

2

Q. 1) Attempt the following

[36 MARKS]

- 1) The coefficient of ab^2c^2 in the expansion of $(a + b + c)^5$
 - a) 15
 - b) 30
 - c) 10
 - d) 20
- 2) In the expansion of $(x_1 + x_2 + \dots + x_r)^n$, number of terms are _____.
 - a) $\binom{n}{r}$
 - b) $\binom{n+r-1}{r-1}$
 - c) $\binom{n+r-1}{r}$
 - d) $\binom{n-r}{r-1}$
- 3) ____ non-negative solution are there for an equation $a+b+c=10$, where $a,b,c \geq 0$
 - a) 10
 - b) 12
 - c) 11
 - d) 66
- 4) In ____ many ways can the letters of the word 'LEADER' be arranged.
 - a) 72
 - b) 144
 - c) 360
 - d) 6!
- 5) At a party there are n men and n women. How many ways are there for the dance if everyone has to change partners?
 - a) $n!$
 - b) D_n
 - c) $(n-2)!$
 - d) $(n-1)!$
- 6) $\phi(11)$ is:
 - a) 13
 - b) 10
 - c) 14
 - d) 12
- 7) Degree of recurrence relation $a_n = a_{n-3} + a_{n-1}$ is ____
 - a) 1
 - b) 2
 - c) 3
 - d) 4
- 8) If $a_n = a_{n-1} + 4$ and $a_0 = 2$, then $a_2 =$
 - a) 6
 - b) 8
 - c) 10
 - d) 12
- 9) Which of the following is a linear non homogenous recurrence relation?
 - a) $a_n = 3a_{n-2} + 2a_{n-1}$
 - b) $a_n - 7a_{n-1} + 12a_{n-2} = 0$
 - c) $a_n - 3a_{n-1} = 4n$
 - d) $a_n - 12a_{n-5} = 0$
- 10) Characteristic roots of recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$ are
 - a) 7, 10
 - b) 2, 5
 - c) -7, -10
 - d) -2, -5
- 11) A one - to - one mapping of a set $A = \{a_1, a_2, \dots, a_n\}$ onto itself is called _____.
 - a) Permutation
 - b) combination
 - c) disjoint cycle
 - d) cyclic Permutation
- 12) Two cycles of a set A are said to be disjoint if there is ____ common element in both cycles.
 - a) no
 - b) one
 - c) two
 - d) many
- 13) The set of all subsets of set A is called the _____ set of A .
 - a) Proper subset
 - b) universal set
 - c) singleton set
 - d) power set
- 14) A set which is not countable is called _____ set.
 - a) Denumerable
 - b) countable
 - c) uncountable
 - d) proper
- 15) Rising factorial $[9]^{\uparrow} =$ _____.
 - a) $9 \times 9 \times 9$
 - b) $9 \times 8 \times 7$
 - c) 9^3
 - d) $9 \times 10 \times 11$
- 16) $S(n, 1) =$ _____.
 - a) 0
 - b) 1
 - c) n
 - d) $n-1$
- 17) In class of 25 students ____ minimum students are there who were born in same month.
 - a) 2
 - b) 3
 - c) 4
 - d) 25

18) The total number of partitions of a set with n elements is called _____ number.

- a) Bell b) sterling c) countable d) proper

Q. 2 A) Attempt the following (Solve any 01)

[08 MARKS]

- 1) i) If A and B are countable sets then prove that $A \times B$ is countable.
ii) A and B are two non-empty finite sets with $A \cap B = \emptyset$ prove that $|A \cup B| = |A| + |B|$
- 2) i) Prove that \mathbb{Q} is countable.
ii) In how many ways can we draw a heart or a spade from an ordinary deck of playing cards? A heart or an ace? A card numbered 2 through 10?

B) Attempt the following (Solve any 01)

[05 MARKS]

- 1) Prove that : $S(n, 2) = 2^{(n-1)} - 1$
- 2) How many minimum people are required to guarantee that (i) atleast 2 of them are born exactly at same hour, minute and second? (ii) atleast 5 of them are born exactly at same hour, minute and second?

Q. 3 A) Attempt the following (Solve any 01)

[08 MARKS]

- 1) i) State and prove the Vandermonde's Identity.
ii) State the Binomial theorem. Find the coefficient of x^3y in the expansion of $(x + y)^4$.
- 2) i) State and prove Pascal's Identity.
ii) State the Multinomial number. Find the coefficient of x^2y^2z in expression $(2x - 3y + 4z)^5$.

B) Attempt the following (Solve any 01)

[05 MARKS]

- 1) Prove Derangement formula $d_n = n! \sum_{k=0}^n \frac{(-1)^k}{k!}$.
- 2) How many non- negative solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 < 21$?

Q. 4 A) Attempt the following (Solve any 01)

[08 MARKS]

- 1) i) Prove that the product of two even permutation is even.
ii) Define the terms a) cyclic Permutation b) Even and odd permutation
- 2) Let $A = \{1, 2, 3, 4, 5, 6\}$ and $P_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 2 & 6 & 5 \end{pmatrix}$, $P_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 4 & 6 \end{pmatrix}$, $P_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 5 & 4 & 1 \end{pmatrix}$.
Compute (1) $(P_2 \circ P_3)^{-1}$ (2) $P_3^{-1} \circ P_2^{-1}$ (3) $(P_1 \circ P_2) \circ P_3$

B) Attempt the following (Solve any 01)

[05 MARKS]

- 1) Solve the recurrence relation $a_n = -2a_{n-1} + 2a_{n-2} + 4a_{n-3}$, $a_1 = 0$, $a_2 = 2$, $a_3 = 8$.
- 2) Solve the recurrence relation $a_n = 2a_{n-1} - 2a_{n-2}$, $a_1 = 1$, $a_2 = 4$.