

Duration:[2½Hours]

[Total Marks: 75]

N.B. 1) All questions are compulsory.

2) Figures to the right indicate full marks.

1. (a) Attempt any **ONE** question: (8)
- Let G be a graph with vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$ and adjacency matrix $A = [a_{ij}]$. Then show that the entry $a_{ij}^{(k)}$ in i^{th} row and j^{th} column of A^k is the number of distinct $v_i - v_j$ walks of length k in G .
 - State and prove *Havel – Hakimi* theorem for degree sequence of a graph G .
- (b) Attempt any **TWO** questions: (12)
- Define isomorphism of graphs. Give an example of non isomorphic graphs that has equal number of vertices and equal number of edges. Justify your answer.
 - Show that in a party of 6 or more people, either there are 3 persons who know one another or there are three persons who do not know one another.
 - If G is simple graph with p vertices, q edges and k components, then prove that $q \geq p - k$.
2. (a) Attempt any **ONE** question: (8)
- State and prove Cayley's formula for spanning trees.
 - Define Cut Edge in a simple graph G . Show that an edge e of a graph G is a cut edge if and only if there exists two vertices x and y such that e lies on every $x - y$ path in G .
- (b) Attempt any **TWO** questions: (12)
- Show that there exist a tree with degree sequence $d_1 \geq d_2 \geq \dots \geq d_n$ if and only if
 - $d_i \geq 1$, for $1 \leq i \leq n$ and
 - $\sum_{i=1}^n d_i = 2n - 2$
 - Show that every non-trivial graph contains at least two vertices which are non-cut vertices.
 - State Huffman's algorithm for prefix code.
3. (a) Attempt any **ONE** question: (8)
- If a connected graph G contains exactly two vertices of odd degree say x and y , then show that it contains a (x, y) -Eulerian trail.
 - If G is a graph on p vertices with $p \geq 3$ such that $deg(u) + deg(v) \geq p$ for every pair of non adjacent vertices u and v in G , then prove that G is Hamiltonian.
- (b) Attempt any **TWO** questions: (12)
- Define closure of a graph $C(G)$ and show that a simple graph is Hamiltonian if and only if its closure is Hamiltonian.

- ii. Prove that the cube graph Q_k is bipartite k -regular graph with 2^k vertices.
- iii. Describe Fluery's Algorithm to find a closed Eulerian trail.

4. Attempt any **THREE** questions:

(15)

- (a) Let G be a simple graph and $\delta(G) \geq 2$, then show that there exists a cycle of length at least $\delta(G) + 1$ in G .
- (b) Explain Dijkstra's algorithm and show that Dijkstra's algorithm produces the shortest path.
- (c) Show that any two longest paths in a connected graph G has a vertex in common.
- (d) Describe Depth First Search (DFS) algorithm. Use DFS to find spanning tree for the Complete graph K_5 .
- (e) Show that the line graph a simple graph G is a path if and only if G is a path.
- (f) Let G be a connected graph with $2n$ odd vertices with $n \geq 1$. Show that $E(G)$ can be partitioned into subsets E_1, E_2, \dots, E_n so that $\langle E_i \rangle$ is an open trail for each i .
