(ii) $\int_{-1}^{1}[P_2(x)]^2\,dx$ (5)

5. (a) Find the general solution near x = 0 using Frobenius method of :

$$x\,y'' + (1-2x)\,y' + (x-1)\,y = 0 \qquad (10)$$

(b) Identify and name the nature of singularities

$$(1-x^2)^2\,y'' + x(1-x)\,y' + (1+x)\,y = 0 \qquad (5)$$

[This question paper contains 4 printed pages.]

Your Roll No. ....

Sr. No. of Question Paper : 1206

F

Unique Paper Code : 2222011201

Name of the Paper : Mathematical Physics - II (DSC - 4)

Name of the Course : **B.Sc. (Hons.) Physics- core**

Semester : II

Duration : 2 Hours

Maximum Marks : 60

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt FOUR questions in all
3. Question No. 1 is compulsory.
4. Use of non-programmable scientific calculator is allowed.

P.T.O.

1. Attempt ALL questions. Each question carries equal marks. (3×5=15)

(a) Let u_1, u_2, u_3 be orthogonal coordinates. Prove that $|\nabla u_p| = h_p^{-1}, p = 1,2,3$

(b) Write the expression only of the general solution near x = −1 using Frobenius method of

$$y'' + x y' + (2x - 1) y = 0$$

(c) Using the expression of the generating function of the Legendre Polynomials $P_n(x)$ find the expression for $P_2(x)$ and $P_3(x)$.

(d) Evaluate using Beta function property

$\int_0^\infty \frac{z^{m-1}}{1+z}\, dz = \frac{m}{\sin m\pi}$ the integral $\int_{-\infty}^{\infty} \frac{e^{2u}}{1+e^{3u}}\, du$

(e) Is the given function periodic, $f(t) = \sin\,(10+\pi)t$. If yes, what is its period?

2. (a) Find the Fourier series expansion of the function

$f(x) = x^2 \;,\; 0 < x < 2\pi$ (10)

(b) Plot the even and odd components of a function

defined by $f(t) = \begin{cases} e^{-t} & ,t > 0 \\ 0 & ,t < 0 \end{cases}$ (5)

3. (a) Derive the expression for $\nabla^2\varphi$ in cylindrical coordinates. (10)

(b) Represent the vector $\vec{A} = z\hat{\imath} - 2x\hat{\jmath} + y\hat{k}$ in cylindrical coordinates (ρ, ϕ, z). Thus determine A_ρ, A_ϕ and A_z (5)

4. (a) Prove that $P_n(x)$ is the coefficient of t^n in the expansion of $\frac{1}{\sqrt{1-2xt+t^2}}$ in the ascending powers of t. Hence find the value of $P_n(1)$ (10)

(b) Evaluate using the orthonormalization property of Legendre polynomial

(i) $\int_{-1}^{1} P_3(x)\, P_4(x)\, dx$,