$\frac{dx}{dt} = 1 + \frac{x}{t}, 1 \le t \le 2, x(1) = 1$ taking the step size as $h = 0.5$. (6.5)

03.01.24(M)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4469 G

Unique Paper Code : 32357501

Name of the Paper : DSE-I Numerical Analysis (LOCF)

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All **six** questions are compulsory.
3. Attempt any **two** parts from each question.
4. Use of non-programmable scientific calculator is allowed.

1. (a) Discuss the order of convergence of the Newton Raphson method. (6)

(b) Perform three iterations of the Bisection method in the interval (1, 2) to obtain root of the equation $x^3 - x - 1 = 0$. (6)

(c) Perform three iterations of the Secant method to obtain a root of the equation $x^2 - 7 = 0$ with initial approximations $x_0 = 2$, $x_1 = 3$. (6)

2. (a) Perform three iterations of False Position method to find the root of the equation $x^3 - 2 = 0$ in the interval (1, 2). (6.5)

(b) Find a root of the equation $x^3 - 5x + 1 = 0$ correct up to three places of decimal by the Newton's

(c) Approximate the derivative of $f(x) = 1 + x + x^3$ at $x_0 = 0$ using the first order forward difference formula taking $h = ½$, ¼ and 1/8 and then extrapolate from these values using Richardson extrapolation. (6)

6. (a) Using the trapezoidal rule, approximate the value of the integral $\int_3^7 \ln x \, dx$. Verify that the theoretical error bound holds. (6.5)

(b) Derive the Simpson's 1/3rd rule to approximate the integral of a function. (6.5)

(c) Apply the modified Euler method to approximate the solution of the initial value problem

x	−1	0	1	2
f(x)	3	−1	−3	1

(6)

5. (a) Derive second-order backward difference approximation to the first derivative of a function f given by

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}. \quad (6)$$

(b) Use the formula

$$f''(x_0) \approx \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}$$

to approximate the second derivative of the function $f(x) = e^x$ at $x_0 = 0$, taking h = 1, 0.1, 0.01 and 0.001. What is the order of approximation. (6)

Raphson method with $x_0 = 0$. In how many iterations does the solution converge? Also write down the order of convergence of the method used. (6.5)

(c) Explain the secant method to approximate a zero of a function and construct an algorithm to implement this method. (6.5)

3. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$$

and use it to solve the system $AX = [0 \quad 4 \quad 1]^T$. (6.5)

(b) Set up the Gauss-Jacobi iteration scheme to solve the system of equations :

$$5x_1 + x_2 + 2x_3 = 10$$

$$-3x_1 + 9x_2 + 4x_3 = -14$$

$$x_1 + 2x_2 - 7x_3 = -33$$

Take the initial approximation as $X^{(0)} = (0,0,0)$ and do three iterations. (6.5)

(c) Set up the Gauss-Seidel iteration scheme to solve the system of equations :

$$6x_1 - 2x_2 + x_3 = 11$$

$$-2x_1 + 7x_2 + 2x_3 = 5$$

$$x_1 + 2x_2 - 5x_3 = -1$$

Take the initial approximation as $X^{(0)} = (1, 0, 0)$ and do three iterations. (6.5)

4. (a) Construct the Lagrange form of the interpolating polynomial from the following data :

x	0	1	3
f(x)	1	3	55

(6)

(b) Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial.

x	0	1	2	3
y	–1	0	15	80

Hence, estimate the value of f(1.5). (6)

(c) Obtain the piecewise linear interpolating polynomials for the function f(x) defined by the data :