Section III

1. Find the mass of a wire in the shape of curve C: $x = 3\sin t$, $y = 3\cos t$, $z = 2t$ for $0 \le t \le \pi$ and density at point (x, y, z) on the curve is $\delta(x, y, z) = z$.

2. Find the work done by force $\vec{F} = x\hat{i} + y\hat{j} + (xz - y)\hat{k}$ on an object moving along the curve C given by $R(t) = t^2\hat{i} + 2t\hat{j} + 4t^3\hat{k}$.

3. Use Green's theorem to find the work done by the force field $\vec{F}(x,y) = y^2\hat{i} + x^2\hat{j}$ when an object moves once counterclockwise around the circular path $x^2 + y^2 = 2$.

4. State and prove Green's Theorem.

5. Evaluate $\oint\left(2xy^2z\,dx + 2x^2yz\,dy + \left(x^2y^2 - 2z\right)dz\right)$ where C is the curve given by $x = \cos t$, $y = \sin t$, $z = \sin t$, $0 \le t \le 2\pi$ traversed in the direction of increasing t.

6. Use divergence theorem to evaluate $\iint_S \vec{F}.N\,ds$ where
$$\vec{F} = \left(x^5 + 10xy^2z^2\right)\hat{i} + \left(y^5 + 10yx^2z^2\right)\hat{j} + \left(z^5 + 10zy^2x^2\right)\hat{k}$$
and S is closed hemisphere surface $z = \sqrt{1 - x^2 - y^2}$ together with the disk $x^2 + y^2 \le 1$ in x-y plane.

[This question paper contains 4 printed pages.]

04.01.2024(M)

Your Roll No..............

Sr. No. of Question Paper : 4518 **G**

Unique Paper Code : 32351303

Name of the Paper : Multivariate Calculus

Name of the Course : **B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** sections are compulsory.
3. Attempt any **Five** questions from each section.
4. **All** questions carry equal marks.

Section I

1. Find the following limits :

(i) $\lim\limits_{(x,y)\to(0,0)} \left(1 + x^2 + y^2\right)^{\frac{1}{x^2+y^2}}$

(ii) $\lim\limits_{(x,y)\to(0,0)} x \log\sqrt{\left(x^2 + y^2\right)}$

2. Find an equation for each horizontal tangent-plane to the surface

$z = 5 - x^2 - y^2 + 4y$

3. The output at a certain factory is $Q = 150K^{\frac{2}{3}}L^{\frac{1}{3}}$ where K is the capital investment in units of \$1000, and L is the size of Labor force measured in worker-hours. The current capital investment is \$500,000 and 150 worker hours of Labor are used. Estimate the change in output that results when capital investment is increased by \$500 and Labor is decreased by 4 worker-hours.

4. Let $w = f(t)$ be a differentiable function of t where $t = (x^2 + y^2 + z^2)^{1/2}$. Show that

$(dw/dt)^2 = (\partial w/\partial x)^2 + (\partial w/\partial y)^2 + (\partial w/\partial z)^2$.

5. Let $f(x, y, z) = xyz$ and let $\hat{u}$ be a unit vector perpendicular to both $\vec{v} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} - \hat{k}$. Find the directional derivative of f at $P_0(1, -1, 2)$ in the direction of $\hat{u}$.

6. Find the absolute extrema of the function $f(x, y) = e^{x^2 - y^2}$ over the disk $x^2 + y^2 \le 1$.

Section II

1. Evaluate the double integral $\iint_D \frac{dA}{y^2 + 1}$ where D is triangle bounded by $x=2y$, $y=-x$ and $y=2$.

2. Evaluate $\int_0^2 \int_0^{\sqrt{2x - x^2}} y\sqrt{(x^2 + y^2)}\, dy\, dx$ by converting to polar coordinates.

3. Find the volume of tetrahedron T bounded by plane $2x + y + 3z = 6$ and co-ordinate planes.

4. Use spherical co-ordinates to verify that volume of a half sphere of radius R is $\frac{2}{3}\pi R^3$.

5. Use cylindrical co-ordinates to compute the integral $\iiint_D z\left(x^2 + y^2\right)^{-\frac{1}{2}} dx\, dy\, dz$ where D is the solid bounded above by the plane $z=2$ and below by the surface $2z = x^2 + y^2$.

6. Use a suitable change of variables to compute the double integral $\iint_D \left(\frac{x - y}{x + y}\right)^2 dy\, dx$, where D is the triangular region bounded by line $x + y = 1$ and co-ordinate axes.