6. (i) Prove that δ_{ij} (in Cartesian Co-ordinates) is an isotopic tensor of order two. (3)

(ii) Using Cartesian tensors, prove that

$$(\vec{A}\times\vec{B})\times(\vec{C}\times\vec{D}) = \vec{B}(\vec{A}\cdot\vec{C}\times\vec{D}) - \vec{A}(\vec{B}\cdot\vec{C}\times\vec{D}) \quad (8)$$

(iii) Evaluate $\varepsilon_{ijk}\,\varepsilon_{puk}\,\delta_{ju}$ (In Cartesian Co-ordinates). (4)

7. (i) A covariant tensor has components xy, $2y - z^2$, xz in Cartesian Co-ordinates. Find its covariant components in spherical co-ordinates. (12)

(ii) Check whether an inner product in the form $A_\mu B_\mu$ (in General Co-ordinates) is possible or not. (3)

8. (a) Show that $\frac{\partial A_p}{\partial_x q}$ (in general co-ordinates) is not a tensor even though A_p is a covariant tensor of rank one. (5)

(b) If A_q is a vector, show that $F_{pk} = \frac{\partial A_p}{\partial x_k} + \frac{\partial A_k}{\partial x_p}$ is a second order tensor. (6)

(c) Prove that velocity is a contravariant tensor of order one. (4)

[This question paper contains 4 printed pages.]

Your Roll No..............

05.01.2024(M)

Sr. No. of Question Paper : 4594 G

Unique Paper Code : 32227502

Name of the Paper : Advanced Mathematical Physics – I (DSE)

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all taking at least **two** questions from each section.
3. **All** questions carry equal marks.

Section – A

1. (a) Show that the set of three cube roots of unity i.e. $\{1, \omega, \omega^2\}$ is a group under multiplication? (5)

(b) If V be the set of all ordered pairs (x, y) $\forall$ x, y $\in$ R, the addition and scalar multiplication defined as

$$(x,y) + (x_1,y_1) = (x + x_1,\ y + y_1)$$

$$\text{and } \alpha(x, y) = (\alpha x, y),\ \alpha \in R$$

then show that V is not a vector space. (5)

(c) If V = $\{(x, y, z)\ \forall x, y, z \in R\}$ be a vector space under addition and scalar multiplication. W = $\{(x,y,z): x - 4y + 3z = 0\ \forall x,y,z\}$, verify whether W is a subspace of V. (5)

2. (a) Let V be the vector space of polynomials of degree ≤ 3 over R. Determine whether polynomials $u = t^3 - 3t^2 + 5t + 1$, $v = t^3 - t^2 + 8t + 2$, $w = 2t^3 - 4t^2 + 9t + 5$ are independent or not? (5)

(b) A linear transformation T of R^2 maps [1,2] into [–2,1] and also maps [–1,1] into [5,–7], compute the image of [x, y]. (5)

(c) Let T be the transformation such that

$$T(x,y,z) = (2x, 4x - y, 2x + 3y - z)$$

(i) Show that T is invertible.

(ii) Find T^{-1}. (2,3)

3. (i) Find the eigenvalues and normalized eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$ (7)

(ii) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$$

Hence find A^{-1}. (8)

4. (i) Evaluate A^{20}, where $A = \frac{1}{3}\begin{pmatrix} 4 & \sqrt{2} \\ \sqrt{2} & 5 \end{pmatrix}$. (5)

(ii) Solve the following system of differential equations using matrix method

$$y_1' = y_1 + y_2$$
$$y_2' = 4y_1 + y_2$$

subject to the initial conditions $y_1(0) = y_2(0) = t$. (10)

Section – B

5. (i) For an asymmetric object, obtain an expression for the moment of inertia tensor. Prove that it is a symmetric tensor of order two. (6,4)

(ii) Using tensors, show that scalar product of two vectors is invariant. (5)