VCD 10/10/19 SYBSC(PM) (ATKT) SEM-IV(Choice Based) MATHS- III Marks:100

3 Hours

Instructions: 1) All questions are compulsory.

2) All questions carry equal marks.

3) Figures to right Indicate full marks of subquestion.

Q.1 Choose correct alternative in each of the following. (20)

a) If n= ________ then the Bernoulli differential equation reduce to variable separable form.

i) 0 ii) 1 iii) 2 iv) None of these

b) The order and degree of differential equation $\frac{d3y}{dx3} - \frac{d2}{dx2} + 2\frac{dy}{dx} - 3y = cosx - 1$ is ________

i) 3 and 2 ii) 2 and 3 iii) 3 and 1 iv) 1 and 3

c) ________ of the following is homogenous differential equation.

i) $(x+y)dy + (5x-3y+8)dx=0$ ii) $(6x-4y+3)dx=(3x-2y+1)dy$

iii) $(2x+y-3)dy = (x+2y-3)dx$ iv) $x3dy=(y^3+y^2\sqrt{y2 - x2)dx}$

d) Orthogonal trajectories of the family of curves $xy=k$ represents a family of ________

a) Parabolas b) Straight lines c) Hyper bolas d) Circles

e) If(e^{4t} , e^{4t}) and (e^{-2t},$-e^{-2t}$)are linearly independent solutions of $\frac{dx}{dt} = x + 3y, \frac{dy}{dt} = 3x + y$ then Particular solution for which $x(0)=5, y(0)=1$ is________

a) ($3e^{4t}+2e^{4t}$, $3e^{-2t}-2e^{-2t}$) b) ($4e^{4t}+e^{4t}$,$4e^{-2t}-3e^{-2t}$)

c) ($3e^{4t}+2e^{-2t}$, $-2e^{-4t}+e^{-2t}$) d) ($3e^{4t}+2e^{2t}$,$3e^{4t}-2e^{-2t}$)

f) The equation D2 + PD + Q =0 is called ________

a) Auxiliary equation b) Characteristic equation

c) both a & b d) None of these

g) If $y_1= x^2 1\ x1$ and $y_2= x^3$, $x \in 1R$ then Wcy . ıy2) is ________

a) 0 b) 1 c) -1 d) None of these

h) The general solution of 2y''+5y'-12y=0 is ________

a) $y=c,e^{3x}+c2e^{4x}$ b)$y=c.e^{-3x}+c2e^{-4x}$

b) $y=c,e^{-4x}+c2e^{3x12}$ d) None of these

i) A ______________ order first degree differential equation of the form $\frac{dy}{dx}$ +py=Qy^n known as Bernoulli different equation.

a) First b) Second c) Third e) None of these

j) If μ (x,y)dx+ μNc(x,y)dy=0 is an exact differential equation then μ is called______

a)Leibnitz constant b) Bernoulli constant c)Integrating factor d) None of these

Q.2 A) Attempt any ONE of the following (8)

a) Prove that the Bernoulli Differential equation $\frac{dy}{dx}$ +py=Qy^n is reduces to linear differential equation by the transformation z=y^{1-n}

b) The general solution of the linear differential equation $\frac{dy}{dx}$ +py=Q is $ye\int pdx = \int Qe\int pdx\, dx + c$ prove the above result.

Q.2 (B) Attempt any two of the following.

a) Find the time required for the sum of money to double itself at 5% p.a compounded continuously.

b) Use the substitution y=vx to solve $\frac{dy}{dx} = \frac{x+y}{x-y}$

c) Solve $(x + 2y)\, dx - dy = dx + dy$

d) Solve $\frac{dy}{dx} = \frac{1}{(x2y3+01xy}$

Q.3 (A) Attempt Any ONE of the following. (8)

a) Let $y,(x)$ and $y2(x)$be any two solutions to the differential equation $y'' + py' + Qy = 0$ on the interval $[a, b]$, then their wronskian $w(y,, y2) = y, y,' - y2y'$ is identically zero if and only if $y, (x)$, and $y2(x)$ are linearly dependant on $[a, b]$

b) Consider the differential equation $y" + py' + Qy = R - (1)p, Q$ constant and R iş function of x with $cf = c, y + c2y2$ where y, and $y2$ are solution of $y" + py' + Qy = 0 - (2)$ let PI is $y = uy, +vy2$ where u and v are functions of x then prove $u = -\int \frac{y2R}{W} dx, v = \int \frac{y,R}{W} dx$ and W is wronskian.

Q.3 (B) Attempt any Two of the following . (12)

a) Solve $y" - 5y' - 6y = e3x$ given $y(0) = 2, y'(0) = 1$

b) Solve the IVP $y' + 12y' + 36y = 0, y(1) = 0, y'(1) = 0$

c) Find the other linearly independent solution to the differential equation $(1 - x2)y" - 2xy' + 2y = 0$ given $y, = x$ is a solution .

d) If $y" + py' + Qy = 0$ then prove that $c, y, (x) + c2y2(x)$ is also a solution for any constant $c1$ *and* $c2$

Q.4 (A) Attempt any ONE of the following . (8)

a) If w(t) wronskian of two solutions $x = x_1(t) y = y_1(t)$ and $x = x2(t), y = y2(t)$ to the homogeneous linear system $\frac{dx}{dt} = a_1(t)x + b_1(t)y, \frac{dy}{dt} = a_1(t)x + b2(t)y$ then prove either w(t) is identically equal to zero or w(t) is never zero on $[a, b]$

b) If the auxiliary equation to the homogeneous system of equation with constant coefficients $\frac{dx}{dt} = a_1x + b_1y, \frac{dy}{dt} = a2x + b2y$ (1)

has real equal roots m,m then there exist non trivial constant A_1,B_1,A_2,b_2,A_3,B_3 such that $x = A_1e^{mt}. y = B_1e^{mt}$ and $x = (A2 + A3t)e^{mt}. y = (B2 + B3t)e^{mt}$ are linearly independent solutions to the system (1)

Q.4(B) Attempt any Two of the following (12)

a) show that $x = 3t - 2, y = -2t + 3$ is particular solution to the non homogeneous system $\frac{dx}{dt} = x + 2y + t - 1, \frac{dy}{dt} = 3x + 2y - 5t - 2$

b) Find the third order differential equation whose equivalent system of linear equation is $\frac{dx1}{dt} = x2\ \frac{dx2}{dt} = x3, \frac{dx3}{dt} = -\frac{8}{t}x3 + \frac{16}{t2}x2 - \frac{5}{t3}x1$

c)Find the general solutions to the system $\frac{dx}{dt} = 4x - 3y, \frac{dy}{dt} 8x - 6y$

d) Prove that $x = 3t + 2, y = 2t - 1$ is a particular solution to the non-homogeneous linear system.

Q.5 Attempt any Four of the following (20)

a) Find the general solution to the given system $\frac{dx}{dt} = x - 4y, \frac{dy}{dt} = x + y$

b) Find the general solution to the system satisfying initial conditions $\frac{dx}{dt} = x + y, \frac{dy}{dt} = y$ with $x(0) = -3, y(0) = 3$

c) solve $y'' + 3y' + 2y = sine^{x}$

d)Solve the IVP for $\frac{d2y}{dx2} - 4\frac{dy}{dx} - 12y = 3e^{5x}$ with $y(0) = \frac{18}{7}$ and $y'(0) = -\frac{1}{7}$

e) Solve $x\frac{dy}{dx} = 4 - 2y$ with initial condition $y(1) = 0$

f) If $(x + y)^n$ is an integrating factor of $94x^2+2x + 6y)dx + (2x^2 +9y + 3x) = 0$ then find n and solve the equation.