(b) Prove the recurrence relation :

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$

for the *Chebyshev polynomials.* (5)

4. (a) Decompose the matrix $A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$ into the form LU where L is a unit lower triangular and U is an upper triangular matrix. (10)

(b) Use the factors obtained above to solve the system $AX = B$ where $B = \begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}$. (5)

5. (a) Given

$$\frac{dy}{dx} = y - x$$

Where y(0) = 2. Find y(0.2) with h = 0.1 using Runge-Kutta second order formula up to three decimal places. (8)

(b) Solve the boundary value problem $y''(x) = y(x)$; $y(0) = 0$, $y(1) = 1.1752$ by the shooting method,, taking the two initial approximations for $y'(0)$ as 0.7 and 0.8 and h = 0.5. Perform only first iteration. (7)





[This question paper contains 4 printed pages.]

09.01.24(M)

Your Roll No..............



Sr. No. of Question Paper : 1708 G

Unique Paper Code : 2223012001

Name of the Paper : Numerical Analysis (DSE)

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : III

Duration : 2 Hours Maximum Marks : 60

## Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt total **four** questions in all with question no. **1** being compulsory.
3. **All** questions carry equal marks.
4. Scientific non-programmable calculators are allowed.

1. All questions are compulsory (5×3=15)

(a) Find the relative error in

$$u = \frac{5xy^2}{z^3}$$

P.T.O.

at $x = y = z = 1$ when the relative error in each of x, y, z is 0.01.

(b) Given a vector $A = [-1, 4, 2]$ calculate the 1-norm, 2-norm and the $\infty$-norm.

(c) Find the eigen values and corresponding normalised eigen vectors of the matrix :

$$\begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}$$

(d) Evaluate the following integral using Trapezoidal Rule with $n = 4$ correct to three decimal places.

$$\int_0^1 x^2 dx$$

(e) Given $\frac{dy}{dx} = x^2$ with $y(0) = 1$.

Find the value of y(l) using Euler's method with $h = 0.5$.

2. (a) Using Gauss Elimination method, solve the following system of equations :



$$2x + 2y + z = 6$$
$$4x + 2y + 3z = 4$$
$$x - y + z = 0$$ (8)

(b) Using power method determine the largest eigenvalue and the corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Let the initial eigenvector be

$$X^0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

Perform 3 iterations. (7)

3. (a) Find the value of f(8) using Newton's *Divided Difference Method* for the following dataset.

| x | 4 | 5 | 7 | 10 | 11 | 13 |
|---|---|---|---|---|---|---|
| f(x) | 48 | 100 | 294 | 900 | 1210 | 2028 |

(10)