Ct 24/9/16 (old)

FYBSC- SEM II -MATHS II - $2\frac{1}{2}$HRS - 75 MARKS -

NOTE : 1) All questions are compulsory.

2) For Q.1. Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any three subquestions (each 4 marks) from part (b).

3) For Q.4 , attempt any three.(each 5 marks)

Q.1. (a) Attempt any one. [each 8]

1) Define S(n, k) the stirling number of the second kind and find S(7, 3).

2) Prove that If R is an equivalence relation on X then R induces a partition on X and Check whether following relation is an equivalence or not and write corresponding partition if exist.

i) $R_1 = \{(a,a),(b,b),(c,c),(b,c),(c,b)\}$

ii) $R_2 = \{(a,a),(b,b),(c,c),(a,c),(c,b),\}$

(b) Attempt any three. [each 4]

1) Solve the following recurrence relation

$$a_n = 3a_{n-1,} - 2a_{n-2,}, n \geq 3, a_1 = [illegible], a_2 = 3$$

2) Prove that if R is an equivalence relation on a non empty set X then For any $a, b \in X$, either $[a] = [b]$ or $[a] \cap [b] = \emptyset$ is true where $[a]$ = equivalence class of a

3) Check whether following relation R is an equivalence or not

X = the set of integer $\mathbb{Z}$

For any $a, b \in \mathbb{Z}$ a R b iff $a - b$ is divisible by 4

4) Define the following term

i) Equivalent set

ii) Finite and infinite set

iii) Countably infinite set

Q.2. (a) Attempt any one. [each 8]

1) State and prove Multinomial Theorem.

2)Write down all partitions of an integer n = 5& n=7

(b) Attempt any three. [each 4]

1) Given any integer $n, n_1, n_2, \ldots, n_k$ satisfying $n_1 + n_2 + \cdots + n_k = n$

Then Prove that $\binom{n}{n_1, n_2, \ldots, n_k} = \frac{n!}{n_1! n_2! n_3! \ldots\ldots n_k!}$

2) Using Principle of Inclusion and Exclusion Solve following

In a class of 150 students, 70 have offered Mathematics, 80 have offered Physics and 90 have offered Chemistry. Of these, 40 students are for Maths and Physics, 30 are for Maths and Chemistry and 50 are for Physics and Chemistry. If 10 students have offered all these three subjects. Show that ,there are 20 students from this class, who have neither of these subjects.

3) Show that the number of integers from the set 1 to 100, which are not divisible 4,6,10 is 64.

4) Define the signature of a permutation and find the signature of $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$

Q.3. (a) Attempt any one. [each 8]

1) State and prove Remainder Theorem for polynomial $f(x)$ in $\mathbb{R}[x]$

2) i) Define an irreducible polynomial in $\mathbb{R}[x]$ and prove that $x^2 - 1$ is reducible in $\mathbb{R}[x]$ but not in $\mathbb{Q}[x]$

ii) Find g.c.d (greatest common divisor) of following pairs of polynomials over $\mathbb{Q}[x]$

$$x^2 - 1, \quad x^3 + 2x^2 - x - 2$$

b) Attempt any three. [each 4]

1) Define an unit in $\mathbb{R}[x]$ and prove that The only unit polynomials in $\mathbb{R}[x]$ are the nonzero constant polynomials i.e nonzero real numbers

2) By diving $f(x)$ by $g(x)$ find the quotient and remainder in $\mathbb{R}[x]$

$$f(x) = x^3 - 4x^2 + x + 6, \quad g(x) = x^2 - 1$$

3) Prove that if $p(x)$ is an irreducible polynomial such that $p(x) | a(x)b(x)$ for $a(x), b(x)$ in $\mathbb{R}[x]$ then $p(x) | a(x)$ or $p(x) | b(x)$ in $\mathbb{R}[x]$

4) Express following complex number in the polar form $\sqrt{3} + i$

Q.4. Attempt any three. [each 5]

1) If S(n,k) denotes number of partitions of an n-set X into the k-parts where $n \geq 1$ and $1 \leq k \leq n$ then prove following

i) S(n,1) = 1, S(n,n) = 1

ii) S(n, k) = S(n-1, k-1) + k S(n-1, k), $2 \leq k \leq n - 1$

2) Prove that if R is an equivalence relation on a non empty set X then

$\bigcup_{a \in X}[a] = X$ where $[a]$ = equivalence class of a

3) i) Write down $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 2 & 5 & 1 \end{pmatrix}$ in cyclic form also as a product of Transposition.

ii) Write down (1 3 2)(4 5) cycle of S_5 in the standard form

iii) Find the product $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 2 & 3 & 5 \end{pmatrix}$

iv) Find invers of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 1 & 4 \end{pmatrix}$

4) For any integer $n \geq 2$ exactly half of permutations in S_n are odd and half are even

5) Define an associates in $\mathbb{R}[x]$ and prove that if $f(x), g(x)$ are associates in $\mathbb{R}[x]$ then $f(x) = c.g(x)$ where c is suitable constant in $\mathbb{R}$.

6) State Demoivre's theorm and using it prove that

$$\cos 3\theta = cos^3\theta - 3\cos\theta sin^2\theta$$

$$\sin 3\theta = 3cos^2\theta sin\theta - sin^3\theta$$