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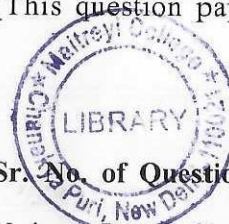
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Section III

- Find the mass of a wire in the shape of curve $C: x = 3 \sin t, y = 3 \cos t, z = 2t$ for $0 \leq t \leq \pi$ and density at point (x, y, z) on the curve is $\delta(x, y, z) = z$.
- Find the work done by force $\vec{F} = x\hat{i} + y\hat{j} + (xz - y)\hat{k}$ on an object moving along the curve C given by $R(t) = t^2\hat{i} + 2t\hat{j} + 4t^3\hat{k}$.
- Use Green's theorem to find the work done by the force field $\vec{F}(x, y) = y^2\hat{i} + x^2\hat{j}$ when an object moves once counterclockwise around the circular path $x^2 + y^2 = 2$.
- State and prove Green's Theorem.
- Evaluate $\oint (2xy^2z \, dx + 2x^2yz \, dy + (x^2y^2 - 2z) \, dz)$ where C is the curve given by $x = \cos t, y = \sin t, z = \sin t, 0 \leq t \leq 2\pi$ traversed in the direction of increasing t .
- Use divergence theorem to evaluate $\iint_S \vec{F} \cdot \vec{N} \, ds$ where $\vec{F} = (x^5 + 10xy^2z^2)\hat{i} + (y^5 + 10yx^2z^2)\hat{j} + (z^5 + 10zy^2x^2)\hat{k}$ and S is closed hemisphere surface $z = \sqrt{1 - x^2 - y^2}$ together with the disk $x^2 + y^2 \leq 1$ in x - y plane.

(1000)

[This question paper contains 4 printed pages.]



04.01.2024(M)
Your Roll No.....

Sr. No. of Question Paper : 4518

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Unique Paper Code : 32351303

Name of the Paper : Multivariate Calculus

Name of the Course : B.Sc. (H) Mathematics

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

- Write your Roll No. on the top immediately on receipt of this question paper.
- All sections are compulsory.
- Attempt any **Five** questions from each section.
- All questions carry equal marks.

Section I

- Find the following limits :

(i) $\lim_{(x,y) \rightarrow (0,0)} (1 + x^2 + y^2)^{\frac{1}{x^2 + y^2}}$

(ii) $\lim_{(x,y) \rightarrow (0,0)} x \log \sqrt{x^2 + y^2}$

P.T.O.

2. Find an equation for each horizontal tangent-plane to the surface

$$z = 5 - x^2 - y^2 + 4y$$

3. The output at a certain factory is $Q = 150K^{\frac{2}{3}}L^{\frac{1}{3}}$ where K is the capital investment in units of \$1000, and L is the size of Labor force measured in worker-hours. The current capital investment is \$500,000 and 150 worker hours of Labor are used. Estimate the change in output that results when capital investment is increased by \$500 and Labor is decreased by 4 worker-hours.

4. Let $w = f(t)$ be a differentiable function of t where $t = (x^2 + y^2 + z^2)^{1/2}$. Show that

$$(dw/dt)^2 = (\partial w/\partial x)^2 + (\partial w/\partial y)^2 + (\partial w/\partial z)^2.$$

5. Let $f(x, y, z) = xyz$ and let \hat{u} be a unit vector perpendicular to both $\vec{v} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} - \hat{k}$. Find the directional derivative of f at $P_0(1, -1, 2)$ in the direction of \hat{u} .

6. Find the absolute extrema of the function $f(x, y) = e^{x^2 - y^2}$ over the disk $x^2 + y^2 \leq 1$.

Section II

1. Evaluate the double integral $\iint_D \frac{dA}{y^2 + 1}$ where D is triangle bounded by $x=2y$, $y=-x$ and $y=2$.

2. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{x^2 + y^2} dy dx$ by converting to polar coordinates.

3. Find the volume of tetrahedron T bounded by plane $2x + y + 3z = 6$ and co-ordinate planes.

4. Use spherical co-ordinates to verify that volume of a half sphere of radius R is $\frac{2}{3}\pi R^3$.

5. Use cylindrical co-ordinates to compute the integral $\iiint_D z(x^2 + y^2)^{-\frac{1}{2}} dx dy dz$ where D is the solid bounded above by the plane $z=2$ and below by the surface $2z = x^2 + y^2$.

6. Use a suitable change of variables to compute the double integral $\iint_D \left(\frac{x-y}{x+y}\right)^2 dy dx$, where D is the triangular region bounded by line $x + y = 1$ and co-ordinate axes.