(c) Consider the linear transformation $L: P_3 \to P_2$ given by $L(p) = p'$ where $p \in P_3$

Is L an isomorphism?

(1000)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1271 F

Unique Paper Code : 2352571201

Name of the Paper : ELEMENTARY LINEAR ALGEBRA

Name of the Course : **B.Sc. (Prog.) DSC-B2**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. **All questions** carry equal marks.

P.T.O.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove that:

$$\|x+y\| \le \|x\| + \|y\|$$

(b) Define norrn ofa vector. Find a unit vector in the same direction as the vector $\left[\frac{1}{5}, -\frac{2}{5}, -\frac{1}{5}, \frac{1}{5}, \frac{2}{5}\right]$. Is the normalized (resulting) vector longer or shorter than the original? Why?

(c) Use Gaussi.an elimination method to solve föllowing systems of linear equations. Give the complete solution set, and if the solution set is infinite, specify two panicular solutions.

$$\begin{aligned} 3x + 6y - 9z &= 15 \\ 2x + 4y - 6z &= 10 \\ -2x - 3y + 4z &= -6 \end{aligned}$$

Find the kernel of L and range of L.

6. (a) Consider the linear transformation $L: R^3 \to R^3$ given by $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 & 5 \\ -2 & 3 & -13 \\ 3 & -3 & 15 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Find the basis for kernel of L.

(b) Consider the linear operator $L: R^2 \to R^2$ given by

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Show that L is one-to-one and onto operator.

$$f([a_1, a_2, a_3]) = [a_1, a_2, -a_3]$$

Prove that f is a linear transformation.

(b) Find the matrix for the linear transformation $L: P_3 \rightarrow R^3$ given by

$$L(a_3x^3 + a_2x^2 + a_1x + a_0) = [a_0 + a_1, 2a_2, a_3 - a_0]$$

With respect to the bases $B = (x^3, x^2, x, 1)$ for P_3 and $C = (e_1, e_2, e_3)$ for R^3

(c) Consider the linear operator $L: R^n \rightarrow R^n$ given by

$$L([a_1, a_2, \cdots, a_n]) = [a_1, a_2, 0 \cdots, 0]$$

2. (a) Determine whether the two matrices are row equivalent?

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 1 \\ 5 & -1 & 5 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 10 \\ 2 & 0 & 4 \end{bmatrix}$$

(b) Find. the rank of the following matrix.

$$\begin{bmatrix} -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 4 & 0 & -2 & 1 \\ 3 & -1 & 0 & 4 \end{bmatrix}$$

(c) Express the vector X = [2, –5, 3] as a linear combination of the vectors a_1= [1, –3,2], a_2= [2, –4,–1], and a_3= [1, –5,7] if possible.

3. (a) Determine the characteristic polynomial of the following matrix.

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

(b) Show that the set of vectors of the form [a, b, 0, c, a - 2b + c] in $\mathbb{R}^5$ forms a subspace of $\mathbb{R}^5$ under the usual operations.

(c) For S = $\{x^3 + 2x^2, 1 - 4x^2, 12 - 5x^3, x^3 - x^2\}$, use the Simplified Span Method to find a simplified general form for all the vectors in span(S), where S is the given subset of P_3, the set of all polynomials of degree less than or equal to 3 with real coefficients.

4. (a) Use the Independence Test Method to determine whether the given set S is linearly independent or linearly dependent.

s = {1, -1, 0, 2], [0, -2, 1, 0], [2, 0, -1, 1}

(b) Let the subspace W of $\mathbb{R}^5$ be the solution set to the matrix equation AX = 0 where A is

$$\begin{bmatrix} 1 & 2 & 1 & 0 & -1 \\ 2 & -1 & 0 & 1 & 3 \\ 1 & -3 & -1 & 1 & 4 \\ 2 & 9 & 4 & -1 & -7 \end{bmatrix}$$

Find the basis and the dimension for W. Show that dim(W) + Rank(A) = 5.

(c) Show that P_n, the set of all polynomials of degree less than or equal to n with real coefficients, is a vector space under the usual operations of addition and scalar multiplication.

5. (a) Consider the mapping $f: R^3 \to R^3$ given by