(b) Prove that:

$$\int_{-1}^{1} P_m(x)\,P_n(x)\,dx = \frac{2}{2n+1}\,\delta_{mn} \qquad (7)$$

6. (a) Prove that:

$$\int_{0}^{1} x\,J_n(ax)\,J_n(bx)\,dx = \frac{1}{2}\,J_{n+1}^2(x)\,\delta_{ab} \qquad (12)$$

where $J_n(a) = J_n(b) = 0$

(b) Show that:

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\,\sin x \qquad (3)$$

7. Determine the solution of one-dimensional wave equation (15)

$$\frac{\partial^2 u}{\partial t^2} = c^2\,\frac{\partial^2 u}{\partial x^2}, \qquad 0 \le x \le l, \qquad t > 0$$

under the boundary conditions $u(0, t) = u(l, t) = 0$ and initial conditions

$$u(x, 0) = \begin{cases} x, & 0 \le x \le l/2 \\ l - x, & l/2 < x \le l \end{cases}$$

and $\left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$

[This question paper contains 4 printed pages.]

28.12.2023(M)

Your Roll No..............

Sr. No. of Question Paper : 4352 **G**

Unique Paper Code : 32221301

Name of the Paper : Mathematical Physics II

Name of the Course : **B. Sc. (Hons) Physics (CBCS-LOCF)**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt five questions in all.
3. Question No. 1 is compulsory.

1. Attempt any *five* questions: (5 × 3 = 15)

(a) Determine if the following functions are odd, even or neither of them:

(i) $f(x) = \sin x \qquad \text{if } -\pi < x < \pi$

(ii) $f(x) = x^2 \qquad \text{if } 0 < x < 2\pi$

(iii) $f(x) = \begin{cases} 2 - x & \text{if} \quad 0 < x < 4 \\ x - 6 & \text{if} \quad 4 < x < 8 \end{cases}$

(b) Given $f(x + 2\pi) = f(x)$ and $f(x) = |x|$, $-\pi \le x \le \pi$ Plot $f(x)$ in the interval $(-3\pi, 3\pi)$.

(c) Evaluate: $\int_0^\infty x^2 e^{-2x^2} dx$

(d) If $\Gamma(1/2) = \sqrt{\pi}$, find the value of $\Gamma(-1/2)$.

(e) Find the value of $P_n(-1)$

2. (a) Given, $f(x) = \pi - x,\ 0 \le x \le \pi$. (7, 3)

Find Fourier sine series and hence show that:

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$$

(b) Prove that even function has no sine terms in its Fourier expansion. (5)

3. Find the Fourier series expansion of the function:

$f(x) = x^2,\ -\pi < x < \pi$

Hence, prove that $1 + \frac{1}{9} + \frac{1}{25} + \ldots = \frac{\pi^2}{8}$..

(12, 3)

4. Using Frobenius Method, solve the following differential equation:

$$x^2 y'' + 4xy' + (x^2 + 2)y = 0 \qquad (15)$$

5. (a) The generating function of Legendre polynomials is given by:

$$(1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} t^n P_n(x),$$

,

Using this generating function, find

$P_0(x), P_1(x), P_2(x), P_3(x), P_4(x)$. (8)