(c) What is space quantization? Calculate the possible orientations of the total angular momentum vector J corresponding to j=3/2 with respect to a magnetic field along the z axis. (5)

Useful integral

$$\int_{-\infty}^{+\infty} dx\, exp(-\alpha x^2 + \beta x) = \sqrt{\frac{\pi}{\alpha}} \exp(\beta^2/4\alpha)$$

$$\int_{0}^{+\infty} dx x^n\, exp(-\alpha x) = \frac{n!}{\alpha^{n+1}}$$

[This question paper contains 8 printed pages.]

27.12.2023 (M)

Your Roll No...............

Sr. No. of Question Paper : 4334 G

Unique Paper Code : 32221501

Name of the Paper : Quantum Mechanics and Applications

Name of the Course : **B.Sc. (Hons)- Physics CBCS**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **FIVE** questions in all .
3. **Question No. 1** is compulsory.
4. All questions carry equal marks
5. Non programmable calculators allowed

1. Attempt **any five** of the following

(a) Verify whether the following operators are Hermitian:

(i) $-i\hbar\frac{d}{dx}$

(ii) Hamiltonian of free particle

(iii) Hamiltonian of a particle in a I-d harmonic oscillator potential.

(b) Evaluate the following commutators : if $[\hat{x}, \hat{p}_x] = i\hbar$

(i) $[\hat{P}_x, \hat{L}_x]$

(ii) $[\hat{L}_i, \hat{L^2}]; i = x, y, z$

(iii) $[\hat{H}, \hat{L^2}]$

(c) What are the stationary states? Why they are so called?

(d) Find the momentum space wave function corresponding to $e^{-\alpha x}$.

(b) Apply L_Z on the wave function $\psi_{2}1-1$ for the hydrogen atom and find the eigen value of L_Z and corresponding eigen function.

Given : $\psi_{21-1} = \frac{1}{8\sqrt{\pi}} \cdot (\frac{1}{a_0})^{3/2} \frac{r}{a_0} \cdot e^{-r/2a_0} \cdot \sin\theta e^{-i\phi}$

(9, 6)

6. (a) Draw the energy level diagram to show the Zeeman splitting of the ground and first excited states of sodium in a week magnetic field. Also show the transitions allowed by the selection rules.

(b) Describe Stern Gerlach Experiment with necessary theory. What does it demonstrate? (7, 8)

7. (a) Find the terms for 3p4d configuration system in LS Coupling. Show them in diagram. (5)

(b) Is $^2D_{1/2}$ a possible term? Calculate the angle between vector I and vector s in the $^2P_{3/2}$ state of a one electron atom. (5)

to $\frac{1}{n^3}$ for a large value of n . Explain the meaning of degeneracy. Show that degree of degeneracy of nth energy level is given by $2n^2$ in hydrogen atom. (9, 6)

5. (a) The ground state wave function of hydrogen atom is : $\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} \exp(-r/a_0)$

(i) Calculate the expectation value $<r>$ and $<\frac{1}{r}>$ in the ground state of Hydrogen atom.

(ii) Calculate the probability of finding the particle at a distance less than a_0.

(iii) Write the wave function $\psi(r)$ and energy eigen value E_n for hydrogenic atom having at Z charge and principle quantum n.

(e) A particle is represented by the wave function $\psi(x, 0) = A(a^2 - x^2)$ if $-a \leq x \leq a$ and $\psi(x, 0) = 0$, otherwise. Find the uncertainty in p.

(f) Assume that a magnetic dipole, whose moment has magnitude μ_l, is aligned parallel to external magnetic field whose strength has magnitude B. Take $\mu_l = 1$ Bohr magneton and $B = 1T$. Calculate the energy required to turn the magnetic dipole so that it is aligned antiparallel to the field.

(5×3=15)

2. (a) Consider a one-dimentional infinite potential well of width L. The wave function is given by

$$\Psi(x,0) = \begin{cases} 3\phi_1(x) + 4\phi_2(x) & \textit{inside the well} \\ 0 & \textit{outside the well} \end{cases}$$

where $\phi_1(x)$ and $\phi_2(x)$ are normalized wavefunctions of the ground state and first excited state.

(i) Normalize $\psi(x, 0)$.

(ii) Find the average energy of the particle.

(iii) Write down $\psi(x, t)$.

(iv) Find the probability of finding the particle in the interval [0, L/2] at two different times,

$$t = \frac{\hbar}{3E_1} \cdot \frac{\pi}{2} \text{ and } t = \frac{\hbar}{3E_1} \cdot \pi . 4$$

(b) A particle of mass m, which moves freely inside an infinite potential well of length 'a', is initially in the state: $\Psi(x,0) = \sqrt{\frac{1}{5a}} \sin(\frac{\pi x}{a}) + \sqrt{\frac{3}{5a}} \sin(\frac{3\pi x}{a})$

(a) Find $\psi(x, t)$ at any later time t.

(b) If the energy is measured, what are the possible values of energies with what probabilities? (9, 6)

3. (a) Solve the Schrodinger equation for a Linear Harmonic Oscillator. Obtain and plot first three eigenfunctions. Calculate the number of node in first three eigen functions. Also explain why they are symmetric and antisymmetric.

(b) Calculate the uncertainty in momentum and position for the ground state eigenfunction of linear Harmonic Oscillator and hence obtain uncertainty relation. (8, 7)

4. (a) Write the Schrodinger equation for a 3D hydrogen atom in spherical polar coordinates. Derive three separate equations for r, θ, ϕ using the method of separation of variables. Solve the equation for ϕ to obtain the normalized eigenfunctions and show that they are orhogonal.

(b) Show that the fractional difference in the energy between adjacent eigenvalues $\frac{\Delta E_n}{E_n}$ proportional