

ATKT
2015-16

VCD - 30/5115 - MATHS II - FYBSC - SEM II EXAM - 75 MARKS - 2.5HRS - 220

Note : 1) All questions are compulsory.

- 2) For Q.1, Q.2, Q.3, attempt any one subquestion (each 8 mks) from part (a), and any three subquestions (each 4 mks) from part(b)
- 3) For Q.4 Attempt any three.(each 5 mks)

Q.1 (a) Attempt any one [Each 8]

1) Solve following equations using Gaussian elimination method

$$\begin{aligned}x + 7y + 3z &= 11 \\x + y + z &= 3 \\4x + 10y - z &= 13\end{aligned}$$

2) Write the note on elementary transformation and define following with example.

- i) Transpose of matrix
- ii) Diagonal matrix
- iii) Symmetric matrix
- iv) Upper and lower triangular matrix

Q.1 (b) Attempt any three. [Each 4]

- 1) Find Parametric equation of a plane passing through points (1,2,3), (4,5,6), (7,8,9)
- 2) Give geometric interpretation of solution of system of m homogeneous linear equations in n unknowns.
- 3) Define an invertible matrix and prove that $(A^{-1})^T = (A^T)^{-1}$ where A^{-1}, A^T are inverse and transpose of A respectively.
- 4) Define addition and multiplication of matrices and find AB, BA for following

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 5 \\ 2 & 7 \end{bmatrix}, B = \begin{bmatrix} 4 & 2 & 3 \\ 7 & 1 & 5 \end{bmatrix}$$

Q.2 (a) Attempt any one [Each 8]

1) Prove that $(\mathbb{R}[x], +, \circ)$ is a vector space over \mathbb{R} where $\mathbb{R}[x]$ = Set of all polynomials in x with real coefficients.

2) Let V be a vector space over \mathbb{R} and W be a nonempty subset of V . Then prove that W is a subspace of V iff $ax + by \in W$ whenever $x, y \in W, a, b \in \mathbb{R}$

Q.2 (b) Attempt any three. [Each 4]

1) Show that intersection of finitely many subspaces of a vector space V is also a subspace of V .

2) Express given vector $x = (1, 2, 0) \in \mathbb{R}^3$ as linear combination of given vectors $x_1 = (1, 0, 0), x_2 = (0, 1, 1), x_3 = (0, 1, 3)$

3) Find Linear Span of $S = \{(1, 1), (0, 2)\}$ of \mathbb{R}^2

4) Define a subspace W of a vector space V and prove that

$W = \{(x, y, z) : z = x + y, x, y, z \in \mathbb{R}\}$ a subset of \mathbb{R}^3 is a subspace of \mathbb{R}^3 .

Q.3 (a) Attempt any one [Each 8]

1) Prove that rotation of a vector X through an angle θ in anticlockwise direction is a linear transformation.

2) State Rank - Nullity Theorem and verify it for following

$T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(x, y) = (x, x + 2y, y + 3x)$

Q.3 (b) Attempt any three. [Each 4]

1) When is the map $T: V \rightarrow U$ where V, U are vector spaces over \mathbb{R} said to be a linear transformation? check whether following map is a linear transformation or not.

$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x, y) = (y, 0)$

2) Let V, U be a vector spaces over \mathbb{R} . If a transformation $T: V \rightarrow U$ is such that $T(ax + by) = aT(x) + bT(y) \quad \forall x, y \in V, a, b \in \mathbb{R}$ then prove that T is a linear transformation.

3) Let V, V' be a vector spaces over \mathbb{R} and $T: V \rightarrow V'$ be a linear transformation then prove following

i) $\text{Ker } T$ is a subspace of V

ii) $\text{Img } T$ is a subspace of V' .

4) Let V, U be vector spaces over \mathbb{R} . If $T: V \rightarrow U$ is a linear transformation then prove that

i) $T(0) = 0$ ii) $T(-x) = -T(x) \quad \forall x \in V$

Q.4 (a) Attempt any three [Each 5]

1) Define the row echelon form of a matrix and reduce following matrix in row echelon form.

$$A = \begin{bmatrix} 3 & 3 & 9 \\ 0 & 5 & 0 \\ 5 & 8 & 1 \end{bmatrix}$$

2) Find inverse of $A = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & -3 \end{bmatrix}$ using elementary row transformation.

3) If $a \neq 0, a \in \mathbb{R}$ then show that $\{(1+a, 1-a), (1-a, 1+a)\}$ is linearly independent subset of \mathbb{R}^2

4) Express the function $1 + 2t + 3t^2$ as linear combination of $1, 1+t, 1+t^2$

5) Prove that every finitely generated vector space has a finite basis.

6) Prove that Sum of Linear Transformation is also a Linear Transformation.