

(c) For the linear transformation $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, defined as :

$$L\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

Find L^{-1} , if it exists.

(7.5)

(700)

[This question paper contains 8 printed pages.]

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Your Roll No.

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Name of the Paper : Elementary Linear Algebra

Name of the Course : B.A. (Prog.)

Semester : II – DSC

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator not allowed.

P.T.O.

1. (a) If x and y are vectors in R^n , then prove that
 $\|x + y\| \leq \|x\| + \|y\|$.

Also verify it for the vectors $x = [-1, 4, 2, 0, -3]$
 and $y = [2, 1, -4, -1, 0]$ in R^5 . (5.5+2)

- (b) Prove that for vectors x and y in R^n ,

(i) $x \cdot y = \frac{1}{4} (\|x + y\|^2 - \|x - y\|^2)$

(ii) If $(x + y) \cdot (x - y) = 0$, then $\|x\| = \|y\|$.
 (4+3.5)

- (c) Solve the systems $AX = B_1$ and $AX = B_2$
 simultaneously, where

$$A = \begin{bmatrix} 9 & 2 & 2 \\ 3 & 2 & 4 \\ 27 & 12 & 22 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -6 \\ 0 \\ 12 \end{bmatrix}, \quad \text{and} \quad B_2 = \begin{bmatrix} -12 \\ -3 \\ 8 \end{bmatrix}$$

(7.5)

- (c) Let $L: V \rightarrow W$, be a linear transformation, then
 define $\text{Ker}(L)$, $\text{Range}(L)$. Further show that
 $\text{Ker}(L)$ is a subspace of V and $\text{Range}(L)$ is a
 subspace of W . (1.5+1.5+2.5+2)

6. (a) For the linear transformation $L: R^3 \rightarrow R^3$ defined

as

$$L \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & -1 & 5 \\ -2 & 3 & -13 \\ 3 & -3 & 15 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Find $\text{Ker}(L)$ and $\text{Range}(L)$. (4+3.5)

- (b) Let $L: V \rightarrow W$ be a one-to-one linear
 transformation. Show that if T is a linearly
 independent subset of V , then $L(T)$ is a linearly
 independent subset of W . (7.5)

P.T.O.

Consider the set of all real polynomials denoted by $P(x)$, and the set of all real polynomials of degree at most n denoted by $P_n(x)$. Describe a basis of $P(x)$ and $P_n(x)$ and mention if these are finite dimensional or infinite dimensional.

(2+4+1.5)

5. (a) Show that the mapping $L : M_{nn} \rightarrow M_{nn}$, defined as $L(A) = A + A^T$ is a linear operator, where M_{nn} is set of $n \times n$ matrices and A^T denotes the transpose of the matrix A . Find the Kernel of L .

(3+4.5)

- (b) Let $L: R^2 \rightarrow R^3$ be a linear transformation defined as $L\{[a, b]\} = [a - b, a, 2a + b]$. Find the matrix of linear transformation A_{BC} of L , with respect to the basis $B = \{[1, 2], [1, 0]\}$ and $C = \{[1, 1, 0], [0, 1, 1], [1, 0, 1]\}$.

(7.5)

2. (a) Find the reduced row echelon form of the following matrix :

$$A = \begin{bmatrix} 2 & -5 & -20 \\ 0 & 2 & 7 \\ 1 & -5 & -19 \end{bmatrix} \quad (7.5)$$

- (b) Express the vector $x = [2, -1, 4]$ as a linear combination of vectors $v_1 = [3, 6, 2]$ and $v_2 = [2, 10, -4]$, if possible.

(7.5)

- (c) Define the rank of a matrix and determine it for the following matrix :

$$B = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 4 \\ -1 & -3 & 1 \end{bmatrix} \quad (1.5+6)$$

P.T.O.

3. (a) Check if the following matrix is diagonalizable or not :

$$\begin{bmatrix} 3 & 4 & 12 \\ 4 & -12 & 3 \\ 12 & 3 & -4 \end{bmatrix} \quad (7.5)$$

- (b) Show that the set of all polynomials $P(x)$ forms a vector space under usual polynomial addition and scalar multiplication. (7.5)

- (c) Give an example of a finite dimensional vector space. Check if the following are a vector space or not :

- (i) $\mathbb{R}^2$ with the addition $[x, y] \oplus [w, z] = [x + w + 1, y + z - 1]$ and scalar multiplication $a \otimes [x, y] = [ax + a - 1, ay - 2]$.

- (ii) set of all real valued functions $f: \mathbb{R} \rightarrow \mathbb{R}$

such that $f\left(\frac{1}{2}\right) = 1$, under usual function addition and scalar multiplication.

(1.5+3+3)

4. (a) Define subspace of a vector space. Further show that intersection of two subspaces of a vector space V is a subspace of V . (1.5+6)

- (b) Define a linearly independent set. Check if $S = \{(1, -1, 0, 2), (0, -2, 1, 0), (2, 0, -1, 1)\}$ is linearly independent set in $\mathbb{R}^4$ or not.

(1.5+6)

- (c) Define an infinite dimensional and finite dimensional vector space.

P.T.O.