VCD: 011222-FYCS-SEM-I- Discrete Mathematics-$2\frac{1}{2}$hrs-MARKS-75 (174)

Note: (i) All questions are compulsory.
(ii) Figures to the right indicate marks.
(iii) Illustrations, in-depth answers and diagrams will be appreciated.
(iv) Mixing of sub-questions is not allowed.

Q1. Attempt any four of the following: **(20 marks)**

a. Given $A = \{1,2,3,4\}$ and $B = \{x, y, z\}$. Let R be the following relation from A to B: $R = \{ (1,y), (1,z), (3,y), (4,x), (4,z) \}$
   (i) Determine the matrix of relation.
   (ii) Find the inverse relation of R.
   (iii) Determine domain and range of R.

b. Let $A = \{1,2,3,4,5\}$ and R be partial order relation defined as
   $R = \{ (1,1), (2,2), (3,3), (4,4), (5,5), (5,3), (5,1), (4,3), (4,2), (4,1), (3,1), (2,1)\}$.
   Find Hasse diagram of poset A.

c. Check whether the function $f : \boldsymbol{R} \to \boldsymbol{R}$ $s.t. f(x) = 3x - 2$ is bijective or not. If yes, find the inverse of f.

d. $f(x) = x^2, g(x) = x - 3$.
   (i) Find fog and gof
   (ii) Find $fog(5)$ and $gof(7)$

e. Solve the following recurrence relation:
   $$a_n = 6\,a_{n-1} - 9\,a_{n-2}\ ;\quad a_0 = 1, a_1 = 6$$

f. Formulate and solve Tower of Hanoi problem.

Q2. Attempt any four of the following: **(20 marks)**

a. How many 3 digit numbers can be formed using the digits $0 - 9$ if
   (i) Repetition of digits is allowed.
   (ii) Repetition of digits is not allowed.

b. What is the coefficient of $a\,b^3\,c^2$ in the expansion of $(a + 2b - 3c)^6$

c. How many minimum numbers must be selected from the set $\{1,2,3,4,5,6,7,8\}$ to guarantee that atleast one pair of these numbers sums to 9?

d. Among 100 students, 55 students passed in Mathematics, 30 passed in Science and 15 passed in both Mathematics and Science. How many students passed in
   (i) Atleast one subject
   (ii) Only in Mathematics.

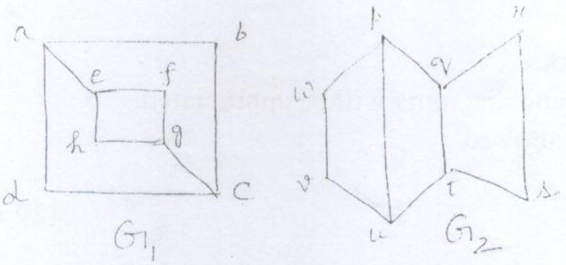
e. Let G be the phase structure grammar where $T = \{a, b, c\}, S$ is the starting symbol and productions are $\{S \to a\,Sb, aS \to Aa, Aab \to c\}$. Find $L(G)$.

f. State and prove Pascal's identity.

Q3. Attempt any four of the following: **(20 marks)**

a. Define binary search tree, complete binary tree and extended binary tree using examples.

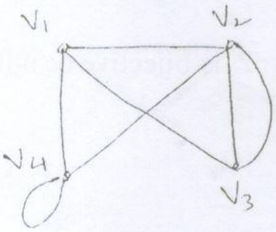
b. Check whether the following 2 graphs are isomorphic or not.



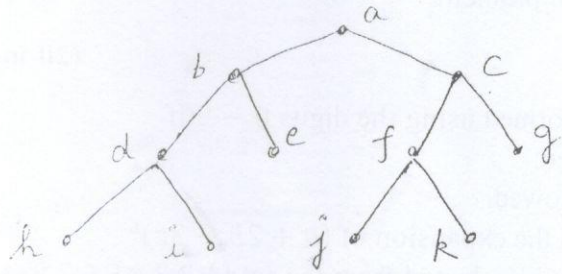
c. (i) What do you mean by chromatic number of a graph?

(ii) What is the chromatic number of K_5(complete graph on 5 vertices) and a complete bipartite graph $K_{3,3}$?

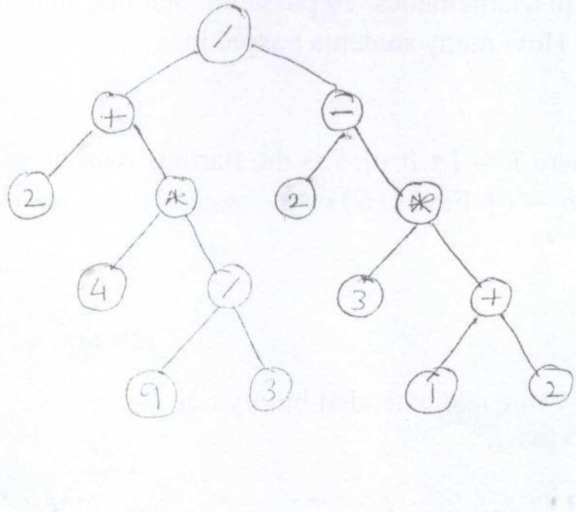
d. For the following graph, find the degree of all the vertices and the degree of graph.



e. Traverse the following tree using pre order and post order algorithm



f. Determine the value of the expression.



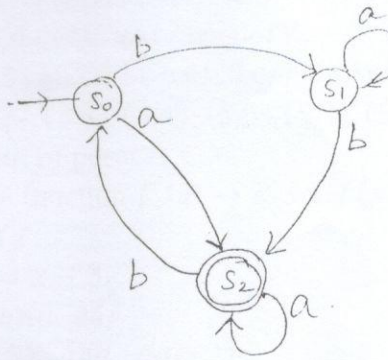
Q4. Attempt any three of the following: **(15 marks)**

a. Formulate and solve recurrence relation representing number of regions in which plane is divided by n number of lines where no two lines are parallel and no three lines intersect at a common point.

b. Let A be the set of lines in a plane. Define the relation R on A by $a\ R\ b$ if and only if line a is parallel to line b. Show that R is an equivalence relation.

c. In how many ways can a committee of 3 men and 2 women be chosen from 7 men and 5 women?

d. Consider the following state diagram of finite state automata. Find (a) states (b) input letters (c) Initial State (d) accepting state (e) $f(S_1, b)$ (f) Find State table



e. Define the following with examples (i) Isolated Vertex (ii) Simple graph (iii) Regular graph (iv) Complete graph (v) Planar Graph

f. Represent the following expressions using binary tree :

(i) $(a - b) \div (cd + e)$

(ii) $(x - xy) + \left(\frac{x}{y}\right)$