(iii) Prove

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = -\nabla^2 \vec{A} + \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) \qquad (6)$$

5. (i) Verify Divergence Theorem for

$$\vec{F} = (x^2)\,\hat{\imath} + (y^2)\,\hat{\jmath} + (z^2)\,\hat{k}$$

taken over the cube $0 \le x, y, z \le 1$. (9)

(ii) Verify Green' theorem in the plane for

$$\oint (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$$

over C which is boundary of the region defined by $y = \sqrt{x}$, $y = x^2$. (9)

6. (i) Show that scalar product of two vectors is invariant under rotation of axes. (4)

(ii) Find an expression for the mean and variance of Poisson distribution. (8)

(iii) Evaluate $\iiint (2x + y)\,dV$ where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0$, $y = 0$, $y = 2$ and $z = 0$. (6)

Physics

[This question paper contains 4 printed pages.]

28.12.2023 (M)

Your Roll No..............

Sr. No. of Question Paper : 1555 **G**

Unique Paper Code : 2222011101

Name of the Paper : Mathematical Physics – I

Name of the Course : **B.Sc. Hons. Physics**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question **1** is Compulsory.
3. Attempt any **four** questions from question Numbers **2-6**.
4. **All** questions carry equal marks.

1. (a) By calculating the Wronskian of the functions x, x^2 and x^3 check whether the functions are linearly dependent or independent.

(b) Find the coordinates P(1,2) with reference to the new axes, when the axes are rotated by 30° in anticlockwise direction.

P.T.O.

(c) Find the unit tangent vector to any point on the curve

$x = (t^2+1),\ y = (4t-3),\ z = (2t^2-6t) \quad t>0$

(d) Show that if $\Phi(x, y, z)$ is any solution of Laplace equation $\nabla^2 \Phi = 0$, then $\vec{\nabla}\Phi$ is a vector which is both solenoidal and irrotational.

(e) Show that $\oiint (\overrightarrow{\nabla} r^2).\overrightarrow{dS} = 6V$ where V is the volume enclosed by surface S.

(f) The probability distribution function is defined by

X:	0	1	2	3	4	5	6
P(X):	k	3k	5k	7k	9k	11k	13k

Find $P(3 < X \leq 6)$. (3×6)

2. (a) Solve by Method of Variation of Parameters:

$d^2y/dx^2 - y = 2/(1+e^x)$ (6)

(b) Consider an LCR circuit, governed by the differential equation

$$d^2I/dt^2 + \frac{R}{L}\, dI/dt + \frac{1}{LC} I = \frac{1}{L}\, dE(t)/dt$$

It is connected in series and has R = 10 ohms, C = 10^{-2} farad, L = 1/2 henry and an applied voltage E = 12 V. Assuming no initial current

and no initial charge at t = 0 when the voltage is first applied, find the subsequent current for the problem. (6)

(c) Solve the differential equation :

$x^2\, d^2y/dx^2 - 2x\, dy/dx + 2y = x \log x$ (6)

3. (i) Solve by Method of Undetermined Coefficients:

$d^2y/dx^2 + 10\, dy/dx + 25y = 14\, e^{-5x}$ (6)

(ii) Show that following equation is inexact equation and solve it :

$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$ (6)

(iii) Solve the following differential equation:

$dy/dx + y/x = y^2$ (6)

4. (i) Show that

$\vec{\nabla} f(r) = f'(r)\, \vec{r}/r$ where $f'(r) = df(r)/dr$

where $\vec{r} = x\,\hat{\imath} + y\hat{\jmath} + z\,\hat{k}$. (6)

(ii) Show that

$\vec{F} = (y^2 \cos x + z^3)\,\hat{\imath} + (2y \sin x - 4)\,\hat{\jmath} + (3xz^2 + 2)\,\hat{k}$ is a conservative force field and then evaluate $\oint \vec{F}.\overrightarrow{dr}$ over any contour C from (0, 1, −1) to (π/2, −1, 2). (6)