

$$y(x, 0) = \begin{cases} \frac{3ax}{L}, & 0 < x < \frac{L}{3} \\ \frac{3a}{2L}(L-x), & \frac{L}{3} < x < L \end{cases}$$

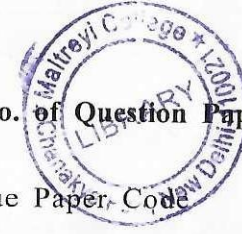
Using the 1-D wave equation, find the deflection

$y(x, t)$ at any time t . (12)

Momin

27/12/2023

[This question paper contains 8 printed pages.]



Your Roll No.....

Sr. No. of Question Paper : 1536

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Unique Paper Code : 2222012301

Name of the Paper : Mathematical Physics – III

Name of the Course : B.Sc. (H) Physics

Semester : III

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. **All** questions carry equal marks.
4. Question number **1** is compulsory.
5. Attempt any **two** questions from **Section A**, any **one** question from **Section B** and any **one** question from **Section C**.

1. Attempt **all** questions :

(6×3=18)

(a) Solve $z^3 + 1 = 0$ and plot the roots.

(b) Discuss the analyticity of $|z|^2$ in finite z -plane.

(c) Show that $1 + \cos 72^\circ + \cos 144^\circ + \cos 216^\circ + \cos 288^\circ = 0$.

(d) Find the principal value of $\ln(\sqrt{3} - i)$.

(e) Obtain the Fourier Sine integral representation of $f(x)$, where

$$f(x) = \begin{cases} 4, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

(f) A flexible string of length L is stretched between its fixed ends lying at $x = 0$ and $x = L$. It is released from rest. Write the initial and boundary conditions for the displacement of the string $g(x, t)$.

(b) A rectangular plate ($a \times b$) with insulated surfaces has its temperature $u(x, y)$ as

$$u(0, y) = 0, u(a, y) = 0,$$

$$u(x, 0) = 0, u(x, b) = F(x)$$

Using 2-D Laplace equation, determine the steady state temperature distribution within the plate. The temperature of the plate has an upper bound $|u(x, y)| < M$. (12)

8. (a) Using method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial y} + u$$

$$\text{such that } u(x, 0) = 4e^{-3x} \quad (6)$$

(b) A string is stretched between the fixed points $(0, 0)$ and $(L, 0)$ and released at rest from the initial deflection given by (a is some constant)

6. (a) If Fourier transform of $f(x)$ is given by

$$\mathfrak{F}[f(x)] = F(k)$$

then show that

$$\mathfrak{F}\{f(bx)\} = \frac{1}{|b|} F\left(\frac{k}{b}\right); b \text{ is some constant.} \quad (6)$$

- (b) Find the Fourier transform of Dirac-Delta function,

$$\delta(x). \quad (6)$$

- (c) Plot the function and find its Fourier transform

$$f(x) = e^{-a|x|}, a > 0 \quad (6)$$

Section C

7. (a) Using method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$$

$$\text{such that } u(0, y) = 8e^{-3y} \quad (6)$$

Section A

2. (a) Use de Moivre's theorem to prove that :

$$\frac{\sin(4\theta)}{\sin(\theta)} = 2\cos(3\theta) + 2\cos(\theta) \quad (7)$$

- (b) Find all the roots of

$$(1 + z)^5 = (1 - z)^5 \quad (7)$$

- (c) Locate and state the nature of the singularities of the following function in the finite complex plane :

$$f(z) = \frac{(z + 2i)^{1/3}}{z(z^2 + 1)^2} \quad (4)$$

3. (a) Determine the value of a such that the function $u(x, y) = ax^2 - 3xy$ is harmonic and find its conjugate function $v(x, y)$ such that $f(z) = u + iv$ is analytic. (8)

- (b) Evaluate the following integrals where the closed curve C is the positively oriented boundary of square whose sides lie along the lines $x = \pm 1$ and $y = \pm 1$ (5,5)

$$(i) \frac{1}{2\pi i} \oint_C \frac{\sin(z)}{(z^2 - 4)} dz$$

$$(ii) \oint_C \frac{z \cos(z)}{(2z + 1)^2} dz$$

4. (a) Use residue theorem to evaluate the integral :

(8)

$$\int_0^\infty \frac{x^4}{(x^2 + 9)(x^2 + 4)} dx$$

- (b) Expand the function $f(z) = \frac{1}{z^2(z+1)}$ in a

Laurent series valid for

(5,5)

$$(i) 0 < |z| < 1$$

$$(ii) |z| > 1$$

Section B

5. (a) Find the Fourier Sine transform of e^{-x} and hence evaluate the integral

$$\int_0^\infty \frac{k \sin(kx)}{1 + k^2} dk \quad (9)$$

- (b) Fourier Sine transform of $g(x)$ is given by the following equation

$$\sqrt{\frac{2}{\pi}} \int_0^\infty g(x) \sin(kx) dx = \begin{cases} 0; & k < 0 \\ 1 - k; & 0 \leq k \leq 1, \\ 0; & k > 1 \end{cases}$$

find $g(x)$.

(9)

P.T.O.