Duration: $2\frac{1}{2}$ Hours **OLD COURSE** **Max. Marks : 75**

1) **All questions are compulsory**
2) **Figures to the right indicate marks.**

Q.1 (a) Attempt any ONE of the following **(8)**

(i) If $\lim_{(x,y)\to(a,b)} f(x,y) = L$ and if $\lim_{x\to a} f(x,y)$ and $\lim_{y\to b} f(x,y)$ both exists, then prove that
$$\lim_{x\to a}(\lim_{y\to b} f(x,y)) = \lim_{y\to b}(\lim_{x\to a} f(x,y)) = L.$$
Give an example to show that the converse is not true.

(ii) Let S be an open subset of $\mathbb{R}^n$ and f, g : S → $\mathbb{R}^m$ and let a ∈ S, $\lambda \in \mathbb{R}$. If $\lim_{x\to a} f(x)$ = b and $\lim_{x\to a} g(x)$= c then using $\epsilon - \delta$ definition prove that p) $\lim_{x\to a}(\lambda f(x)) = \lambda b$ q) $\lim_{x\to a}(f(x).\, g(x)) = b.\, c$

(b) Attempt any TWO of the following **(12)**

(i) Let f: $\mathbb{R}^2 \to$: $\mathbb{R}^2$ be defined by f(x, y) = $(x^2 - y^2, x^2 + y^2)$. Using $\epsilon - \delta$ definition show that each component of f is continuous at (1, 2).

(ii) If f: $\mathbb{R}^2 \to \mathbb{R}$ be defined by f(x, y) = |x| + |y| then show that $f_x(0,0)$ *and* $f_y(0,0)$ do not exist. Check whether f is continuous at (0, 0)

(iii) Find $\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{\sqrt{(x^2+y^2)}}$ using polar co-ordinates.

(iv) Using $\varepsilon - \delta$ definition, discuss the continuity of $f: \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x,y) = (x^2 + 2y, y^2 + 2x)$ at (2, 3)

Q.2 (a) Attempt any ONE of the following **(8)**

(i) Let S be an open subset of $\mathbb{R}^2$ and $f: S \to \mathbb{R}$ be such that $D_1 f, D_2 f, D_{12} f, D_{21} f$ exists on S. If $(a,b) \in S$ and $D_{12} f, D_{21} f$ are continuous on S, then show that $D_{12} f(a,b) = D_{21} f(a,b)$.

(ii) Let S be an open subset of $\mathbb{R}^n$ and $f: S \to \mathbb{R}$ be differentiable at $a \in S$ with total derivative $Df(a)$. Show that $f'(a;y)$ exists for all $y \in \mathbb{R}^n$ and $f'(a;y) = Df(a)$ and $f'(a;y) = \sum_{k=1}^{n} D_k f(a) y_k = \nabla f(a) \cdot y$ for all $y \in \mathbb{R}^n$.

(b) Attempt any TWO of the following **(12)**

(i) State and prove the Mean Value Theorem for a scalar field

(ii) Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function. Let A(1,3), B(3,3), C(1,7), D(6,15). The directional derivative of f at A in the direction of AB is 3 and in the direction of AC is 26. Find the directional derivative of f at A in the direction of AD.

(iii) If $f: \Re^2 \to \Re^2, g: \Re^2 \to \Re^3$ $f(x,y) = (xy^2, xy)$, $g(u,v) = (u+v, u-v, uv)$.
(p)Compute the Jacobian matrices $Df(x,y), Dg(u,v)\, \&\, D(g \circ f)(x,y)$
(q)Verify that $D(g \circ f)(1,1) = Dg(1,1) \cdot Df(1,1)$

(iv) Let $\boldsymbol{z} = \boldsymbol{e^{u+v+w}}$, where $u = x^2 \sin^2 y$, $v = 2x \sin x \sin y$, $w = y^2$. Use chain rule to find $\boldsymbol{z_x}, \boldsymbol{z_y}$.

Q.3 (a) Attempt any ONE of the following **(8)**

(i) State and prove Stoke's Theorem for an oriented smooth simple parametrized surface in $\Re^3$ bounded by simple ,closed, curve traversed counter clockwise assuming general form of Green's Theorem.

(ii) State Divergence Theorem for a solid in 3 – space bounded by an orientable closed surface with positive orientation and prove the Divergence theorem for cubical region.

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(b) Attempt any TWO of the following **(12)**

(i) Compute the surface area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 9$

(ii) Evaluate the surface integral $\iint_S \overline{F}\cdot\hat{n}dS$ if $\overline{F}(x,y,z) = x\hat{i} + y\hat{j} + z\hat{k}$ and S is the sphere $x^2 + y^2 + z^2 = 1$.

(iii) Use Stokes' Theorem to compute the integral $\iint_s curl\,\bar{F}\cdot\hat{n}ds$, where $\bar{F}(x,y,z) = yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ above the xy –plane.

(iv) Verify Divergence Theorem for vector field $\overline{F}(x,y,z) = 3x\hat{i} + xy\hat{j} + 2xz\hat{k}$ and V is the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.

Q.4 Attempt any THREE of the following **(15)**

(i) If $f(x, y) = \frac{x^2-y^2}{x^2+y^2}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$, then find $\lim_{x\to0}(\lim_{y\to0} f(x,y))$ and $\lim_{y\to0}(\lim_{x\to0} f(x,y))$. Also find $\lim_{(x,y)\to(0,0)} f(x,y)$ if exists

(ii) Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right) & ,if\ xy \neq 0 \\ 0 & ,if\ xy = 0 \end{cases}$$

Find $D_{12}f(0,0), D_{21}f(0,0)$ and check whether they are equal.

(iii) Find all differentiable vector fields $f:\Re^3 \to \Re^3$ for which the Jacobian matrix D f (x,y,z) = diag (p(x) , q(y) , r(z)) where p, q, r : $\Re \to \Re$ are continuous functions .

(iv) Given $u = f(x,y)$ has continuous partial derivatives with respect to x and y. If $x = r\,cos\theta, y = r\,sin\theta$, then show that $u_x{}^2 + u_y{}^2 = u_r{}^2 + \frac{1}{r^2}\ u_\theta{}^2$.

(v) Evaluate the surface integral $\iint_S xzdS$,where S is the triangle with the vertices (1,0,0),(0,1,0) and (0,0,1).

(vI) Define the Fundamental Vector Product for a surface S whose vector equation is $\bar{r}(u,v) = X(u,v)\hat{\imath} + Y(u,v)\hat{\jmath} + Z(u,v)\hat{k}$ $\forall$(u,v) $\in$ T in uv-plane. Compute $\left\|\frac{\partial\bar{r}}{\partial u} \times \frac{\partial\bar{r}}{\partial v}\right\|$ for $\bar{r}(u,v) = vsin\boldsymbol{\alpha}cos\boldsymbol{u}\hat{\imath} + vsin\boldsymbol{\alpha}sin\boldsymbol{u}\hat{\jmath} + vcos\boldsymbol{\alpha}\hat{k}$