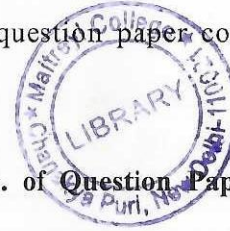


6. (i) Prove that  $\delta_{ij}$  (in Cartesian Co-ordinates) is an isotropic tensor of order two. (3)
- (ii) Using Cartesian tensors, prove that  

$$(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D}) = \vec{B}(\vec{A} \cdot \vec{C} \times \vec{D}) - \vec{A}(\vec{B} \cdot \vec{C} \times \vec{D})$$
 (8)
- (iii) Evaluate  $\epsilon_{ijk} \epsilon_{puk} \delta_{ju}$  (In Cartesian Co-ordinates). (4)
7. (i) A covariant tensor has components  $xy$ ,  $2y - z^2$ ,  $xz$  in Cartesian Co-ordinates. Find its covariant components in spherical co-ordinates. (12)
- (ii) Check whether an inner product in the form  $A_\mu B_\mu$  (in General Co-ordinates) is possible or not. (3)
8. (a) Show that  $\frac{\partial A_p}{\partial x^q}$  (in general co-ordinates) is not a tensor even though  $A_p$  is a covariant tensor of rank one. (5)
- (b) If  $A_q$  is a vector, show that  $F_{pk} = \frac{\partial A_p}{\partial x^k} + \frac{\partial A_k}{\partial x^p}$  is a second order tensor. (6)
- (c) Prove that velocity is a contravariant tensor of order one. (4)

(1000)

[This question paper contains 4 printed pages.]



Your Roll No.....

05.01.2024(M)  
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Sr. No. of Question Paper : 4594

Unique Paper Code : 32227502

Name of the Paper : Advanced Mathematical  
Physics – I (DSE)

Name of the Course : B.Sc. (Hons.) Physics

Semester : V

Duration : 3 Hours

Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all taking at least **two** questions from each section.
3. **All** questions carry equal marks.

**Section – A**

1. (a) Show that the set of three cube roots of unity i.e.  $\{1, \omega, \omega^2\}$  is a group under multiplication? (5)

P.T.O.

- (b) If  $V$  be the set of all ordered pairs  $(x, y) \forall x, y \in \mathbb{R}$ , the addition and scalar multiplication defined as

$$(x, y) + (x_1, y_1) = (x + x_1, y + y_1)$$

$$\text{and } \alpha(x, y) = (\alpha x, \alpha y), \alpha \in \mathbb{R}$$

then show that  $V$  is not a vector space. (5)

- (c) If  $V = \{(x, y, z) \mid \forall x, y, z \in \mathbb{R}\}$  be a vector space under addition and scalar multiplication.  $W = \{(x, y, z) : x - 4y + 3z = 0 \mid \forall x, y, z\}$ , verify whether  $W$  is a subspace of  $V$ . (5)

2. (a) Let  $V$  be the vector space of polynomials of degree  $\leq 3$  over  $\mathbb{R}$ . Determine whether polynomials  $u = t^3 - 3t^2 + 5t + 1$ ,  $v = t^3 - t^2 + 8t + 2$ ,  $w = 2t^3 - 4t^2 + 9t + 5$  are independent or not? (5)

- (b) A linear transformation  $T$  of  $\mathbb{R}^2$  maps  $[1, 2]$  into  $[-2, 1]$  and also maps  $[-1, 1]$  into  $[5, -7]$ , compute the image of  $[x, y]$ . (5)

- (c) Let  $T$  be the transformation such that

$$T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$$

- (i) Show that  $T$  is invertible.

(ii) Find  $T^{-1}$ . (2,3)

3. (i) Find the eigenvalues and normalized eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad (7)$$

- (ii) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$$

Hence find  $A^{-1}$ . (8)

4. (i) Evaluate  $A^{20}$ , where  $A = \frac{1}{3} \begin{pmatrix} 4 & \sqrt{2} \\ \sqrt{2} & 5 \end{pmatrix}$ . (5)

- (ii) Solve the following system of differential equations using matrix method

$$y_1' = y_1 + y_2$$

$$y_2' = 4y_1 + y_2$$

subject to the initial conditions  $y_1(0) = y_2(0) = t$ . (10)

### Section - B

5. (i) For an asymmetric object, obtain an expression for the moment of inertia tensor. Prove that it is a symmetric tensor of order two. (6,4)
- (ii) Using tensors, show that scalar product of two vectors is invariant. (5)