

2) Let $S \neq \emptyset$, open subset of \mathbb{R}^2 , $(a, b) \in S$ be stationary point of f . Suppose $f(x, y)$ possesses continuous second order partial derivative in some neighborhood of (a, b) . Let $A = f_{xx}(a, b)$, $B = f_{xy}(a, b)$, $C = f_{yy}(a, b)$. Let $\Delta = AC - B^2$. Prove that i) if $A > 0$, $\Delta > 0$ then f has local minimum at (a, b) ii) if $A < 0$, $\Delta > 0$ then f has local maximum at (a, b)

(b) Attempt any two. [each 6 Mks]

1) Find the total derivative of f at $(1, 1, 1)$ in Jacobian form and also in linear transformation form where $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ as $f(x, y, z) = (x+y, y+z, z+x)$

2) Define the Differentiability of a vector valued function $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$ at $a \in \mathbb{R}^n$. Let $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$ be differentiable at $a \in \mathbb{R}^n$ then prove that αf is differentiable at a , $\alpha \in \mathbb{R}$ and $D(\alpha f)(a) = \alpha Df(a)$

3) Find the maximum possible rate of change $f(x, y, z) = I_n(x+y+z)$ at $(1, 2, 3)$. Also find the direction in which such a maximum rate of change occurs.

Q.4. Attempt any three. [each 5 Mks]

1) Find f_x, f_y at $(0, 0)$ if exists for $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ is defined as $f(x, y) = x^3/(x^2+y^2)$ if $(x, y) \neq (0, 0)$
 $= 0$ if $(x, y) = (0, 0)$

2) For following function f , find the real $\theta \in (0, 1)$ if exists satisfying $f(b) - f(a) = \nabla f(a + (b-a)\theta) \cdot (b-a)$ where $f(x, y) = x^2 + x + y$, $a = (0, 0)$, $b = (1, -1)$

3) Using chain rule find the total derivative of $f(x, y, z) = xy^2 + yz^2 + zx^2$, $x(t) = e^t$, $y(t) = \sin t$, $z(t) = \cos t$.

4) Find directional derivative of f at a in direction of u $f(x, y, z) = z^2 - x^2 - y^2$, $a = (1, 0, 1)$, $u = (4, 3, 0)$

5) Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}^3$, $g: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ as $f(x, y) = (3x^3 + 4xy, y^2 + 3x, x^2 + y^2) = (f_1, f_2, f_3)$, $g(u, v, w) = (uvw, u^2 + v^2 + w^2) = (g_1, g_2)$ Find $J(f(x, y))$, $J(f(g(a)))$, $a = (1, -1, -1)$

6) Locate all critical points of $f(x, y) = x^3 - 6xy + 3y^2 - 2yx + 4$

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NOTE : 1) For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any two subquestions (each 6marks) from part (b). For Q.4 , attempt any three. (each 5 marks)

Q.1. (a) Attempt any one. [each 8Mks]

1) Define a continuity of a vector valued function $f: S \rightarrow \mathbb{R}^m, S \neq \emptyset$ subset of \mathbb{R}^n . Prove that for nonempty subset S of $\mathbb{R}^n, f, g: S \rightarrow \mathbb{R}^m$ continuous at $a \in S$ then $f-g$ is continuous at $a \in S$.

2) Prove that sequence $w_n = (s_n, t_n)$ in \mathbb{R}^2 converges to a limit $w = (s, t) \in \mathbb{R}^2$ iff $(s_n) \rightarrow s$ & $(t_n) \rightarrow t$

(b) Attempt any two. [each 6Mks]

1) Define norm of x where $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ and prove that $\|x+y\| \leq \|x\| + \|y\|, x, y \in \mathbb{R}^n$

2) Show that $S = \{(x, y) \in \mathbb{R}^2 / 3x + 4y < 12\}$ is an open set.

3) Define $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $f(x, y) = x \sin \frac{1}{y} + y \cos \frac{1}{x}$ for $(x, y) \neq (0, 0)$

= 0. otherwise Find $\lim_{(x, y) \rightarrow (0, 0)} f(x, y)$ if exists

Q.2. (a) Attempt any one. [each 8Mks]

1) State and prove Euler's theorem for function of two variable.

2) Define a Differentiability of a scalar valued function $f: S \rightarrow \mathbb{R}, S \neq \emptyset$ subset of \mathbb{R}^n at point $a \in S$. Prove that for nonempty subset S of $\mathbb{R}^n, f: S \rightarrow \mathbb{R}$ be differentiable at $a \in S$ then $\partial f(a) / \partial x_i$ exists for $i=1, 2, \dots, n$

(b) Attempt any two. [each 6Mks]

1) Evaluate total derivative of f using definition at the mentioned point $f(x, y, z) = x^2 + 2y^2 + 3z, a = (1, -1, 0)$

2) Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ as $f(x, y) = 3 \sin x + y \cos x$. Find $f_x, f_y, f_{xx}, f_{yx}, f_{xy}, f_{xyy}$.

3) Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ be non constant differentiable function, $k \in \mathbb{R}, f(x, y) = k$ describes the curve C having tangent at each of its points then prove that i) gradient vector ∇f is normal to C ii) the directional derivative of f is zero along C

Q.3. (a) Attempt any one. [each 8Mks]

1) Let $S \neq \emptyset$, open subset of $\mathbb{R}^n, a \in S$ & $f: S \rightarrow \mathbb{R}$ be a scalar field. Let f be differentiable at a . If f has a local maximum or local minimum at a then prove that $\nabla f(a) = 0$.