$$y(x,0)=\begin{cases}\dfrac{3ax}{L}, & 0<x<\dfrac{L}{3}\\ \dfrac{3a}{2L}(L-x), & \dfrac{L}{3}<x<L\end{cases}$$

Using the 1-D wave equation, find the deflection y(x, t) at any time t. (12)

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Monty 27/12/2023

[This question paper contains 8 printed pages.]

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Your Roll No.....

Sr. No. of Question Paper : 1536 **G**

Unique Paper Code : 2222012301

Name of the Paper : Mathematical Physics – III

Name of the Course : **B.Sc. (H) Physics**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. **All** questions carry equal marks.
4. Question number **1** is compulsory.
5. Attempt any **two** questions from **Section A**, any **one** question from **Section B** and any **one** question from **Section C**.

P.T.O.

1. Attempt **all** questions : (6×3=18)

(a) Solve $z^3 + 1 = 0$ and plot the roots.

(b) Discuss the analyticity of $|z|^2$ in finite z-plane.

(c) Show that $1 + \cos 72° + \cos 144° + \cos 216° + \cos 288° = 0$.

(d) Find the principal value of $\ln(\sqrt{3} - i)$.

(e) Obtain the Fourier Sine integral representation of $f(x)$, where

$$f(x) = \begin{cases} 4, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

(f) A flexible string of length L is stretched between its fixed ends lying at x = 0 and x = L. It is released from rest. Write the initial and boundary conditions for the displacement of the string g(x, t).

(b) A rectangular plate (a × b) with insulated surfaces has its temperature u(x, y) as

$$u(0, y) = 0,\ u(a, y) = 0,$$

$$u(x, 0) = 0,\ u(x, b) = F(x)$$

Using 2-D Laplace equation, determine the steady state temperature distribution within the plate. The temperature of the plate has an upper bound $|u(x, y)| < M$. (12)

8. (a) Using method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y} + u$$

such that $u(x, 0) = 4e^{-3x}$ (6)

(b) A string is stretched between the fixed points (0, 0) and (L, 0) andreleased at rest from the initial deflection given by (a is some constant)

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6. (a) If Fourier transform of f(x) is given by

$$\Im[f(x)] = F(k)$$

then show that

$$\Im\{f(bx)\} = \frac{1}{|b|} F\left(\frac{k}{b}\right);\ b \text{ is some constant.} \quad (6)$$

(b) Find the Fourier transform of Dirac-Delta function, $\delta(x)$. (6)

(c) Plot the function and find its Fourier transform

$f(x) = e^{-a|x|},\ a > 0$ (6)

Section C

7. (a) Using method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$$

such that $u(0, y) = 8e^{-3y}$ (6)

Section A

2. (a) Use de Moivre's theorem to prove that :

$$\frac{\sin(4\theta)}{\sin(\theta)} = 2\cos(3\theta) + 2\cos(\theta) \quad (7)$$

(b) Find all the roots of

$$(1 + z)^5 = (1 - z)^5 \quad (7)$$

(c) Locate and state the nature of the singularities of the following function in the finite complex plane :

$$f(z) = \frac{(z + 2i)^{1/3}}{z(z^2 + 1)^2} \quad (4)$$

3. (a) Determine the value of a such that the function $u(x, y) = ax^2 - 3xy$ is harmonic and find its conjugate function $v(x, y)$ such that $f(z) = u + iv$ is analytic. (8)

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(b) Evaluate the following integrals where the closed curve C is the positively oriented boun-dary of square whose sides lie along the lines $x = \pm 1$ and $y = \pm 1$ (5,5)

(i) $\frac{1}{2\pi i}\oint_C \frac{\sin(z)}{(z^2-4)}dz$

(ii) $\oint_C \frac{z\cos(z)}{(2z+1)^2}dz$

4. (a) Use residue theorem to evaluate the integral : (8)

$$\int_0^\infty \frac{x^4}{(x^2+9)(x^2+4)}dx$$

(b) Expand the function $f(z) = \frac{1}{z^2(z+1)}$ in a Laurent series valid for (5,5)

(i) $0 < |z| < 1$

(ii) $|z| > 1$

Section B

5. (a) Find the Fourier Sine transform of e^{-x} and hence evaluate the integral

$$\int_0^\infty \frac{k\sin(kx)}{1+k^2}dk$$ (9)

(b) Fourier Sine transform of g(x) is given by the following equation

$$\sqrt{\frac{2}{\pi}}\int_0^\infty g(x)\sin(kx)\,dx = \begin{cases} 0; & k < 0 \\ 1-k; & 0 \le k \le 1, \\ 0; & k > 1 \end{cases}$$

find g(x). (9)

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