(iii) Muons are unstable particles that spontaneously decay into an electron and two neutrinos. If the number of muons at time $t = 0$ is N_0, the number at time t is given by $N = N_0 e^{-t/\tau}$ where $\tau = 2.2 \mu s$ is the mean lifetime of a muon in a frame of reference in which the muon is at rest. If a bunch of muons are moving at speed 0.95c, what is the observed mean lifetime? What fraction of muons remain after the bunch has travelled a distance of 10.0 km? (6)

Morning 02/01/2024

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1593 G

Unique Paper Code : 2222011102

Name of the Paper : Mechanics

Name of the Course : **B.Sc. Hons. (Physics)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** Questions carry equal marks.
3. Q. No. **1** is compulsory.
4. Answer **any four** of the remaining five questions.
5. Use of non-programmable scientific calculators are allowed.

1. Attempt all parts of this question.

(i) A block of mass 2 kg is placed on a frictionless platform and the coefficient of static friction between theblock and the platform is 0.6. The platform is subjected to an acceleration a. Determine the maximum acceleration of the platform such that the block does not slip on it. (3)

(ii) Find the work done in moving a particle from the point A(–2, 1, 3) to B(1, –2, –1) in the force field

$$\vec{F} = \left(y^2z^3 - 6xz^2\right)\hat{i} + 2xyz^3\hat{j} + \left(3xy^2z^2 - 6x^2z\right)\hat{k}.$$

(3)

(iii) A cosmic ray proton with energy 10^{20} eV crosses our galaxy, which has diameter of about 10^5 light years. How long does it take the proton to traverse the galaxy, in its own rest frame? Use the following : 1 eV = 1.6×10^{-19} J, mass of proton is about $m_p = 1.67 \times 10^{-27}$ kg. (3)

(iv) The centre of mass of three particles of masses 10 gm, 20 gm and 30 gm is at (1, –2, 3). Where should the fourth particle of mass 40 gm be placed so that the centre of mass of the combined system is at (1,1,1)? (3)

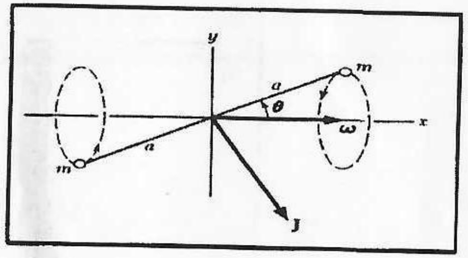
(ii) A planet of mass m and angular momentum l moves in a circular orbit in a central potential $U(r) = -kr^{n+1}$, where k is a constant and r is the distance of the particle from the origin. Find the angular frequency of radial oscillations if the particle is slightly perturbed radially. Determine the value of n for which the orbit can be stable. (6)

(iii) A particle of mass 50 g moves under an attractive central force of magnitude $4r^3$ dynes. The angular momentum is equal to 1000 g cm^2s^{-1}. Find the effective potential energy of the particle. If the radius of the particle's orbit varies between r_0 and $2r_0$ then determine r_0. (6)

6. (i) A frame of reference is rotating relative to an inertial frame with a constant angular speed ω. Determine expressions for the Centrifugal and the Coriolis forces acting on a particle of mass *m*, observed to move in the rotating frame of reference. (6)

(ii) Two particles of rest mass m_0 approach each other with equal and opposite velocity *v*. What is the total energy of one particle as measured in the rest frame of the other? (6)

at angle θ with the length of the rod as shown in the figure. Derive an expression for the angular momentum $\vec{J}$ of the rod and hence show that $\vec{J}$ rotates with the rod always inclined at a fixed angle relative to the rod and to the axis of rotation. (6)



(iii) A bowling ball is thrown down the alley with speed v_0. Initially it slides without rolling, but due to friction it begins to roll. What is its speed when it rolls without sliding. (6)

5. (i) Using the radial and tangential equation of motion, show that the angular momentum l and the energy E of a particle of mass m moving in a potential U(r) can be expressed as $l = mr^2\dot{\theta}$ and $E = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} + U(r)$. Hence, obtain the effective force acting on the particle. (6)

(v) A particle of mass m moves in a central force field defined by $\vec{F} = -\frac{k}{r^4}\vec{r}$. If E is the total energy of the particle then find the speed of the particle. (3)

(vi) A particle of mass 10 kg is moving in a circle of radius 4 m with a constant speed of 5 ms^{-1}. What is its angular momentum about a point on the axis passing through the centre of the circle and perpendicular to its plane, at a distance of 3 m from its centre? (3)

2. (i) Explain the principle of rocket propulsion. Formulate the equation of motion of a rocket and hence deduce an expression for the instantaneous velocity of the rocket that takes off vertically upwards from rest under the influence of gravity. (6)

(ii) A rocket launched vertically expels mass at a constant rate equal to 0.05 m_0 kg ms^{-1}, where m_0 is its initial mass. The exhaust velocity of the gases relative to the rocket is 5 km s^{-1}. Find the velocity & the height of the rocket after 10 s. (6)

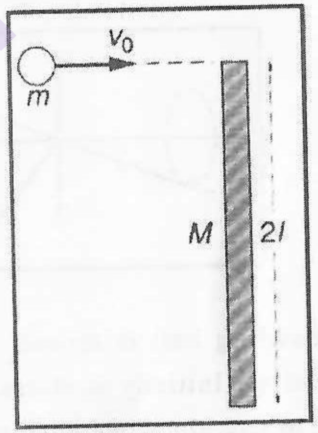
(iii) Determine the centre of mass of a circular plate of uniform thickness t and diameter 2R. If a circular hole of diameter R is drilled into the plate at its boundary, will the centre of mass of the remaining portion of the plate change? If yes, then determine the centre of mass of the remaining portion of the plate. (6)

3. (i) State and derive an expression for Work-Energy Theorem. (6)

(ii) A commonly used potential energy function to describe the interaction between two atoms is the Lennard-Jones potential defined as

$U(r)=\varepsilon\left[\left(\frac{r_0}{r}\right)^{12}-2\left(\frac{r_0}{r}\right)^{6}\right]$, where r is the separation between the two atoms. Are the two atoms interacting with each other by a central force? Show that the radius at the potential minimum is r_0 and that the depth of the potential is ε. Find the frequency of small oscillations about equilibrium for two identical atoms of mass m bound to each other by the Lennard-Jones interaction. (6)

(iii) A plank of length $2l$ and mass M lies on a frictionless table. A ball of mass m and speed v_0 strikes its end as shown in the figure. Find the final velocity of the ball, v_f, assuming that mechanical energy is conserved and that v_f is along the original line of motion.



(6)

4. (i) Derive an expressions for Moment of Inertia of a uniform mass distribution in the form of a spherical shell about one of its tangential axis. (6)

(ii) Consider a rigid body consisting of two equal masses m joined by a massless rod of length 2a. The rod is made to rotate about a fixed axis through the Centre of Mass (CM) and oriented