5. (a) Find all the asymptotes of the curve

$$x^3 + x^2y - xy^2 - y^3 - 2x^2 + 2y^2 + x + y + 1 = 0.$$

(b) Trace the curve

$$x^2(a^2 - x^2) = a^2y^2,\ a > 0.$$

(c) If $u_n = \int_0^{\pi/2} \sin^n x\, dx$, show that $u_n = \frac{n-1}{n} u_{n-2}$.

Hence evaluate u_5.

6. (a) Prove that the curve

$$(a + y)^2(b^2 - y^2) = x^2y^2,\ a > 0,\ b > 0$$

has at $x = 0$, $y = -a$, a node if $b > a$, a cusp if $b = a$ and a conjugate point if $b < a$.

(b) Trace the curve

$$x(x - 3a)^2 = 9ay^2,\ a > 0.$$

(c) Determine the intervals of concavity and points of inflexion of the curve

$$y = 3x^5 - 40x^3 + 3x - 20.$$

Mathematics 2/01/2024 B.Sc (P) Morning

[This question paper contains 4 printed pages.]

Your Roll No. 467

Sr. No. of Question Paper : 1768 **G**

Unique Paper Code : 2352571101

Name of the Paper : DSC: Topics in Calculus

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **Two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Let $f(x) = \begin{cases} \frac{xe^{1/x}}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Show that f is continuous but not differentiable at $x = 0$.

(b) If $y = \tan^{-1} x$, prove that

$$(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$$

(c) State Euler's theorem and if $z = \sec^{-1}\dfrac{x^3+y^3}{x+y}$, show that $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 2\cot z$.

2. (a) Let $f(x) = |x-5|$, show that f is continuous but not differentiable at $x = 5$.

(b) Find n^{th} derivative of

(i) $\dfrac{1}{1-5x+6x^2}$

(ii) $\sin 3x \sin 2x$

(c) If $u = x^2 \tan^{-1}\dfrac{y}{x} - y^2 \tan^{-1}\dfrac{x}{y}$, prove that

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{x^2-y^2}{x^2+y^2}.$$

3. (a) State Rolle's theorem. Show that there is no real no. k for which the equation $x^3 - 3x + k = 0$ has two distinct roots in [0,1].

(b) Verify Lagrange's Mean Value Theorem for the following functions :

(i) $f(x) = \sqrt{x^2-4}, \quad x \in [2,4]$

(ii) $f(x) = x(x-1)(x-2), \quad x \in \left[0, \frac{1}{2}\right]$

(c) Determine the values of a and b for which.

$\lim\limits_{x \to 0} \dfrac{x(1+a\cos x) - b\sin x}{x^3}$ exists and equals 1.

4. (a) State Maclaurin's theorem. Also, find the Maclaurin's series for

$f(x) = \log(1+x), \; x \in (-1, 1]$.

(b) State Cauchy's mean value theorem. Verify it for the following functions :

(i) $f(x) = x^2, \; g(x) = x$ in $[-1,1]$.

(ii) $f(x) = \dfrac{1}{x^2}, \; g(x) = \dfrac{1}{x}$ in $[2,3]$.

(c) Use Lagrange's Mean Value theorem to prove that

$$\frac{x}{1+x^2} < \tan^{-1}x < x, \; x > 0.$$