**[Time: 2 ½ Hours ]** **[ Total Marks : 60 ]**

**N.B.:**(1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate **full** marks.
(3) **Symbols** have their usual **meanings** unless otherwise **stated**.
(4) Use of **log tables / non-programmable** calculator is **allowed**.

1. (a) Attempt any **one : -** **8**

(i) a) Derive Cauchy Riemann equations in cartesian form.
b) Determine whether $\frac{1}{z}$ is analytic or not?

(ii) If f(z) is analytic in a closed curve C, except at a finite number of poles within C, then

$$\int f(z)dz = 2\pi i\ (sum\ of\ residues)$$

Hence Evaluate $\int_c \frac{4-3z}{z(z-1)(z-2)}dz$ where c is the circle $|z| = \frac{3}{2}$

(b) Attempt any **one : -** **4**

(i) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function $v$ such that $f(z) = u + iv$ is analytic.

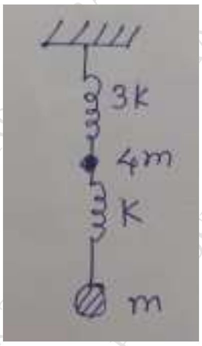
(ii) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series for

a) $1 < |z| < 3$ b) $|z| > 3$

2. (a) Attempt any **one : -** **8**

(i) Find the inertia tensor about the origin for the mass distribution consisting of mass 1 at (0, 1, 1) and mass 2 at (1, −1, 0). Find the principal moment of inertia and the principal axis.

(ii) Find the characteristic frequencies and characteristic mode of vibration for the system of masses (m & 4m) and springs having spring constants (3k & k) as shown in the figure the motion is along a vertical line.
(Neglect mass of the spring.)b



(b) Attempt any **one : -** **4**

(i) Using the Levi-Civita symbol, show that, $A \times (B \times C) = B(A.C) - C(A.B)$

(ii) Find the eigenvalues of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

3\. (a) Attempt any **one : -** **8**

(i) Solve the Laguerre equation using Frobenius method

$$x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + ny = 0$$

(ii) Solve the Bessel equation

$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)\, y = 0$ using Frobenius method.

(b) Attempt any **one : -** **4**

(i) Using the generating function for Hermite polynomials

G (x ,+) = $e^{2tx-t^2} = \sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!}$ pove that,

$2xH_n(x) = 2_n\, H_{n-1}(x) + H_{n+1}(x)$

(ii) Using Green's Theorem, Find the area of the region in the first quadrant bounded by the curves $y = x$, $y = \frac{1}{x}$ , $y = \frac{x}{4}$ .

4\. (a) Attempt any **one : -** **8**

(i) State the expression of Fourier transform and its inverse in three-dimensional space. Find the Fourier transform of Yukawa potential $\frac{e^{-\alpha r}}{r}$

(ii) Solve the differential equation using Laplace transform

$y'' - y = t$ subject to initial conditions $y(0) = 1$ & $y'(0) = 1$

(b) Attempt any **one : -** **4**

(i) Find the Laplace transform of $f(t) = t \sinh at$.

(ii) State and prove Fourier Convolution Theorem.

5. Attempt any **four:** - **12**

(a) Determine the poles of the function and residue at the poles: $f(z) = \frac{z}{\sin z}$

(b) Use Cauchy's integral formula to calculate

$$\int_c \frac{2z+1}{z^2+z} dz \qquad where\ C\ is\ |z| = \frac{1}{2}$$

(c) What is the rank of the tensor $T_{ijkl}$ ? Write its transformation equation.

(d) For matrices M, C and D, show that $M^n = C\, D^n C^{-1}$ where $C^{-1}\, M\, C = D$ and D is diagonal.

(e) Prove that $P_n(1) = 1$

(f) Find regular singular point of the differential equation

$$2x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + (x^2 - 4)y = 0$$

(g) Find the inverse Laplace transform of $F(s) = \frac{7}{(s-1)^3}$

(h) Show Fourier transform property: $[\nabla f(\vec{r})]^T(\vec{k}) = -i\vec{k}g(\vec{k})$

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