

- NOTE :
- 1) All questions are compulsory.
 - 2) For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any three subquestions (each 4 marks) from part (b).
 - 3) For Q.4 , attempt any three.(each 5 marks)

Q.1. (a) Attempt any one. [each 8]

1) Let $\sum a_n, \sum b_n$ be convergent series converging to a & b respectively then prove following

i) If $c_n = a_n - b_n \quad \forall n \in \mathbb{N}$ then $\sum c_n$ is convergent, converging to $a - b$.

ii) If $\alpha \in \mathbb{R}$ is fixed and $c_n = \alpha a_n$ then $\sum c_n$ is convergent, converging to αa .

2) Examine convergence of following series.

i) $\sum \frac{n}{n^2 - \cos^2(n)} \quad \text{ii) } \sum \frac{n^2+2}{n^4+5}$

(b) Attempt any three. [each 4]

1) Find n^{th} partial sum and determine if the series converge or diverge.

$$1 - 3 + 9 - 27 + \dots + (-1)^{n-1}(-3)^{n-1}$$

2) State Leibnitz theorem and examine the convergence of $\sum_{n=2}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$

3) Define an absolute convergence of series and prove that absolute convergent series are convergent.

4) State Modified root test and examine the convergence of $\sum_{n=1}^{\infty} \left(\frac{n+3}{n}\right)^{n^2}$

Q.2. (a) Attempt any one. [each 8]

1) Define the derivative of a real valued function f at $x \in I$ an open interval subset of \mathbb{R} and show that following functions f are differentiable on \mathbb{R}

i) $f: \mathbb{R} \rightarrow \mathbb{R}$ as $f(x) = c \quad c \in \mathbb{R}$.

ii) $f: \mathbb{R} \rightarrow \mathbb{R}$ as $f(x) = a^x \quad a > 0$

2) State Leibnitz theorem and find n^{th} derivative of following

(b) Attempt any three. [each 4]

1) Find y_n , n^{th} order derivative of y if $y = \frac{1}{x^2 - 5x + 6}$

2) If $xy - \log y = 1$ then prove that $y^2 + (xy - 1)y_1 = 0$

3) Find equation of tangent and normal to the following

$$x^2 + xy + 2y^2 = 28 \text{ at } (2,3)$$

4) If $-1 \leq x \leq 1$ then find derivative of $y = \sin^{-1} x$ $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Q.3. (a) Attempt any one. [each 8]

1) State Rolle's Theorem and give their geometrical interpretation.

Verify Rolle's Theorem for $\frac{x^2 - 4x}{x + 2}$ in $[0, 4]$

2) State Lagranges Mean Value Theorem and give their geometrical interpretation.

Verify Lagranges Mean Value Theorem for $f(x) = \sqrt{x^2 - 4}$ in $[2, 3]$

(b) Attempt any three. [each 4]

1) Find the Taylor's Polynomials of degree n at $x = a$ for the mentioned values of a & n

$$f(x) = \cos x \quad a = \frac{\pi}{4}, n = 5$$

2) State L-Hospital's Rule and evaluate following

$$\lim_{x \rightarrow 0} \frac{\log(1+x^3)}{\sin^3 x}$$

3) Find absolute maximum and minimum values of each function on the given interval. $f(x) = \frac{-1}{x^2}$ $[0.5, 2]$

4) Divide the number 22 into two parts so that the sum of their square is minimum

Q.4. Attempt any three. [each 5]

1) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$

2) Prove that the necessary and sufficient condition for a series $\sum a_n$ to be convergent is for any $\epsilon > 0$ $\exists n_0 \in \mathbb{N}$ such that $|\sum_{p=n+1}^m a_p| < \epsilon$ $m > n \geq n_0$

3) If a function $f: I \rightarrow \mathbb{R}$, $I \subseteq \mathbb{R}$ open interval, is differentiable at $c \in I$ then prove that f is continuous at $c \in I$. What about converse?

4) Verify Cauchy Mean Value Theorem for $f(x) = \sqrt{x}$, $g(x) = \frac{1}{\sqrt{x}}$ in $[a, b]$

5) Sketch the graph of $y = x^3 - 3x^2 + 4$

6) Is $f(x) = |x|$ differentiable on \mathbb{R} ?