

1555

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(iii) Prove

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = -\nabla^2 \vec{A} + \vec{\nabla} (\vec{\nabla} \cdot \vec{A}) \quad (6)$$

5. (i) Verify Divergence Theorem for

$$\vec{F} = (x^2) \hat{i} + (y^2) \hat{j} + (z^2) \hat{k}$$

taken over the cube $0 \leq x, y, z \leq 1$. (9)

(ii) Verify Green's theorem in the plane for

$$\oint (3x^2 - 8y^2) dx + (4y - 6xy) dy$$

over C which is boundary of the region defined

$$\text{by } y = \sqrt{x}, y = x^2. \quad (9)$$

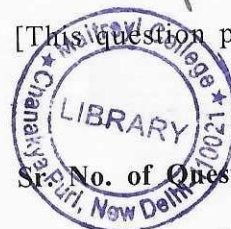
6. (i) Show that scalar product of two vectors is invariant under rotation of axes. (4)

(ii) Find an expression for the mean and variance of Poisson distribution. (8)

(iii) Evaluate $\iiint (2x + y) dV$ where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2$ and $z = 0$. (6)

(500)

[This question paper contains 4 printed pages.]

28.12.2023 (M)
Your Roll No.....

Sr. No. of Question Paper : 1555

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Unique Paper Code : 2222011101

Name of the Paper : Mathematical Physics - I

Name of the Course : B.Sc. Hons. Physics

Semester : I

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

- Write your Roll No. on the top immediately on receipt of this question paper.
 - Question 1 is Compulsory.
 - Attempt any **four** questions from question Numbers 2-6.
 - All** questions carry equal marks.
- (a) By calculating the Wronskian of the functions x, x^2 and x^3 check whether the functions are linearly dependent or independent.
(b) Find the coordinates P(1,2) with reference to the new axes, when the axes are rotated by 30° in anticlockwise direction.

P.T.O.

- (c) Find the unit tangent vector to any point on the curve

$$x = (t^2+1), y = (4t-3), z = (2t^2-6t) \quad t > 0$$

- (d) Show that if $\Phi(x, y, z)$ is any solution of Laplace equation $\nabla^2 \Phi = 0$, then $\vec{\nabla} \Phi$ is a vector which is both solenoidal and irrotational.

- (e) Show that $\oint_S (\vec{\nabla} \cdot \vec{r}) \cdot \vec{dS} = 6V$ where V is the volume enclosed by surface S .

- (f) The probability distribution function is defined by

X:	0	1	2	3	4	5	6
P(X):	k	3k	5k	7k	9k	11k	13k

Find $P(3 < X \leq 6)$. (3×6)

2. (a) Solve by Method of Variation of Parameters:

$$d^2y / dx^2 - y = 2 / (1 + e^x) \quad (6)$$

- (b) Consider an LCR circuit, governed by the differential equation

$$d^2I/dt^2 + \frac{R}{L} dI/dt + \frac{1}{LC} I = \frac{1}{L} dE(t)/dt$$

It is connected in series and has $R = 10$ ohms, $C = 10^{-2}$ farad, $L = 1/2$ henry and an applied voltage $E = 12$ V. Assuming no initial current

and no initial charge at $t = 0$ when the voltage is first applied, find the subsequent current for the problem. (6)

- (c) Solve the differential equation :

$$x^2 d^2y/dx^2 - 2x dy/dx + 2y = x \log x \quad (6)$$

3. (i) Solve by Method of Undetermined Coefficients:

$$d^2y/dx^2 + 10 dy/dx + 25y = 14 e^{-5x} \quad (6)$$

- (ii) Show that following equation is inexact equation and solve it :

$$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0 \quad (6)$$

- (iii) Solve the following differential equation:

$$dy/dx + y/x = y^2 \quad (6)$$

4. (i) Show that

$$\vec{\nabla} f(r) = f'(r) \vec{r} / r \quad \text{where } f'(r) = df(r)/dr$$

$$\text{where } \vec{r} = x \hat{i} + y \hat{j} + z \hat{k}. \quad (6)$$

- (ii) Show that

$$\vec{F} = (y^2 \cos x + z^3) \hat{i} + (2y \sin x - 4) \hat{j} + (3xz^2 + 2) \hat{k}$$

is a conservative force field and then evaluate $\oint_C \vec{F} \cdot d\vec{r}$ over any contour C from $(0, 1, -1)$ to $(\pi/2, -1, 2)$. (6)