

(3 hours)

Total marks:80

N.B.: (1) Question No. 1 is compulsory.  
(2) Attempt any Three from remaining.

- Q1 a) If  $\tanh x = 1/2$  then find value of  $x$  and  $\sinh 2x$  [3]  
 b) If  $u = \log(x^2 + y^2)$  Find  $\frac{\partial^2 u}{\partial x \partial y}$  [3]  
 c) If  $x = u - uv$ ,  $y = uv - uvw$ ,  $z = uvw$  find  $\frac{\partial(x,y,z)}{\partial(u,v,w)}$  [3]  
 d) Using Maclaurin's series, Prove  $\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} + \dots$  [3]  
 e) Check if the matrix  $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$  is unitary [4]  
 f) Find  $n^{\text{th}}$  derivative of  $\frac{2}{(x-1)(x-2)(x-3)}$  [4]
- Q2. a) Solve  $x^5 = 1 + i$  and find the continued product of the roots. [6]  
 b) Reduce the matrix  $A = \begin{bmatrix} 3 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$  to the normal form [6]  
 and find its Rank  
 c) State and Prove Euler's theorem for two variables hence [8]  
 find value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  where  $u = \frac{xy}{x+y}$
- Q3 a) Investigate for what values of  $\lambda$  and  $\mu$  the equations [6]  
 $x + y + z = 6$ ,  $x + 2y + 3z = 10$  and  $x + 2y + \lambda z = \mu$  have  
 i) No solution ii) Unique solution iii) Infinite solutions  
 b) Examine the function for its extreme values [6]  
 $f(x, y) = y^2 + 4xy + 3x^2 + x^3$   
 c) If  $\tan(\alpha + i\beta) = \sin(x + iy)$  then Prove  $\frac{\tan x}{\tanh y} = \frac{\sin 2\alpha}{\sinh 2\beta}$  [8]
- Q4 a) If  $x = u \cos v$ ,  $y = u \sin v$  then [6]  
 Prove  $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} = 1$   
 b) Prove that  $\log \left( \frac{\sin(x+iy)}{\sin(x-iy)} \right) = 2i \tan^{-1}(\cot x \tanh y)$  [6]  
 c) Solve by Gauss Jordan method [8]  
 $2x + 3y + 4z = 1$ ,  $x + 5y + z = 1$ ,  $x + y + 6z = 5$
- Q5. a) Prove  $\cos^6 \theta - \sin^6 \theta = \frac{1}{32} [\cos 6\theta + 15 \cos 2\theta]$  [6]  
 b) Evaluate  $\lim_{x \rightarrow 0} \left[ \frac{x - \sin x}{x^3} \right]$  [6]  
 c) If  $y = \cos(m \sin^{-1} x)$  then [8]  
 prove that  $(1 - x^2)y_{n+2} - (2n + 1)x y_{n+1} + (m^2 - n^2)y_n = 0$

Q6 a) Check if the following vectors [6]  
 $X_1 = [3, 1, 2, 1], X_2 = [4, 6, 2, -4], X_3 = [-6, 0, -3, -4]$   
 $X_4 = [1, 0, 2, 1]$  are linear dependent hence find the relation between them if any.

b) If  $f\left(\frac{z}{x^3}, \frac{y}{x}\right) = 0$  then [6]  
 prove that  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 3z$

c) Fit a second degree parabola  $y = ax^2 + bx + c$  to the following data [8]

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9