(3 Hours) **[Total Marks: 100]**

Note: (i) All questions are compulsory.
(ii)Figures to the right indicate marks for respective parts.

Q.1 Choose correct alternative in each of the following **(20)**

$i.$ A D.E. is considered to be ordinary if it has
(a) more than one dependent variable
(b) one independent variable
(c) more than one independent variable
(d) None of these

$ii.$ The order and degree of the differential equation $\frac{d^2y}{dx^2} + 5xy\frac{dy}{dx} = 6x^2$, is
(a) 2 and 1
(b) 2 and 2
(c) 1 and 2
(d) 1 and 1

$iii.$ The function $f(x, y) = 4x^2 - 7xy + \frac{x^2}{y}\tan\left(\frac{y}{x}\right)$
(a) is homogenous of degree 1
(b) is homogenous of degree 2
(c) is homogenous of degree 3
(d) not homogenous

$iv.$ The differential equation $2x\frac{dy}{dx} - y = 3$, represents a family of
(a) straight lines
(b) circles
(c) parabolas
(d) ellipses

$v.$ General solution of $y'' + 4y = 0$ is $y =$
(a) $c_1e^{2x} + c_2e^{-2x}$
(b) $(c_1 + c_2x)e^{2x}$
(c) $c_1 \sin 2x + c_2 \cos 2x$
(d) $c_1x^2 + c_2x^{-2}$

$vi.$ General solution for differential equation $y'' - y' = 0$ is
(a) $y = c_1e^x + c_2e^{-x}$
(b) $y = c_1 + c_2e^x$
(c) $y = c_1\cos x + c_2 sinx$
(d) $y = ce^x$

$vii.$ Wronskian determinant $W(y_1, y_2)$ with usual symbols is equal to
(a) $y_1y_2' - y_2y_1'$
(b) $y_1y_2' + y_2y_1'$
(c) $y_1y_1' - y_2y_2'$
(d) $y_1y_1' + y_2y_2'$

viii. One of the solutions of the homogeneous linear system of differential equations $\begin{cases} \frac{dx}{dt} = 2x \\ \frac{dy}{dt} = 3y \end{cases}$ is

(a) $\begin{cases} x = 3e^{2t} \\ y = e^{2t} \end{cases}$ (b) $\begin{cases} x = 3e^{2t} \\ y = e^{2t} \end{cases}$

(c) $x = 3e^{2t}$, $y = 5e^{3t}$ (d) None of these

ix. The Wronskian of two solutions $(x_1(t), y_1(t))$ & $(x_2(t), y_2(t))$ for the linear system of first order homogeneous differential equations is

(a) $\begin{vmatrix} x_1(t) & y_1(t) \\ x_1'(t) & y_1'(t) \end{vmatrix} \times \begin{vmatrix} x_2(t) & y_2(t) \\ x_2'(t) & y_2'(t) \end{vmatrix}$ (b) $\begin{vmatrix} x_1(t) & y_1(t) \\ x_1'(t) & y_1'(t) \end{vmatrix} + \begin{vmatrix} x_2(t) & y_2(t) \\ x_2'(t) & y_2'(t) \end{vmatrix}$

(c) $\begin{vmatrix} x_1(t) & y_1(t) \\ x_2(t) & y_2(t) \end{vmatrix}$ (d) None of these

x. One of the solutions of the homogeneous linear system $\begin{cases} \frac{dx}{dt} = -4x + 2y \\ \frac{dy}{dt} = 3x - 3y \end{cases}$ is _____

(a) $\begin{cases} x = 3e^{2t} \\ y = e^{2t} \end{cases}$ (b) $\begin{cases} x = 2e^{-t} \\ y = 3e^{-t} \end{cases}$

(c) $\begin{cases} x = e^{t} \\ y = 3e^{t} \end{cases}$ (d) None of these

Q2. Attempt any **ONE** question from the following: **(08)**

a) i. Show that the general solution of the linear first order O.D.E. $\frac{dy}{dx} + Py = Q$, where P and Q are integrable functions of x, is $y = e^{-\int Pdx}\left(\int Qe^{\int Pdx}dx + c\right)$, c being an arbitrary constant. Hence solve the O.D.E. $\frac{dy}{dx} + 2xy = 4x$.

ii. The current $i(t)$ at time t in an electrical circuit containing a source of e.m.f., inductance and resistance is governed by the differential equation $L\frac{di}{dt} + Ri = E(t)$, where the inductance L and the resistance R are constant whereas the e.m.f. $E(t)$ is a function of time t. Determine the current
(a) If the initial current is 0 and the applied e.m.f. is constant.
(b) If the initial current is 0 and the applied e.m.f. is periodic in time t and given as E(t) = $E_0 \cos \omega t$, where E_0 and ω are constants.

Q.2 Attempt any **TWO** questions from the following: **(12)**

b) i. Show that the following differential equation is non-exact. Hence find the I.F. and solve. $\left(\tan y - 3x^4\right)dx - \left(x \sec^2 y - x^2 \cos y\right)dy = 0$.

ii. Solve: $\frac{dy}{dx} = \frac{x+y}{x-y}$.

iii. Solve the following Bernoulli's Differential equation. $2xy\frac{dy}{dx} = y^2 - 2x^3$.

iv. Find the orthogonal trajectories of $ay^2 = x^3$

Q3. Attempt any **ONE** question from the following: **(08)**

$a)$ i. Let m_1 and m_2 be the roots of the auxiliary equation of the differential equation $y'' + py' + q = 0$, where p and q are constants. Discuss the general solution of the differential equation when
$(a) m_1$ and m_2 are real and unequal.
$(b) m_1$ and m_2 are complex roots

ii. Let $y_1(x)$be a non-zero solution to the differential equation $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$. Then show that another linearly independent solution $y_2(x) = y_1(x) \int \frac{e^{\int -p(x)dx}}{{y_1}^2(x)} dx$

Q3. Attempt any **TWO** questions from the following: **(12)**

$b)$ i. Find the general solution for the differential equation $y'' - 5y' + 6y = 0$

ii. Solve the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = 6e^{4x}$

iii. Using the method of variation of parameters solve $\frac{d^2y}{dx^2} + 4y = cosx$

iv. Show that $y(x) = c_1x + c_2x^2$ is solution for the equation $x^2y'' - 2xy' + 2y = 0$, hence find particular solution,if $y(1) = 3$, $y'(1) = 5$.

Q4. a) Attempt any **ONE** question from the following: **(08)**

i. Prove that the two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ of the homogeneous linear system $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$ are linearly dependent on $[a, b]$ iff their Wronskian is identically zero on $[a, b]$.

ii. What do we mean by the general solution of a system of linear homogeneous O.D.E. of the first order in two variables?

Let $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ be two solutions of the following homogeneous linear system $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$ on $[a, b]$. Prove that $\begin{cases} x = c_1x_1(t) + c_2x_2(t) \\ y = c_1y_1(t) + c_2y_2(t) \end{cases}$ is also a solution for any real constants c_1 and c_2.

Q4.b) Attempt any **TWO** questions from the following: **(12)**

i. Solve the linear system: $\begin{cases} \frac{dx}{dt} = 5x + 4y \\ \frac{dy}{dt} = -x + y \end{cases}$

ii. Find the general solution of the following linear system: $\begin{cases} \frac{dx}{dt} = 7x + 6y \\ \frac{dy}{dt} = 2x + 6y \end{cases}$

iii. Define Wronskian of the two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ of the homogeneous system $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$. Show that this Wronskian is either identically zero or nowhere zero on $[a, b]$.

iv. Show that $(-2e^t \sin 2t, e^t \cos 2t)$ and $(2e^t \cos 2t, e^t \sin 2t)$ are linearly independent solutions of $\begin{cases} \frac{dx}{dt} = x - 4y \\ \frac{dy}{dt} = x + y \end{cases}$

Q5. Attempt any **FOUR** questions from the following: **(20)**

a) Check whether the following differential equations are exact and solve. $(x+y-10)dx+(x-y-2)dy=0$.

b) Solve: $\frac{dy}{dx}+x\tan(y-x)=1$.

c) Show that $y = c_1x + c_2x^{-2}$ is a solution of $x^2y'' + 2xy' - 2y = 0$ on any interval not containing the origin

d) Solve the differential equation $y'' + y = x$ by the method of variation of parameters.

e) Find the general solution of the system: $\begin{cases} \frac{dx}{dt} = 4x - 2y \\ \frac{dy}{dt} = 5x + 2y \end{cases}$

f) Show that both $x_1 = 2e^{5t}, y_1 = e^{5t}$ and $x_2 = e^{-t}, y_2 = -e^{-t}$ are solutions of the system $\begin{cases} \frac{dx}{dt} = 3x + 4y \\ \frac{dy}{dt} = 2x + y \end{cases}$. Also show that these two solutions are linearly independent.
