(b) An electron in a hydrogen atom is in a state described by

$$\psi = \frac{1}{\sqrt{6}}[2\psi_{100} + \psi_{211} + \psi_{21-1}]$$

Calculate the expectation value of $\hat{L}_z$ in this state.

(Given $\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$ and

$$\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}} e^{-r/a_0}$$

$$\psi_{211} = \frac{1}{8\sqrt{\pi}a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}\sin\theta\, e^{+i\phi})$$ (3)

6. (a) What is Larmor Precession? Draw the relevant diagram and derive the expression for Larmor frequency.

(b) A beam of silver atoms moving with a velocity 10^7 cm/s passes through a magnetic field of gradient 0.5 Wb/m^2/cm for 10 cm. What is the separation between the two components of the beam as it comes out of the magnetic field? (8,7)

7. (a) What is spin orbit coupling? Explain the fine structure splitting in the energy levels due to this. For the 2p level of the hydrogen atom with $E_n = -3.14$ eV, evaluate the fine structure splitting.

(b) Consider a two-electron system with $l_1 = 1$, $l_2 = 1$. Explain the LS coupling scheme in such a case. Write the spectral notation for each state. (10,5)

[This question paper contains 4 printed pages.]

Your Roll No. ....

Sr. No. of Question Paper : 1015

Unique Paper Code : 32221501

Name of the Paper : Quantum Mechanics and Applications

Name of the Course : **B.Sc. (Honors) Physics**

Semester : V

Duration : 3 Hours   Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **FIVE** questions in all. Question No. **1** is compulsory.
3. **All** questions carry equal marks.
4. Non programmable calculators are allowed.

1. Attempt any **FIVE** of the following :

(a) Calculate the commutator $[\hat{L}_x, \hat{p}_x]$. (given $[\hat{x}, \hat{p}_x] = i\hbar$).

(b) The wave-function of a particle is $\psi(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$ for $0 \le x \le L$. Determine the probability of finding the particle at x = L/3 for n = 3 state.

P.T.O.

(c) Derive the relation between 'magnetic dipole moment' and 'orbital angular momentum' of an electron revolving around a nucleus.

(d) Write the quantum numbers for the state represented by $4^2F_{5/2}$.

(e) Normalize the wave function e^{-ax^2} in a one-dimensional space.

(f) A free particle of mass m is described by the wave-function $\psi(x) = A\exp(i\mu x)$ where A and μ are constants. Determine the probability current density for this particle.

(g) Determine the uncertainty in position for the normalized wave-function $\psi(x) = \left(\frac{2\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2}$ for $-\infty < x < \infty$. (5×3=15)

2. (a) Explain the concept of expectation values. Give expressions for the expectation values of velocity, momentum and energy in terms of respective operators in three dimensions. Mention the difference between expectation values and eigenvalues of an operator corresponding to a dynamical variable.

(b) The wave-function of a particle of mass m is given by

$$\Psi(x) = \left(\frac{\beta^2}{\pi}\right)^{1/4} e^{-\beta^2x^2/2} \text{ for } -\infty < x < \infty.$$

Determine the total energy of the particle, if potential energy is $V(x) = \frac{1}{2}m\omega^2x^2$. (7,8)

3. The Gaussian wave packet for a free particle is defined by the wave function

$$\Psi(x,0) = N\exp\left(-\frac{x^2}{2\sigma^2} + ik_0x\right).$$

Prove that the centre of this Gaussian wave packet travels with a velocity $v = \frac{k_0\hbar}{m}$.

(Use $\int_{-\infty}^{\infty} e^{-x^2/\sigma^2}\,dx = \sigma\sqrt{\pi}$ and $\int_{-\infty}^{\infty} e^{-(ax^2\pm bx)}\,dx = \sqrt{\frac{\pi}{a}}e^{b^2/4a}$). (15)

4. (a) Solve the Schrodinger equation for a linear Harmonic Oscillator and obtain first two eigenfunctions. (10)

(b) Find ΔX and ΔP for the ground state eigenfunction of linear Harmonic Oscillator and obtain the uncertainty principle. (5)

5. (a) The 'θ' equation obtained after applying separation for variables to the Schrodinger equation for a 3D hydrogen atom in spherical polar coordinates, is given by

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) + \left(\lambda - \frac{m_l^2}{\sin^2\theta}\right)\Theta = 0.$$

Solve the above equation for $m_l = 0$ (or otherwise) to show that

$$\lambda = l(l+1), \quad l = 0,1,2,\ldots\ldots\ldots \quad (12)$$