(iii) If X is a random variable that takes only nonnegative values, then Show that for any value $a > 0$,

$$P\{X \geq a\} \leq \frac{E[X]}{a} \qquad (6.5)$$

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Your Roll No..............
05.01.2024(M)

Sr. No. of Question Paper : 4588 G

Unique Paper Code : 32357507

Name of the Paper : DSE – 2 : Probability Theory and Statistics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions selecting any **two** parts from each questions no.'s **1** to **6**.
3. Use of scientific calculator is permitted.

1. (i) If X has the probability density

$$f(x)=\begin{cases} ke^{-2x} & for\ x>0 \\ 0 & elsewhere \end{cases}$$

Find k and $P(0.5 \le X \le 1)$. Also find the distribution function of the random variable X and use it to reevaluate $P(0.5 \le X \le 1)$. (6)

(ii) Let the random variables X_1 and X_2 have the joint pdf

$$f(x_1, x_2)=\begin{cases} x_1+x_2 & \text{if } 0<x_1<1,\ 0<x_2<1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the marginal pdf of X_1 and X_2 and compute $P(X_1 + X_2 \le 1)$. (6)

(iii) Let X be a continuous random variable with pdf $f(x) = ke^{-kx}$, $0 \le x < \infty$. Find E(X), $E(X^2)$, Var(X) and the cumulative distribution function. (6)

(iii) The mean height of 500 students is 151 cm and the standard deviation is 15 cm. assuming that the heights are normally distributed, find how many students have heights between 120 and 155 cm? (6.5)

6. (i) Calculate the correlation coefficient for the following heights (in inches) of father's (X) and their son's (Y) : (6.5)

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

(ii) If $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ constitute a random sample from an infinite population with mean μ, the variance σ^2 and the moment generating function $M_X(t)$ then show that, the limiting distribution of $Z=\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ as $n \to \infty$ is the standard normal distribution. (6.5)

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$$f(x_1, x_2) = \begin{cases} 8x_1x_2 & \text{if } 0 < x_1 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Are X_1 and X_2 independent? (6.5)

5. (i) Suppose the joint moment generating function, $M(t_1, t_2)$, exists for the random variables X and Y. Then X and Y are independent if and only if

$M(t_1, t_2) = M(t_1, 0)M(0, t_2)$. (6.5)

(ii) If the probability density of X is given by

$$f(x) = \begin{cases} 630x^4(1-x)^4, & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability that it will take on a value within two standard deviations of the mean and compare this probability with the lower bound provided by Chebyshev's theorem. (6.5)

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2. (i) If a random variable X has a discrete uniform distribution $f(x) = \frac{1}{k}$ for $x = 1,2,3\ldots k$, then find the mean and variance. (6)

(ii) Let the random variables X and Y have the joint pdf

$$f(x, y) = \begin{cases} e^{-y} & \text{if } 0 < x < y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

Find the moment generating function of the joint distribution. (6)

(iii) Let the random variables X and Y have the joint pdf

$$f(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the conditional pdf of X given Y = y and the conditional pdf of Y given X = x. (6)

3. (i) If X is a Poisson distributed random variable with parameter λ then prove that

$$\mu_{r+1} = \lambda\left[r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right] \quad \text{for} \quad r = 1, 2, 3, \ldots \qquad (6)$$

(ii) If the probability is 0.60 that a girl child exposed to a certain contagious disease will catch it, what is the probability that the eleventh girl child exposed to the disease will be the fifth to catch it? (6)

(iii) Let the random variables X and Y have the joint pdf

$$f(x, y) = \begin{cases} 6y & \text{if } 0 < y < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Define $Z = \frac{2X}{3}$, find the mean and variance of Z. (6)

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4. (i) If a random variable X has a beta distribution then show that its mean and variance are given by :

$$\mu = \frac{\alpha}{\alpha+\beta} \text{ and } \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}. \qquad (6.5)$$

(ii) If the exponent of e of a bivariate normal density of random variables X and Y is

$$\frac{-1}{102}[(x+2)^2 - 2.8(x+2)(y-1) + 4(y-1)^2]$$

then find mean of X, mean of Y, standard deviation of X, standard deviation of Y and the correlation coefficient of X and Y. (6.5)

(iii) Let the random variables X_1 and X_2 have the joint pdf