

120522

VCD - _____ - FYBSC (PCM/PMS) SEM IIATKT-Calculus II-100MARKS - 3HRS

Q.1) Tick mark the correct option (2 mks each)

- 1) The geometric series $\sum ar^{n-1}$ is convergent if
A) $|r|=0$ always B) $|r| > 1$ C) $|r| < 1$ D) $|r|=1$
- 2) The function $f(x) = 10 - 12x - 3x^2 - x^3$ is
A) always increasing B) always decreasing C) both A and B D) neither increasing nor decreasing
- 3) critical points of $f(x) = 5x^2 + 4x$ are
A) $-\frac{2}{5}$ B) $\frac{2}{5}$ C) 4 D) 0
- 4) The derivative of the inverse function of $f(x) = x^2 + 3x - 7$ at $x=1$ is
A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) 3 D) 1
- 5) The sum of the series given below if exist then it is
 $1/9 + 1/27 + 1/81 + \dots$
A) 0 B) $\frac{1}{3}$ C) 7 D) 1
- 6) If the series given $\sum a_n$ is convergent then
 $\lim_{n \rightarrow \infty} a_n =$
A) 1 B) 3 C) 2 D) 0
- 7) If the series $\sum b_n, \sum a_n$ is convergent then
A) $\sum(a_n + b_n)$ is convergent B) $\sum(a_n + b_n)$ is not convergent C) $\sum(a_n b_n)$ is not convergent D) $\sum(a_n + b_n) = 0$
- 8) The series is said to be conditionally convergent if it is
A) convergent but not absolutely convergent
B) convergent and absolutely convergent
C) absolutely convergent but not convergent
D) neither convergent nor absolutely convergent
- 9) The sequence $a_n = 1/n^2$ then
A) a_n and $\sum a_n$ both converge B) a_n and $\sum a_n$ both diverge C) a_n is convergent and $\sum a_n$ does not converge. D) a_n and $\sum a_n$ both undefined
- 10) Every bounded sequence in \mathbb{R} has.....
A) unbounded subsequence
B) convergent subsequence
C) divergent subsequence
D) no subsequence
- 11) If a function $f: [a, b] \rightarrow \mathbb{R}$ is continuous then
A) f is bounded and attains its bounds
B) f is bounded and doesn't attain its bounds
C) f is not bounded and attains its bounds
D) f is neither bounded nor attain its bounds

- 12) The polynomial x^2+1 has
 A) No zero in \mathbb{R} B) two zero in \mathbb{R}
 C) three zero in \mathbb{R} D) four zero in \mathbb{R}
- 13) $f, g: [a, b] \rightarrow \mathbb{R}$ are continuous such that $g(a) < f(a) < 0$, $g(b) > f(b) > 0$ then
 A) $f(x) \cdot g(x)$ is not equal to zero for any x in $[a, b]$ B) $f(x) \cdot g(x)$ is equal to zero for some x in $[a, b]$
 C) $f(x) \cdot g(x)$ is equal to one for some x in $[a, b]$ D) $f(x) \cdot g(x)$ is equal to 2 for some x in $[a, b]$
- 14) The series $\sum (-1)^n$
 A) oscillates between -1 and 0
 B) oscillates between -1 and 2
 C) oscillates between -2 and 0
 D) oscillates between 1 and 0
- 15) The series $\sum 1/n^p$ is convergent if
 A) $p < 1$ B) $p > 1$ C) $p = 1$ D) $p < 0$
- 16) If a function from I to \mathbb{R} where I is an open interval, is differentiable at point p in I then
 A) f is a nonconstant function on \mathbb{R}
 B) f is a constant function on \mathbb{R}
 C) f is continuous on \mathbb{R}
 D) f is discontinuous on \mathbb{R}
- 17) If $y = xe^x$ then n th derivative $y_n =$
 A) e^{x+n} B) $e^x(x+n)$ C) $(x+n)$ D) 0
- 18) If a function from \mathbb{R} to \mathbb{R} is differentiable even function then
 A) f is odd function B) f is even function
 C) f is odd function D) f is zero function
- 19) The function on \mathbb{R} defined as $f(x) = 1/1+|x|$ is
 A) continuous and unbounded
 B) continuous and bounded
 C) neither continuous nor bounded
 D) discontinuous and bounded
- 20) Divide 100 into two parts such that sum of their square is minimum then parts are.....
 A) 50, 50 B) 35, 65 C) 40, 60 D) 30, 70
- 21) --- gives us existence of atleast one point between $x=a$, $x=b$ at which derivative of the function vanishes.
 A) Lagrange's Mean Value theorem
 B) Cauchy's Mean Value Theorem
 C) Rolle's theorem
 D) Bolzano's theorem
- 22) $\sum 1/n(n+1)$
 A) 1 B) 0 C) 3 D) 2
- 23) $\sum ar^{n-1}$
 A) $a/(1-r)$ B) $1/1-r$ C) $a/1+r$ D) 0

24) $\lim_{x \rightarrow 0} x^x$

A)0 B)1 C)3 D)5

25) n^{th} derivative of x^n is

A)n! B) n C)0 D)n-1

Q.2) Attempt any three of the following (5 MKS each)

1) Prove that the geometric series $\sum ar^{n-1}$ is convergent if $|r| < 1$

2) Express the number 5.232323... as a ratio of two integers.

3) Prove that If the series $\sum a_n$ convergent then $\lim_{n \rightarrow \infty} a_n = 0$

4) State The Limit Comparison test for convergence of the series and investigate convergence of $1 + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + \dots$

Q.3) Attempt any three of the following (5 MKS each)

1) Show that $x^3 - 15x + 1 = 0$ has multiple zeros in $[-4, 4]$

2) Prove that If a function $f: [a, b] \rightarrow \mathbb{R}$ is continuous then it is bounded and attains its bounds.

3) Find the derivative of the inverse function of $y = xe^x$

4) Prove that If a function $f: I \rightarrow \mathbb{R}$ where I is an open interval is differentiable at $p \in I$ then f is continuous at p .

Q.4) Attempt any three of the following (5 MKS each)

1) Prove that if a real valued function f is derivable in (a, b) and $f'(x) \neq 0$ for any $x \in (a, b)$ then the function is one one in (a, b)

2) State and prove Lagrange's Mean Value theorem for real valued function.

3) Evaluate $\lim_{x \rightarrow 0} x \log x$

4) Obtain the expansion of $e^{\sin x}$ upto the first four terms.

Q.5) Attempt any one of the following (5 MKS each)

1) Discuss the convergence of alternating harmonic series given by $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$

2) Check whether the following function is differentiable at 0 or not?

$$f(x) = \frac{1}{x} \quad x \neq 0$$

$$= 1 \quad x = 0$$

3) Verify Rolle's theorem for $f: [2, 3] \rightarrow \mathbb{R}$ as $f(x) = x^2$