopsony will generally exist when there is a corresponsing monopoly in the seller's market since. All the firms in the market generally need to purchase similar products. Thus, if monopoly is held in the output market, the monopolist will generally hold maonopsony power in the input market. Vertical integration normally involves a producer's integration into next level of production. That means, a producer may himself, take over distribution of his product. This is forward integration. Backward integration on the other hand, occurs when the producer seeks to integrate into his supply market that means, the producer may himself, take over supply of inputs for his firm. A firm generally uses monopoly power to have forward linkages and monopsony power to have the backward linkages.

7.6 QUESTIONS

1. Explain in detail the social cost incurred due to existence of monopoly power.
2. What are the benefits of monopoly power?
3. Explain the concept of dead-weight loss under monopoly with the help of a diagram.
4. How is the market power measured?
5. What is monopsony? Why does it come into existence?
6. Explain the concept of backward and forward linkages and existence of market power.

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Unit-8

PRICE DISCRIMINATION UNDER MONOPOLY

Unit Structure :

8.0 OBJECTIVES

After going through this unit you will be able to explain the concepts of -

- Monopoly.
- Price Discrimination.
- Monopsony.
- Unions as monopoly input suppliers.
- Bilateral Monopoly.
- First, Second and Third degree Price discrimination.
- Price and output determination under monopoly, price discriminating monopoly and monopsony.

8.1 INTRODUCTION

This unit is designed to explain the concept of monopoly, price discriminating monopoly, monopsony and bilateral monopoly. This unit especially deals with the most important decision of price and output determination under monopoly, monopsony and bilateral monopoly.

8.2 PRICE AND OUTPUT DETERMINATION UNDER MONOPOLY

The monopoly firm is assumed to maximize profit in a stable, known environment, with given technology and market conditions We assume diminishing marginal productivity and so, in the presence of fixed inputs, the firms average and marginal costs will at some point begin to rise with the rate of output per unit time. However, we no longer assume that diminishing returns to scale set in at some point : We leave the question open, and permit any one of increasing constant, or diminishing return to scale to exist over the range of outputs we are concerned with. The essential difference from the competitive model is the assumption that the firm faces a downward sloping demand curve. We curie its demand function in the inverse form.

$$p = D(q) \quad dp/dq < 0 \qquad [8.1]$$

where p is price, q is output per unit time and D is the demand function. We do not place restrictions on the second derivative the function, but do require its first derivative to be negative.

The firms total cost function is

$$c = c(q) \quad c(q) > 0 \qquad [8.2]$$

where c is total cost per unit time. Maxginal cost is always positive, but we do not place restriction on the second derivative, the slope of the marginal cost curve. The profit function of the firm

$$\pi(q) = pq - c(q) \qquad [8.3]$$

where TI is profit per unit time we assume that the profit maximizing output q^* is positive Hence q^* satisfies the conditions.

$$\pi^1(q) = p + qdp/dq - c^1(q) = 0 \qquad [8.4]$$

$$\pi^{11}(q) = 2dp/dq + qd^2p/dq^2 - c^{11}(q) < 0 \qquad [8.5]$$

where [8.4] is the first order and [8.5] the second order condition. The term $(p + qdp/dq)$ is the derivative of total revenue pq with respect to q (taking account 01 [8.1], and is marginal revencue. Thus, [8.4] expresses the condition of equality of marginal cost with marginal revenue. The term $(2dp/dq + qd^2p/dq^2)$ is the derivative of marginal revenue with respect to output and so [8.5] is the condition that the slope of the marginal cost curve must exceed that of the marginal revenue urve at the optimal point. If marginal costs are increasing with output while, by assumption, marginal revenue is diminishing with output [7.5] will necessarily be satisfied since in that case

$$c^{11}(q) > 0 > 2dp/dq + qd^2p/dq^2 \quad [8.6]$$

However, unlike the competitive case, the second-order condition may also be satisfied if $c^{11}(q) < 0$.

More insight into this solution can be gained if we write marginal revenues, MR, as

$$MR = p\left(1 + (q/p)\,dp/dq\right) \quad [8.7]$$

Given the definition of the elasticity of demand

$$e = p\left(dq/dp\right)/q < 0 \quad [8.8]$$

We can write as the relationship between demand elasticity and marginal revenue:

$$MR = P(1 + 1/e) \quad [8.9]$$

Clearly, $e < -1 \Rightarrow MR > 0$ while $e = -1 \Rightarrow Mr = 0,$ and $e > -1 \Rightarrow MR < 0$ combining [8.9] with [8.4], we can write the condition for optimal output as

$$p(1 + 1/e) = c^1(q) = MC \quad [8.10]$$

This equation then establishes immediately the two proposition:

(a) the monopolist's chosen price always exceeds marginal cost since its price elasticity is finite;

(b) optimal output is always at a point on the demand curve at which e < - 1 (given that $c^1(q) > 0$)

Under competition each firm equates marginal cost to price. Hence the extent of the divergence of price from marginal cost under monopoly is often regarded as a measure of the degree of monopoly power enjoyed by the seller. From [8.10]

$$\frac{P-MC}{P}=\frac{-1}{e}-\infty<e<-1 \qquad [8.11]$$

The left - hand side, the price marginal cost difference expressed as a proportion of the price, is the Lerner index of monopoly power. Thus, as $e \rightarrow -\infty$ (the competitive case) monopoly power tends to zero.

The equilibrium position of the firm implied by its choice of output q^* satisfying the above conditions is illustrated in fig [8.1]. In (a) of the figure, the demand curve is D(9) and the corresponding marginal revenue curve is MR. Given the marginal and average cost curves $c^1(q)$ and AC, profit maximizing output is at q^*. Since this must be sold at a market clearing price. choice of q^* requires the prices $p^*=D\left(q^*\right)$

p_1^*

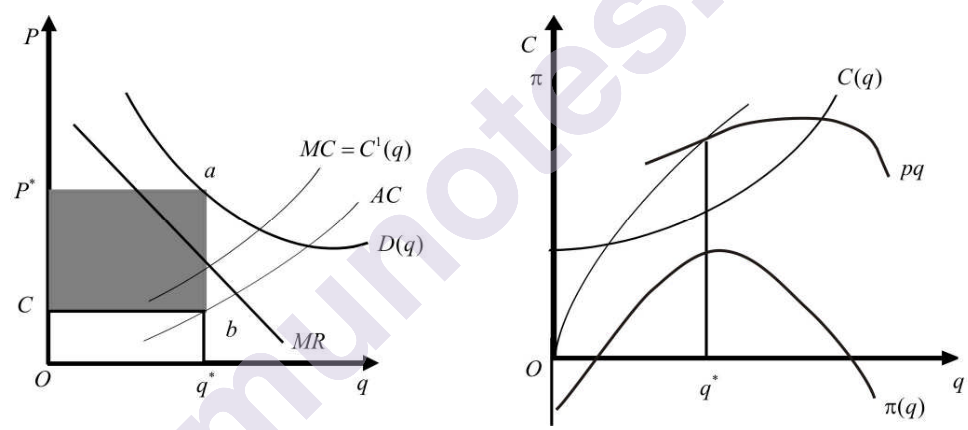

Fig 8.1

We could therefore regard the equilibrium position as a choice either of profit maximizing price p^* or of output q^*, since each implies the other. At output q^*, profit is the difference between total revenue $p^* q^*$ and total cost AC q^* and is shown by the areas p^* above in Fig [7.1] (a). In (b) of the figure, the some equilibrium position is shown in terms of total revenue curve is denoted pa, and its slope at any point measures marginal revenue at that output. Its concave shape reflects the assumption of diminishing marginal revenue. The total cost curve is denoted c (q), and its convex shape reflect the assumption of increasing marginal cost. The total profit function is the vertical difference between these two curves, and is shown as the curve TI (q) in the figure. The maximum of this curve occur at the output q^*, which is also the point at which the tangents to the total revenue and total costs curves respectively are parallel, i.e marginal revenue is equal to marginal cast.

The supernormal profit, i.e. profit in excess of all opportunity costs (including a market determined rate of return on capital which enters into determination of the average and marginal cost avrves), is given by the area $q^*\left(p^*-c\right)$. It can be imputed as a rent to whatever property right confers the monopoly power and prevents the new entry which would compete the profits away. It may be that this right is owned by on individual who leases it to the firm. If the supplied is rational and well informed, she will bid up the price of the base so as just to absorb the supernormal profit, and so the rent is transformed into an opportunity cost of the monopolist. This would be true, for example, if the monopolist rented a party clearly Favorable location. It the monopolist owns the property rights, then he can impute the profits as the return on this property right. Note that the identity of the owners of the right does not affect the price and output which will be set by the monopolist. (Since this is determined by the desire to maximize profit) but simply determines the division of the spoils note also that the term 'property right' is used here in its widest possible sense: it is meant to indude the ownership not only of land but also of such things as brand names, public reputations, manual rights, franchises and patents

8.3 PRICE DISCRIMINATION

Price discrimination exists when different buyers are charged different prices for the same good. It is a practice which could not prevail in a competitive market because of arbitrage. Those buying at lower prices would resell to those offered higher prices and so a seller would not gain from discrimination. Its presence therefore suggests imperfection of competition. The producers charge different prices for the same product, from different units of the same product at different prices is called price discrimination. By doing this, the producer tries to capture more & more consumer surplus, from the buyer to maximize his profit. Price discriminating is followed by grouping the consumers. In some cases, forming groups of consumers is easier as the heterogeneity (differences) of consumers is directly observable. In some cases, however the differences among consumers are not visible. In such cases, the producers have to offer different menus or packages of products at different prices and allow consumers to choose from among alternative choice. Thus, price discrimination refers to the act of manufacturer of selling the same product at different prices to different buyers.

8.3.1 Examples of Price Discrimination:

Monopolist firm many times charges different prices from different consumers, for the same product and without much cost differentials. Following examples would clarify the point.

1) A doctor or a lawyer may charge different fees from different patients / clients for the similar services.
2) A producer may charge different price for the same product at different parts of a country.
3) Same product / services may be sold to the same buyer at different price for varying quantity. For example price for 10 kg of rice may be higher than the price for 100 kg of rice of the same quality.
4) Consumers may be classified into different categories and by changing the quality of services, different rates may be charged. For example first class and second class fares in the train, ordinary or business class in the plane.

All these examples make it clear that price discrimination is quite common while aiming at maximization of profit. Prof. Stigler's definition of price discrimination brings about one more aspect of the concept. According to Prof. Stigler "Price discrimination is defined as the sale of technically similar products at prices which are not proportional to marginal costs."

The concept of price discrimination as indicated in Prof. Stigler's definition may be well understood with the help of following example.

Suppose a hard-bound and color edition of Microeconomics book by Mankiv costs Rs.500 and a soft-bound block and white edition of the same book costs Rs.400 for the publisher. Also suppose that he sells the colored edition for Rs.750 and the black and white edition for Rs.500. in this example, the manufacturer is said to be practicing price discrimination as the price differences between two types of books (750-500) are more than the cost differences (500-400).

8.3.2 POSSIBILITY OF PRICE DISCRIMINATION:

A monopolist can follow price discrimination only under two fundamental conditions.

1) There should be no possibility of transforming any unit of product from one market to the other transferability of commodities.
2) There should be no possibility of buyers transferring themselves from the expensive market to the cheaper market transferability of demand. It is understand from the above points that the monopolist can practice price discrimination only if the units of goods or the units of demand (i.e. the buyers) cannot be transferred from one

market to another. It is possible to discriminate among the buyers only if two types of arbitrage or transaction costs are present.

These are as follows.

1) Arbitrage associated with the transferability of a commodity.
If it is possible to transfer a commodity from one person to another with very less transaction cost, price discrimination is not possible. In other words, price discrimination becomes possible only when it is costlier to resell a product to another consumer may buy goods to resell them to the high price consumer. A low price consumer is the one who gets a commodity at a lower price due to quantity discounts. A high price consumer the one who does such discounts, in case the transaction cost is low, the former (low-price) consumer will buy in bulk and sell it to the latter (high-price) consumer. This does not allow the discriminating monopolist to charge different prices from different consumers. Thus, it is possible to undertake price discrimination in case of services which have a very low or no transferability. For example, doctor can charge discriminating prices to one patient to another. Otherwise, in case of most of the retail products, price discrimination may be difficult. Wherever it is difficult, the monopolist may practice partial discrimination. He may sell his products to retailer at lower price and ensure that his product is sold to the final consumer.

2) Arbitrage associated with the transferability of demand.
In case of such arbitrage, the products physically may not be transferable between the consumers, but the demand for product is transferred between different packages. For example, the consumer may be charged different prices based on price-quantity package or price-quality package makes. The price-quantity package makes consumer choose between say buying two units of a product at certain price or buying one unit of a product at some price. A shirt may cost Rs.250 but if consumer buys two shirts he may be charged Rs.450 (instead of Rs.500). The price-quality package discriminates between the consumers on the basis of quality of a product / service. First class and second class on the train, ordinary or deluxe room accommodation in a hotel are some of the examples of price discrimination which help the monopolist to maximize his profit by charging different price from different consumer.

The two types of arbitrage discussed above are different in terms of their impact on price discrimination. As stated earlier, if there is a possibility of transferring products from one consumer to another, without much arbitrage or transaction cost, one consumer may buy more goods and resell them to the others. Monopolist will not be able to gain from price discrimination in such a case. The transferability of demand, on the other hand, includes the monopolist to discriminate among the consumer by charging different prices from them. In the sections to follow we will try to analyze the welfare effects of price discrimination.

8.3.3 Concluding Remarks:

From the discussion about the possibility and practicality of price discrimination, it is clear that there should not be any seepage or communication between two markets. Thus, price discrimination depends upon the ability of the monopolist to keep two markets quite separate. To conclude, price discrimination is possible under following circumstances:-

1) The nature of product sold is such that there is no possibility of transferring product / service from one market to another.
2) The geographical distance between two markets is very large or the markets are separated by the tariff barriers.
3) Legal section is given to charge different prices from different consumers like electricity for domestic use and for industrial use.
4) Consumer snobbish attitude that higher priced goals are superior to lower priced ones.
5) Monopolistic or oligopolistic market structure.

Check Your Progress:

1) Define the concept price discrimination
2) State the conditions to be followed by a monopoly for price discrimination.
3) Explain the concept of arbitrage and state how it is important factor determining price discrimination.
4) Find out more examples of price quantity and price quality packages offered by the monopolist to discriminate against different consumers.
5) What is the difference between arbitrage associated with commodity transfer and the arbitrage associated with demand transfer?

8.4 FIRST DEGREE DISCRIMINATION

Under third-degree price discrimination the monopolist had some information he could use to partition buyers into sub markets and prevent arbitrage between the sub-markets. This, as the name suggests, is in contrast to.

(a) first degree price discrimination, where the monopolist is able to identify the demand of each individual buyer and prevent arbitrage among all buyers;

(b) Second-degree price discrimination, where the monopolist knows the demand characteristics of buyers in general, but does not know which buyer has which characteristics.

In first degree price discrimination the monopolist can extract all the consumer surplus of each buyer. An interesting aspect of this case is that total output of the good is at the level at which each buyers pays a price equal to marginal cost and monopoly does not distort the allocation of resources. We have a pareto efficient outcome, although the monopolist expropriates all the gains from trade. Any objection to monopoly in this case therefore would have to be on grounds of equity fairness of the income distribution rather than efficiency.

In the second case, the obstacle to price discrimination is that, if one type of buyer is offered a more fausurable price. quantity deal than other types, and the monopoly is not able to identity a buyer's type, then all buyers will take the most fauourable deal. The solution for the monopolist is to offer alternative deals which satisfy a self-selection constraint: a given deal will be preferred to all other by, and only by, the type for which it is designed.

In the rest of this section we explore first-and second-degree price discrimination with a simple model. We assume:

(a) two types of buyer in the market, with n_1 buyers of the first type and n_2 buyers of the second.

(b) a buyers type is determined by her preferences which for each type of buyer can be represented by the quasi linear form.

$$u_i = \bigcup_i (x_i) + y_i \quad i = 1, 2 \qquad [8.14]$$

where x_i is the monopolized good and y_i is a composite commodity representing all other goods;

(c) type 2 buyers have a stronger preference for the good in the sense that for any x

$$MRS_{xy}^2 = \mho_{2x} / u_{2y} = \mho_2^1(x) > \mho_1^1 = \mho_{1x} / \mho_{1y} = MRS_{xy}^1 > 0 \qquad [8.15]$$

$\mho_i(o) = 0$ and $\mho_i^{11}(x) < 0$: buyers have diminishing marginal utility;

(e) the buyers have identical incomes M, and the price of the composite commodity is the some for all consumers and is set at unity. So if $x_1 = x_2 = o$, then $y_1 = y_2 = M$;

Recall, that a quasi-linear utility function implies that a consumers indifference curves in the x, y plane are vertically parcallel, and there is a zero income effect for good x. The consumer's choice problem is

$$\max_{x_1 y_1} \mho_i(x_i) + y_i \; St \; px_i + y_i = M - F \qquad [8.16]$$

p is the price the monopolist charges, and $F \geq 0$ is fixed charge that the monopolist may set for the right to buy the good at price p (examples of such fixed charges are telephone rentals, entrance charges to amusement parks, subscription fees to a book or wine club).

First-order conditions include.

$$\mho_i^1 - \lambda p = 0 \qquad [8.17]$$

$$1 - \lambda = 0 \qquad [8.18]$$

Hence $\mho_i^1(x) = \lambda p = p$, yielding demand functions

$$x_i = \mho_i^{1-1}(p) = x_i(p) \qquad [8.19]$$

$$y_i = M - F - Px_i(p) \qquad [8.20]$$

The indirect utility function is

$$v_i(P, F) = \mho_i(x_i(p)) + M - F - Px_i(P) \qquad [8.21]$$

of particular interest are the derivatives

$$\frac{\partial v_i}{\partial P} = \mho_i^1 x_i^1 - (x_i + Px_i^1) = x_i, \frac{\partial V_i}{\partial F} = -1 \qquad [8.22]$$

where the result for $\partial v_i / \partial p$ is simply Roy's identity. In fig u.3, we show the reservation indifference curves $\bar{u}_i$ for each of the two typesof consumers. Since they have the same income M, they are at the same point when consuming no x, but assumption (c) implies that a type 2 indifference curve is steeper than that of a type 1 at every x (since $MRS_{xy} = -dy_i / dx_i = \mho_i^1$. The budget line market c in the figure corresponds to p=c, so that x_i^c are the respective consumer's.

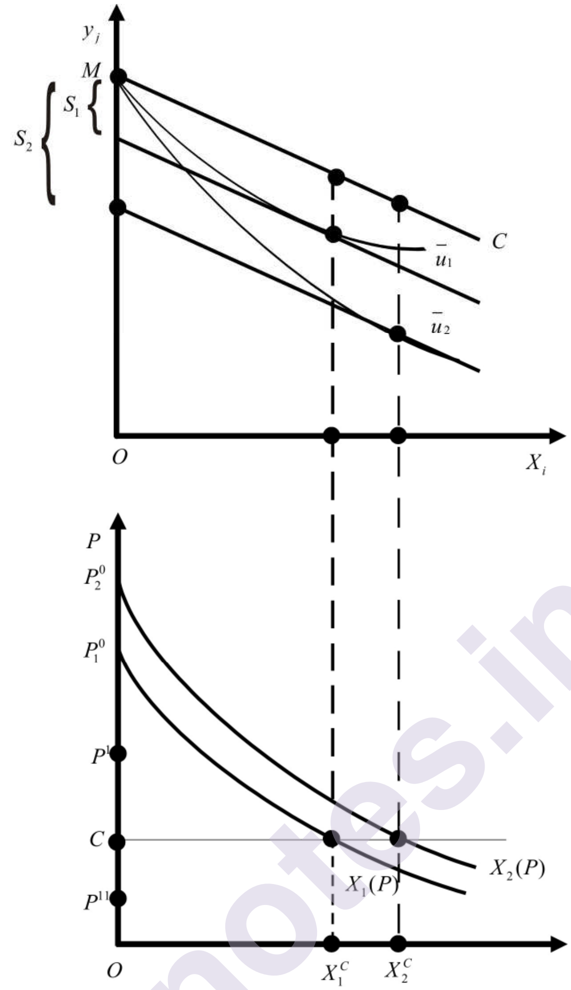

Fig 8.2

demands at that price. In (b) of the figure we show the demand curves derived from these reservation indifference were. Because of the quasi-linearity assumption, these are both Hicksian and Marshallian demand curves, and the area under each between prices p_i^o and p = c gives the type's compensating variation, or maximum uillingness to pay for the right to buy x at price C. These consumer surpluses are denoted by s_i, and correspond to the distances on the y axis shown in (a) of the figure.

We now show that under first degree price discrimination the monopalist's optimal policy is to set a price for each type equal to c, and to set a fixed charge $F_i = S_i\left(i = 1, 2\right)$. The monopolist sells at marginal cost and sets separate fixed charges equal to the total willingness of each type to pay. This requires first that he knows the type of each buyers, and so can prevent a type 2 buyer taking advantage of the lower type 1 fixed charge second he must be able to prevent arbitrage and stop a type 1 buyer reselling to a type 2 buyer at some price between c and $F_2 / x_2^c + c$, which is the average price per unit a type 2 buyer pays in this solution.

The idea underlying this policy can be seen in fig 7.3(b). If the monopolist sets p=c to both types and extracts the total surplus his profit is $s_1 + s_2$. If he sets a higher price, say $P^1 > C$, although he makes a profit on each unit he sells, the sum of these profits and the remaining consumer surpluses is less than $s_1 + s_2$ by the sum of the two shaded triangles. It pays him to expand output and lower price as long as p > c because his own profit increases precisely by the difference p - c, which he can recover through the fixed charge. He will not set a price such as $p^{11} < c$, because the extra surplus he can recover falls short of the extra cost he incures. And clearly it would never be worth while to set o fixed charge $F_1 > S_i$ for only p, because then he sells nothing to type i.

We can derive this result more formally. The monopolist's total profit is

$$\Pi = n_1\left[p_1 x_1\left(p_1\right) + F_1\right] + n_2\left[p_2 x_2\left(p_2\right) + F_2\right] - C\left[n_1 x_1\left(p_1\right) + n_2 x_2\left(p_2\right)\right] \quad [8.23]$$

He must not offer a deal which is curse for each consumer than not buying the good at all. We can express this by the reservation constraints

$$V_i\left(P_i, F_i\right) \geq \bar{u}_i \quad i = 1, 2 \qquad [8.24]$$

where, recall, $\bar{u}_i$ is the utility is obtains by buying non of good x. with β_i as the lagrange multiplier on these constraints, optimal $p_i \; and \; F_i$ are defined by (see Appendix H)

$$n_i\left(x_i + p_i x_i^1 - \left(x_i^1\right)\right) + \beta_i \partial v_i / \partial p_i = 0 \;\; i = 1, 2 \qquad [8.25]$$

$$n_i + \beta_i \, \partial v_i / \partial Fi = o \quad i = 1, 2 \qquad [8.26]$$

$$v_i\left(p_i, F_i\right) \geq \bar{u}_i, \beta_i \geq 0, \beta_i\left[v_i - \bar{u}_i\right] = 0 \;\; \iota = 1, 2 \qquad [8.27]$$

From [8.26] we see that non-zero n_i and $\partial v_i / \partial F_i$ imply $\beta_i > 0$ and so [8.27] implies $v_i = \bar{u}_i$. Both types of consumers receive only their reservation utilities. Then using [8.22] and [8.26] we have $\beta_i = n_i$ and

$$n_i C x_i + P_i x_i^1 - (x_i^1) - n_i x_i = 0 \quad [8.28]$$

implying

$$p_i = c \quad [8.29]$$

The value of F_1 then satisfies $v_i(c, F_i) = \bar{u}_i$ and so must be equal to consumer surplus s_i at price c.

We could interpret third degree price discrimination (analyses in the first part of this section) as the case in which the monopolist can identify each buyer's type and prevent arbitrage between types, but for some reason cannot set fixed charges. He must set a constant price per unit to all buyers of a given type. Then, profit maximization implies a price to each type which is above marginal cost; as we saw earlier. clearly, the monopolist's profits are leuer than under first degree price discrimination. Buyers are better off under third degree price discrimination since, although they face a higher price and so consumer less, they retain some consumer surplus and are on an indifference curve that must be higher than there reservation indifference curve. (use fig 8.3.)

8.5 SECOND DEGREE PRICE DISCRIMINATION

In the case of second degree price discrimination, the monopolist is unable to determine the type of the buyer before she has purchased the good. In that case if he offered only buyer the option of either $(C, S_1)\, or\, (C, S_2)$ every type 2 buyer (a) will as every type 1 buyer) would chose (C, S_1) can the monopolist do better than this by offering options chosen so that only o buyer of type I would want to choose the option designed for her type? In other words, can the monopolist do better by inducing buyers to reveal their type by self-selecting the appropriate deal?

Assume that the monopolist knows the number of buyers of each type, n_i and can specify in a contract both the quantity of output he will supply to a buyer and the total charge for that output. That is, a contract is a pair (x_i, F_i). This implies a price per unit $p_i = F_i / x_i$ and the contract could be equivalently expressed as some combination of a fixed charge and constant price per unit, as in a two-part tariff. The point is that the consumer is offered a quantity and a fixed charge and not a price and a fixed charge. We shall set the reason for this at the end of the following analysis.

The monopolist's profit is

$$\Pi = \sum_{i=1}^{2} n_i (F_i - x_i) \quad [8.30]$$

We again have the reservation constraint, since buyers always have the option of refusing a contract. These are now written in terms of direct utilities, to reflect the fact that quantities are being specified.

$$\mho_i(x_i) + M - F_i \geq \bar{u}_i \qquad \iota = 1, 2 \qquad [8.31]$$

where we use the fact that $y_i = M - F_i$ There are also self - selection constraint which ensure that each type chooses the appropriate deal we write these as

$$\mho_1(x_1) - F_1 \geq \mho_1(x_2) - F_2 \qquad [8.32]$$

$$\mho_2(x_2) - F_2 \geq \mho_2(x_1) - F \qquad [8.33]$$

(M cancels out in these expressions)

If (x_i, F_i) satisfies these constraints, it will only be accepted by type i. (We assume to be able to have a closed feasible set, that if a buyer is indifferent between the two deals she takes the one appropriate to her type.)

In principle we now solve for x_i, F_i by maximizing Π subject to [8.31] - [8.33]. However the first order condition for this would not be instructive. Instead, we first show that, in any optimal solution, (a) the reservation constraint for a type 2 buyer, and (b) the self-selection constraint for a type 1 buyer are non-binding. They can be dropped from the problem thus simplifying the derivation of the optimal contract.

We show this in Fig (8.4), which reproduces the reservation indifference curves from Fig (8.3) (a)

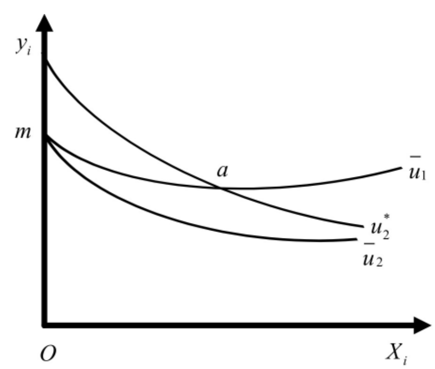

Fig 8.3

(a) Type 2 buyers must be offered $(x_{21}F_2)$ such that $u_2 > \overline{u_2}$. To see that, note that type 1 buyers must be offered a contract (x_1, F_1) that puts them on or above $\overline{u_1}$. But since $\overline{u_1}$ lies above $\overline{u_2}$, such a deal must always be better for type 2 bayers than any contract $(x_2 F_2)$ that puts them on $\overline{u_2}$. So only a point above $\overline{u_2}$ can satisfy their. Self-selection constraint.

(b) Type 1 buyers will always strictly prefer their deal to that offered to type 2 quyers, in an optimal solution. Suppose the optimal deal offered to type 1 buyers is at a in fig 8.3 (it is not relevant to the present argument that a is on $\overline{u_1}$, but me show below that this must be so). Then the deal offered to type 2 buyers must lie on the type 2 indifference curve passing through a, labeled u_2^*. If it were below this, type 2 buyers would prefer a; if above, the monopolist is being need generous to type 2 buyers because, at any given x_2, he could increase F_2 move verticallur derunuared in the figure) without violating either the reservation or self-selection constraints. (This incidentally established that the self-selection constraint for type 2 buyers is strictly binding, as we verify later.) Now if the deal offered to type 2 buyers were on u_2^* at a point to the left of a, it would be preferred to a by type 1 buyers and this violates the self-selection constraint on type 1. It is easy to show that point a it self could not be offered to both types of buyers in equilibrium.

This laves only points on u_2^* to the right of a as possible deals to be offered to type 2 buyers, and since these must be strictly below $\overline{u_1}$ the type. 1 self selection constraint is non-binding. This argument also establishes that at an optimum $x_2 > x_1$.

As a result of these arguments, the monopolist's problem is to find $(x_1, F_1), (x_2, F_2)$ to maximize Π in 7.30 subject only to 7.31 unit I - 1, and 7.33 using β_1 and u_2 for the Lagrange multipliers on 7.31 and 7.33, the first order conditions are,

$$-n_1 c + \beta_1 \mho_1 \left(x_1^*\right) - \mu_2 \mho_2 \left(x_1^*\right) = 0 \qquad [8.34]$$

$$-n_2 C + \mu_2 \mho_2^1 \left(x_2^*\right) = 0 \qquad [8.35]$$

$$n_1 - \beta_1 + \mu_2 = 0 \qquad [8.36]$$

$$n_2 - \mu_2 = 0 \qquad [8.37]$$

$$\mho_1 \left(x_1^*\right) + M - F_1^* - \bar{\mu} \geq 0,\ p_1 \geq 0,\ \beta_1 \left[\mho_1 + M - F_1^* - \overline{\mu_1}\right] = 0 \qquad [8.38]$$

$$\mho_2 \left(x_2^*\right) - F_2^* - \mho_2 \left(x_1^*\right) + F_1^* \geq 0,\ \mu_2 \geq 0,\ \mu_2 \left[\mho_2 - F_2^* - \mho_2 + F_1^*\right] = 0 \qquad [8.39]$$

From [8.37] and [8.38] we see that the type 2 self-selection constraint must bind, and from [8.36] and [8.38] that the type 1 reservation constraint must bind. Substituting for μ_2 in [8.35] gives.

$$\mho_2^1\left(x_2^*\right)=C \qquad [8.40]$$

implying $x_2^*=x_2^c$, o that type 2 consummation is exactly that under first degree price discrimination. Then, substituting for $\beta_1 \; and \; \mu_2$ in [8.34] gives

$$\mho_1^1\left(x_1^*\right)=\frac{n_1 c}{n_1+n_2}+\frac{n_2}{n_1+n_2}\mho_2^1\left(x_1^*\right) \qquad [8.41]$$

Recall that we established in fig 8.4 that we must have $x_2^*>x_1^*$, so that $\mho_2\left(x_1^*\right)>\mho_2^1\left(x_2^*\right)=c$, given diminishing marginal utility. Thus, writing $\mho_2^1\left(x_1^*\right)\equiv c+\delta$ where $\delta > 0$ we have

$$\mho_1^i\left(x_1^*\right)=c+\frac{n_2\,\delta}{n_1+n_2} \qquad [8.42]$$

implying that $x_1^*<x_1^c$, so that type buyers consume less than under first degree price discrimination. the optimal values F_1^* and F_2^* then follow from soluing the constraints as equalities with the optimal x_i^* inserted. We know that F_1^* will have type 1 buyers with their reservation utilities, while F_2^* is such that type 2 buyer retain some consumer surplus. It follows that, compared with first degree price discrimination, type 1 buyers are neither better nor curse off, type 2 buyers are better off, and the monopoly makes less profit.

The optimal second-degree price discrimination equilibrium is illustrated in fig 8.4. The contracts are $\left(x_1^*, F_1^*\right)$ and $\left(x_2^c, F_2^*\right)$ The two most interesting aspects of the solution are first that $x_1^*<x_1^c$, and second that $x_2^*=x_2^c$. These can be rationalized as follows. At any x_1, the total net surplus can be expropriated from type 1 buyers since they can be held to their reservation constraint. Suppose x_1 where set at x_1^c. The contract for type 2 buyers would have to be a point on the indifference were $\overline{\mu_2}$, as show in fig 7.5. Now consider a small reduction in $x_1 \; from \; x_1^c$. Since at x_1^c in a change in net surplus from type 1 buyers of just about zero on the other hand, it permits a downward shift in the indifference curve on which type 2

buyers can be placed, and at any x_2 this results in a strictly positive gain in net surplus to the monopolist. Thus it pays to reduce x_1 below x_1^c of course, for further reductions in x_1 the monopolist will lose some net sueplus from type 1 buyers, but this must be traded off against the gain in surplus from type 2 buiyers, and the optimum x_1^*, just balance these at the margin.

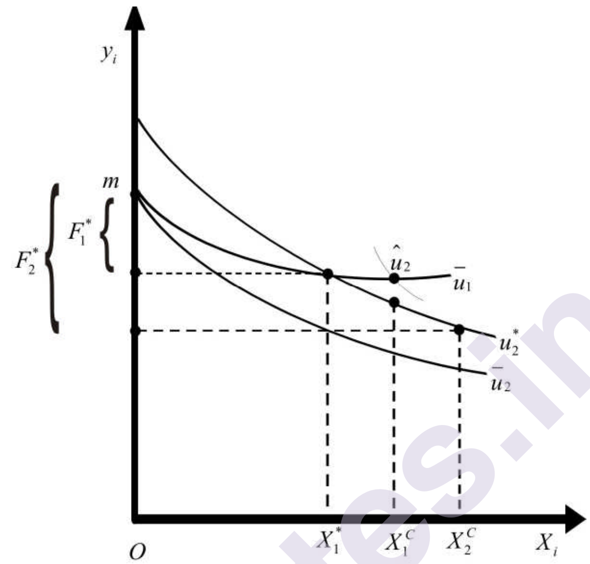

Figure 8.4

To see why $x_2^* = x_2^c$, note that it pays the monopolist to maximize the net surplus of type 2 buyers with respect to outputs since this then maximizes the value of F_2 that can be set, subject to the constraint that type 2 buyers would not prefer the type 1 contract.

There is a qualification to the condition in 8.42. note that as n_1 falls, given n_2, x_2^* must also fall. It is then possible, for suitably small n_1, that 8.42 satisfied cannot be satisfied for any $x_1 > 0,$ in which case F_1 is set sufficiently high that no type 1 buyers enter the market. The monopolist then knows that the only buyers in the market are of type 2, and so he can extract all their consumers surplus, with $F_2^* = s_2$. In terms of fig 8.5, μ_2^* becomes $\overline{\mu_2}$. The intuitive explanation is that, when the proportion of type 1 buyers is sufficiently small, the loss in total profit from reducing x_1 and the corresponding extracted surplus, is small relative to the gain from being able to extract more surplus from type 2 buyers. The equilibrium position in Fig 8.5 depends on the proportions of buyers of the two types as well as on the shapes of the indifference accrues and the value of c.

The importance of the specification of quantities in the contract can be seen if we consider the two part tariffs implied by the implied by the equilibrium in fig 8.5. If type 1 buyers took a contract in which they paid a fixed charge c_1^* and then a price per unit of $p_1^* = \overline{\mu_1^i}\left(x_1^*\right)$, then they would chose consumption x_1^* and pay precisely $c_1^* + p_1^* x_1^* = F_1^*$ likewise, if type 2 buyers were set a fixed charge c_2^* and paid a price per unit $p_2^* = \mho_2^1\left(x_2^c\right) = c$ then they would chose to consumer x_2^c and pay in total. $F_2^* = C_2^* + Cx_2^c$. If the monopolist made these contracts available to all buyers and did not restrict the quantity that could be bought, fig 8.5 should that the self selection constraint would be violated. Type 2 buyers would clearly choose a type 1 contract, which would dominate the contract $\left(x_2^c, F_2^*\right)$ although type 1 buyers still prefer their own contract. On the other land, if the monopolist specified contracts of the form a fixed charge c_1^* and a price per unit p^*, up to a maximum of x_1^* units of consumption, or a fixed charge c_2^* and a price of c for any amount of consumption, then the self's election constraints would continue to hold.

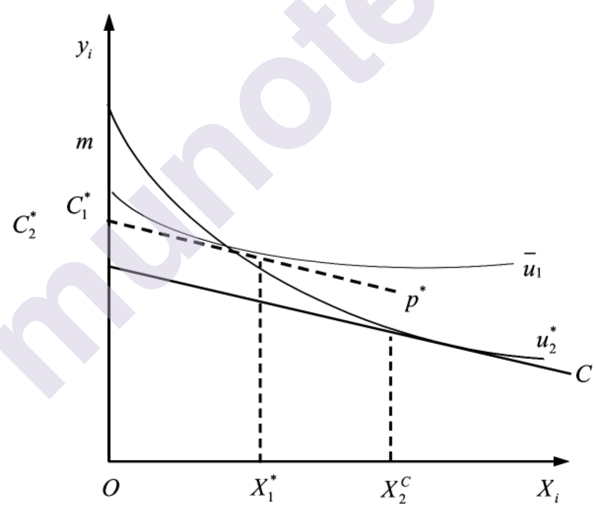

Figure 8.5

In fact, the tariffs or price schedules that firms with market power offer often do specify maximum consumption quantities as well as fixed and variable charge.

A note on terminology, Linear pricing refers to the case in which a buyer is charged a fixed price p per unit bought, so that her total expenditure is E=px a linear function. A two-part tariff consists of a fixed charge. C and a fixed price p per unit bought, so that total expenditure is the affine function. In this case, the average price per unit, $p + c/x$, is a non-linear, decreasing function of the quantity bought. In figure 8.5, the implied unit price F_i^* / x_i^* to each type of buyer will not be the same, implying a kind of non-linearity in the way in which unit price varies with quantity bought. Thus this kind of price discrimination, as well as two-part tariffs, falls under the general heading of 'non-linear pricing'.

To summarize : if a seller can identify each buyer's type (her demand function), and revent arbitrage between types, then he maximizes profit by offering a two part tariff consisting of a unit price equal to marginal cost c, and a fixed charge which expropriates all the consumer surplus of the given type. If a seller cannot identify a buyers type, he must offer optional contract a higher demand type will choose a contract which offers a unit price equal to marginal cost and a fixed charges which leaves her with some consumer surplus, o law demand type will choose a contract which offers a higher price up to a quantity maximum $\left(x_i^*\right)$ and a lower fixed charge which never the less appropriates all her consumer surplus. After natively the contracts may simply specify a quantity supplied and a total charge for that quantity. The aim is to prevent high demand buyers pretending to be low demand buyers, and taking the contract the later would be offered under first degree price discrimination, by making the low demand buyers' contract less attractive to the high demand buyers. Finally, if a buyer's type can be identified and arbitrage between types can be prevented, but the seller is constrained to use linear pricing, we have third degree price discrimination.

8.6 THIRD-DEGREE PRICE DISCRIMINATION: MARKET SEGMENTATION

Suppose that the monopolist can divide the market for his output into two subgroups between which arbitrage can be prevented at zero cost. To concentrate on essentials assume that the costs of supplying the two sub-markets are identical, so that any price difference between the sub markets will arise from dissemination, not differences in say, transport or distribution costs.

The monopolist knows the demand, and therefore marginal revenue, curves, for each graep Let q_1 and q_2 be the quantities sold to the first and second groups respectively, so that total output $q = q_1 + q_2$. Take some fixed total output level, q_0, and consider the division of this between the two sub-markets in such a may as to maximize profit since the total production cost of q_0 is given, profit from the division of this between the two markets is maximized if revenue is maximized. But revenue is maximized only if q_1 and q_2 are chosen such that the marginal revenues in each sub-market are equal. To see this let MR_1 be the marginal revenue in sub-market 1, and MR_2 that in 2. Suppose $MR_1 > MR_2$. Then it would be possible to take one unit of output from market 2, and sell it in market 1, with a net gain in revenue of $MR_1 - MR_2 > 0.$ As long as the marginal revenues were unequal such possibilities for increasing revenue, and therefore profit, would exist. Hence a necessary condition for a profit maximizing allocation of any given total output between the two markets is that marginal revenues in the markets be equal.

In determining the optimal total output level, weare on familiar ground. If $MR_1, (= MR_2)$ differed from marginal cost, it would be possible to vary output in such a way as to increase total profit by increasing output when $MR_1 > MC$, and reducing it in the converse case. Hence a necessary condition for maximum profit is that $MC = MR_1 = MR_2$.

Now let $e_1 \, and \, e_2$ be the price elasticities of demand in the respective sub-markets. Then the basic relation given in 8.9 applies in this case, so that

$$Mc = P_1 (1 + 1/e_1) = p_2 (1 + 1/e_2) \qquad [8.12]$$

From the second equality in [7.12] we have

$$\frac{p_1}{p_2} = \frac{1 + \frac{1}{e_2}}{1 + \frac{1}{e_1}} \qquad [8.13]$$

If $e_1 = e_2$,then clearly $p_1 / p_2 = 1$ and there is no discrimination There will be price discrimination as long as the clasticities are unequal at the profit maximizing point Moreover, if $e_1 < e_2$, then from [8.13] $p_1 < p_2$, and conversely.

(Remember $e_i < 0.$) In maximizing profit the monopolist will set a higher price in the market with the less elastic demand.

The analysis is illustrated in fig 8.6 In (a) of the figure are the demand and marginal revenue curves for such market 1 and in (b) those for 2. The curve MR in (c) is the horizontal sum of the MR_1 and MR_2 curves. MR has the property that at any total output, q^0 the output levels q_1^0 and q_2^0 which have the some marginal revenues in the sub markets as that at q^0 sum exactly to

$q^0, i.e. q_1^0 + q_2^0 = q^0$. The horizontal summation therefore refects the first condition derived above, that any total out put must be divided between the sub markets in such a may as to equalize their marginal revenues. The profit maximizing level of total output is shown at q^*, where $MR = MR\, q^*$ is optimally divided between the submarkets at q_1^* and q_2^* where the sub-market outputs have marginal revenues equal to MC and by construction must sum to q^*. Demand for q_2 is less elastic than that for q_1 so that $p_2^* > p_1^*$.

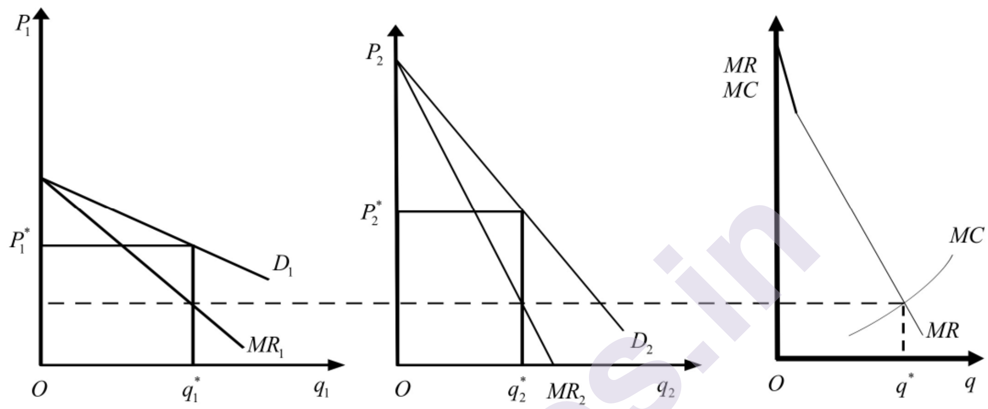

Fig 8.6

8.7 PRICE DISCRIMINATION AND SOCIAL WELFARE:

Whether price discrimination promotes social welfare or not is difficult to say. In other words, there is ambiguity as far as the welfare effect of price discrimination is concerned. It is important to consider whether social welfare is defined in terms of total output or distribution of given output. This is because, the total output effects of price discrimination may have positive welfare effects, whereas distribution may be adversely affected after practicing price discrimination.

As per the Pareto optimality, one of the maximum condition for maximization of social welfare is that the marginal rate of substitution between two goods for different consumers, should be the same. But when a monopolist follows price discrimination, the above-mentioned marginal condition is violated. But another aspect of welfare effects of price-discrimination may be understood by acknowledging the total output effects of price discrimination. According to Joan Robinson, price discrimination sometimes may lead to increase in output. That means, the total output may be more when the monopolist practices discriminating prices rather than a uniform price. Thus, from the point of view of total output, when a society prefers more output to less output, price discrimination may promote social welfare.

To conclude the discussion on welfare effects of pricediscrimination, following points need to be considered.

1) The losses incurred by the consumers in low electricity market in the form of reduction in the consumer surplus as the monopolist charges higher price for them.
2) The gains enjoyed by the consumers in high elasticity market in the form of increased consumer surplus, as the monopolist charges lower price from them.
3) If price discrimination is not exercised by the monopolist and uniform price policy is followed, there is a possibility that some markets may be closed for the monopolist (particularly the high elasticity markets as they have to accept monopoly price).
4) Price discrimination leads to redistribution of income from the consumers in low elasticity markets to the consumers in high elasticity markets and the monopolist. Since the consumers in low elasticity market are generally richer, redistribution may increase social welfare.

8.7 MONOPSONY

Monopsony is defined as a market in which there is a single buyer of a commodity who confronts many sellers. Each of the sellers treats the market price of the good as a parameter and so there is a market supply curve for the good which is derived in the usual way from the supply curves of the individual suppliers. The single buyer of the good faces a market supply function relating total supply to the price he pays. This can be expressed (in the inverse form) as

$$P_1 = P_1(Z_1) \qquad P_1^1 > 0 \qquad \text{[B.1]}$$

where [B.1] shows the price of the commodity which must be paid to generate a particular supply. Note that the buyer is assumed to face on upward-sloping supply curve; the price required is an increasing function of the amount supplied.

The market price of the monopsonized input is determined, given the supply function [B.1], by the buyer's demand for z_1. We assume that the monopsonist is a profit maximizing firm, in which case the demand for z_1, and hence its price, is determined by the firms profit maximizing decision. In the two input, single output case the firms's problem is

$$\max_{Z_1, Z_2} R\left[F(Z_1, Z_2)\right] - P_1(Z_1) - P_2 Z_2 \qquad \text{[B.2]}$$

This is very similar to problem [6.1] except that p_1 depends on z_1 because of [B.1] Input 2 is assumed to be bought on a market in which the firm treats p_2 as a parameter. The firms output may be sold in a competitive or monopolized market. monopoly need not imply monopoly. The firm may, for example, be the only employer of labour in a particular area but be selling its output in a market where it competes with many other firms, and labour may be relatively immobile.

Necessary conditions for a maximum of [B.2] are (when both $Z_1 \; and \; Z_2$ are positive at the optimum)

$$R^1F_1 - \left(P_1 + P_1^l \, Z_1\right) = 0 \qquad [B.3]$$

$$R^1F_2 - P_2 = 0 \qquad [B.4]$$

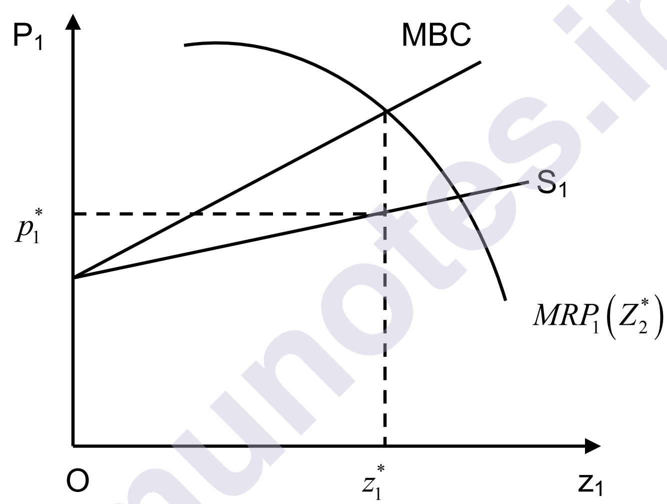

Figure : 8.7

This equilibrium is illustrated for the monopsonized input in Figure 8.7 s_1 is the supply curve of Z_1 and MBC_1 plots the marginal buyer cost $\left(P_1 + P_1^1 \, Z_1\right)$ of the single buyer. $MRP_1\left(Z_2^*\right)$ is the marginal revenue products curve for the input given the optimal level of Z_2 The firm maximizes profit with respect to Z_1 by equating $MRP_1 \; to \; MBC_1 \; at \; Z_1^*$ to generate this supply of Z_1 the firm will set the monopsony price $P_1^* - P_1\left(Z_1^*\right)$.

The analysis of the single buyer confronting many competitive sellers is rather similar to the analysis of the single seller confronting many competing buyers. In each case the firm realizes that it faces a curve relating price to quantity which summarizes the response of the competitive side of the market and the firm sets the quantity or price in the light of this interdependence of price and quantity. In each case the market price overstates the marginal profit contribution of the quantity and in each case this overstatement depends on the responsiveness of quantity to changes in price under monopoly the firm equates $MR = P[1+(1/e)]$ to the marginal cost of output, and the less clastic is demand the greater is the difference between price and marginal cost. [B.5] can be written in a similar may. Defining the clasticity of supply of Z_1 with respect to price as

$$e_1^s = \frac{dz_1}{dp_1} - \frac{P_1}{Z_1} \qquad [B.7]$$

We see that.

$$MBC_1 = P_1 + \frac{dp_1}{dz_1}, z_1 = P_1\left[1 + \frac{1}{x_1^s}\right] \qquad [B.8]$$

and so [B-5] becomes

$$MRP_1 = P_1\left[1 + \frac{1}{e_1^5}\right] \qquad [B-9]$$

The less elastic is supply with respect to price the greater will be the difference between MRP_1 and the price of the input. In other words, the less responsive to price the input supply is, the greater the excess of the value of the marginal unit of the input over the price it receives. This could be regarded as a measure of the degree of 'monopsonistic exploitation'.

8.8 THE EFFECT OF MONOPSONY AND OUTPUT MONOPOLY ON THE INPUT MARKET.

When the output is produced from two or more inputs the analysis of the effect of both monopsony and output monopoly on the price of one of the inputs is complicated, because the use of the other input is likely to change as well, thus shifting the MRP_1 curve. If the output is produced by a single input this complication does not arise, and it is possible to show the implication of monopsony and output monopoly in a single simple diagram such as figures 8.8. Since there is a single output Z_1 is marginal product depends only on Z_1 and so the marginal revenue product MRP_1 and and the value of the marginal product VMP_1 curves in figure 7.8 are fixed, S_1 and MBC_1 are supply and marginal buyer cost curves. There are four possible equilibria in this input market, where suppliers treat the price of Z_1 as a parameter. If the firm also treats P_1 as given, i.e. if it acts as if it has no monopsony power and if it also treats out put price as a parameter then VMP_1 is its demand curve for Z_1 and the market price is P_1^0. If the firm uses its monopsony power but continues to treat output price as a parameter it will equate VMP_1 *to* MBC_1 and set the price P_1^1. If the firm monopolizes its output market but regards P_1 as a parameta its demand curve for Z_1 is MRP_1 and the price of Z_1 is P_1^2. Finally, if the firm exercises both monopoly and monopsony power it equates MRP_1 and MBC_1 and sets a price P_1^3. We see therefore that the price in an input market is reduced below the competitive level P_1^0 by both monopsony and monopoly power. The less elastic are the demand for output and the supply of input functions, the lower will be the price paid to suppliers of the input.

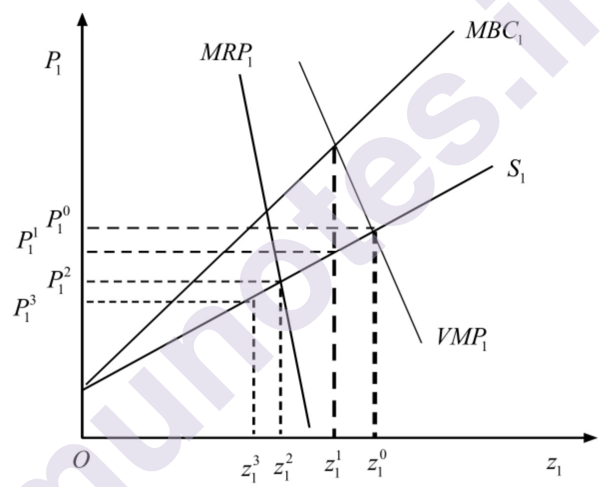

Fig. 8.8

8.9 UNION AS MONOPOLY INPUT SUPPLIERS

We define a union as any association of the suppliers of a partiallar type of labour which is formed with the aim of raising wages or improving working conditions. A union need not, of course, be described as such by its members : Many professional associations such as the British Medical Association and the law society act as unions. Not all unions may be successful in raising the wages of their members above the competitive level. The union, like any would be mnonopolist, must be able to control the supply of labour offered to firms one method of doing this is to ensure that only union members can sell their labour in that particular market, a device known as the closed shop. The closed shop many by itself, reduce the supply of labour to the market if some potential workers dislike being union members as such. In general, however, the closed shop must be coupled with restrictions on the number of union members if all members are to be employed, since higher mages will increase the members of workers wishing to join the union, i.e. become employed at the higher wage.

If the union can act as a monopolist its behaviour will depend on the objective it pursuers. It may be useful to distinguish between the objectives of the officials who run the union and those of the members. In the case of the firm, where conflicts of interest may exist between shareholders and managers, the extent to which the managers rursue the interests of the shareholders depends on the incentive system which relates managerial pay to profits and on the threat of products or capital market competition similar mechanisms may be at work in the case of the union officials salaries can be related to the pay of members of the union unions which do not attend sufficiently closely to their members interests may start to lose member to rival unions officials may be controlled directly through elections, but here the control machinism may be much weaker than in a firm. Each union member has only one vote and so may members must cooperate to change the officials. Shareholders vote in proportion to the numbers of shares held and so a relatively small group of individual shareholders may exercise effective control.

It is by no means obvious that the political structure of a union will generate any well defined preference ordering, let alone one which reflects the interest of its members. (See the discussion of the Arrow Impossibility. Theorem in section 13F). However, we will assume that such a preference ordering exists and can be represented by a utility function $\mho(w, z)$ where w is the wage paid to union members and z is the number of union members employed (we assume that hours of work are fixed). We illustrate the implications of different assumptions about union preferences by specifying there different forms for $\mho$.

The demand side of the labour market monopolized by the union is assumed to be competitive and the union is constrained to choose a wage and employment combination on the labour market demand curve D in fig 8.9. MSR is the corresponding ‘marginal revenue to the sellex’ curve which shows the rate at which the total wage bill wz varies with z 5 is the supply curve ??? the minimum wages necessary to attract different numbers of workers into the industry s plots the reservation wage or supply price of workers. The competitive equilibrium in the absence of an effective union monopoly would be at with a wage rate of Wc.

The economic rent earned by a worker is the difference between the wage paid and the wage necessary to induce that worker to take a job in the industry. The total

$$\mho\left(w,z\right)=\left(w-\widehat{w}\right)z+\widehat{w}z^{0} \quad [C.6]$$

(The union indifference curves are now rectangular hyperbolas with a horizontal axis at $\widehat{w}$). Since $\widehat{w}$ and Z are constrants, [c.5] is maximized by maximizing $\left(w-\widehat{w}\right)$ Z and the union's optimization problem is now analogous to that of a monopolist with a constant 'marginal cost' of $\widehat{w}$.

It is possible to construct many models of the above kinds, each of which may be appropriate to a particular union or industry. A model of the way in which the union's objectives are determined is necessary in order to be able to predict. What objectives will be dominant in what circumstances. This will require a detailed specification of the political constitution of the union, including the frequency and type of auction, whether officials are elected or are appointed and controlled by elected representatives and so on. In addition, the theory could be extended to take account of inter union conflict or cooperation will unions compete for new members? In what circumstances will unions merge or collude? It would be interesting to approach these questions using the concepts of oligopoly theory developed in chapter.

8.10 BILATERAL MONOPOLY

Bilateral monopoly is a market situation in which a single seller confronts a single buyer. For definiteness and continuity, we consider a labour market in which supply is monopolized by a union and there is a single buyer of labour. Z is the sole input in the production of an output y = F (Z). The revenue from sale of y is R (F(Z)) and the MRP curve in fig 7.9 plots the marginal benefit to the buyer of $Z:R^{1}F^{1}=MR.MP$, using the notation of section A. The average revenue product curve ARP plots $R/Z=Py/Z=PAP$ where AP is the average product of z : y / z.

The objective function of the firm is its profit function.

$$\Pi=R\left(F\left(z\right)\right)-wz=\Pi\left(w,z\right) \quad [0.1]$$

and its indifference curves in (w,z) space have slope

$$\left.\frac{dw}{dz}\right|_{d\Pi=0}=-\frac{\Pi z}{\Pi w}=\frac{R^{1}\left(F\left(z\right)\right)F^{1}\left(z\right)-W}{Z}=\frac{MRP-W}{Z} \quad [D.2]$$

Where $R^{1}F^{1}=MRP$ is the firms marginal revenue products for W < MRP its indifference curves are positively sloped and for w > MRP they are negatively sloped. Thus its indifference curves are shaped about the MRP curve. If the firm acted as a monoponist facing competitive labour suppliers, it would announce a wage rate at which it is willing to hire workers and employment would be determined by the supply curves of the workers.

Suppose that the union has the simple objective function.

$$\mho(W, Z) = \left(W - \widehat{W}\, Z + \widehat{W} Z^0\right) \qquad [D.3]$$

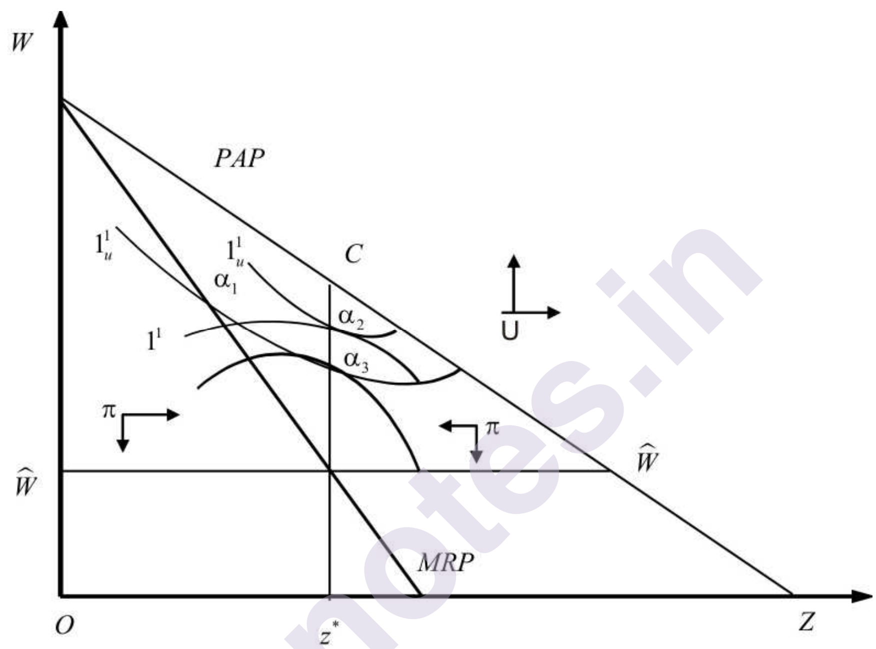

Figure 8.9

examined in section C, where Z^0 is the number of union members and $\widehat{W}$ is the income or wage of those who are unemployed. The unions indifference curves are hyperbolas, rectangular to the $\widehat{W}\,\widehat{W}$ line, with slope.

$$\left.\frac{dw}{dz}\right|_{d\mho=0} = -\frac{W - \widehat{W}}{Z} \qquad [D.4]$$

If the union acted as a monopolist with respect to the labour supply of its members it would announce a wage rate at which its members would be willing to supply labour and employment would be determined by the demand curve for labour.

When a single buyer and a single seller of labour confront each other it seems implausible that either party will treat a wage rate announced by the other as parametric and passively adjust either their supply of demand. Both will realize that they possess market power in the sense that, by refusing to demand or supply labour at a wage announced by the other, they can prevent any gains from trade being achieved and thus impose cost on the other. The two parties must therefore agree on a wage and on employment level before production can occur.

We assume in this section that the agreement between the union and the firm is the outcome of a cooperative game. In such a game all the actions of the parties are controlled by a binding agreement between them specifying what each will do. The cooperative game approach to bargaining is concerned solely with the content of the agreement. It ignores the process of bargaining and negotiation by which agreements are reached (We examine the alternative non-cooperative game approach, which does pay more attention to the bargaining process we attempt to predict the agreement by requiring that it satsfy certain ‘reasonable’ conditions. Two obvious conditions to impose are:

(a) Individual rationality : any agreement should level both parties at least as well off as they would be if there was no agreement;

(b) Efficiency : there should be no other agreement which would make on of them better off and the other no worse off.

If an agreement satisfies these requirements it is an efficient bargaining solution to the cooperative bargaining game.

Applying these conditions provides a partial answer to the question of what agreement will be reached by the union and the firm. If there is no agreement and therefore no employment, the firm will have zero profit. Any agreement which yield a (w,z) combination on or below its average revenue product curve PAP will satisfy the individual rationality constraint for the firm. If the union achievers zero utility if there is no agreement, it will be no worse off with an agreement at any point on or above the line $\widehat{W}\,\widehat{W}$. Thus the set of individually rational agreements which make both parties no worse off is the triangle bounded by the vertical axis, $\widehat{W}\,\widehat{W}$ and pAP in fig 8.9.

Imposing the efficiency requirement further reduces the set of possible pargains. If the parties’ indifference curves interest at a point such as x_1 it is always possible to find another point or bargain which makes at last on of them better off and the other no curse off. Thus moving from the agreement x_1 where the indifference curves J^1 *and* I_u^1 interest to the agreement x_2 will make the union better off since x_2 is on the higher in difference curve I^2u. The firm is no increase off at x_2 since both point x_1 to x_3 would make both union and firm better off.

A necessary condition for efficiency is that the parties indifference curves are tangent.

$$\left.\frac{dw}{dz}\right|_{d\Pi=0} = -\frac{\Pi z}{\Pi w} = \frac{R^1F^1 - W}{Z} = \left.\frac{dw}{dz}\right|_{d\mho=0} = -\frac{W-\widehat{W}}{Z} \qquad \text{[D.5]}$$

which implies

$$R^1F^1 = \widehat{W} \qquad \text{[D.6]}$$

All agreements satisfying [D.6.] are efficient. Notice that [D.6] depends only on the level of employment Z (which enters into R^1R^1) and not on w. the locus of points where [D.6] is satisfied and the agreement is efficient is a vertical line at Z agreement MRP cuts $\widehat{W}\,\widehat{W}$

The set of agreements satisfying individual rationality and efficiency is the contract curve. In the current model to contract curve has a particularly simple form. It is the line cc. in fig 8.9 between the PAP and $\widehat{W}\,\widehat{W}$ curves where indifference curves are tangent and the parties no worse off than if they do not agree.

The efficient bargain model predicts the level of employment Z^* the parties will agree on bt it is unable to predict the wage rate at which the workers will employed. This is perhaps unsurprising the parties can agree to choose on employment level which will maximize their potential gains from agreement the difference between the firm's revenue R(F(Z)) and the 'cost' of labour $\widehat{W}Z$ as perceived by the union A change in z which increases $R-\widehat{W}\,Z$ can make both parties better off and they can therefore agree to it. However, for fixed Z , changes in the wage rate have precisely opposite effects on their utilities.

$$\Pi w = -z, \mho w = z$$

with z held constant changer in w merely make one party better off at the expense of the other. In figure 10.7 the firm will always reefer a bargain lower down cc and the union a bargain higher up CC.

One way to remove the indeterminacy of the bilateral monopoly model is to impose additional requirements on the agreement or solution of the cooperative bargaining game.

8.11 QUESTIONS

1. How the price and output is determined under monopoly?
2. Explain the price and output determination under price discriminating monopoly.
3. Examine the concept of monoposony.
4. Explain the effect of monopsony and output monopoly on the input market.
5. What is bilateral monopoly?

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