6. Solve the differential equations :

(a) $\frac{dy}{dx} = \frac{x^2+2}{y}$

(b) $2xy\,dx + (1+x^2)\,dy = 0$

(c) $\frac{dy}{dx} = \frac{2y^4 + x^4}{xy^3}$ (5+5+5=15)

7. (a) Solve the differential equation by method of undetermined coefficients

$y'' - y' - 2y = 4x^2;\ y(0) = 1,\ y'(0) = 4.$

(b) Solve the differential equation by method of variation of parameters

$y'' - 2y' + y = \frac{e^x}{x}$

(c) A function f(x) is defined as follows :

$$f(x) = \begin{cases} 6x(1-x), & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Show that it is a probability density function and find its mean. (7+5+3=15)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1069 C

Unique Paper Code : 32221101

Name of the Paper : Mathematical Physics – I

Name of the Course : **B.Sc. (Hons.) Physics – CBCS**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question no. **1** is compulsory.
3. Attempt any **five** questions.
4. Use of non-programmable scientific calculator is allowed.

1. Attempt any **five** questions : (5×3=15)

(a) Determine the order and degree of the following differential equation

$$2x\frac{d^2y}{dx^2} + x^2\left(\frac{dy}{dx}\right) - (Sinx)y = 0$$

(b) Define polar and axial vectors. Give one example of each.

(c) Show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint(xdy - ydx)$.

(d) Find a unit vector normal to the surface $xy^3z^2 = 4$ at the point (–1,–1,2).

(e) Determine the constant a so that the vector $\vec{A} = (x + 3y)\hat{\imath} + (y - 2z)\hat{\jmath} + (x + az)\hat{k}$ is Solenoidal.

(f) Prove that a separable equation is always exact.

(g) Find the Wronskian of the set {Sin3x, Cos3x}.

2. (a) Prove that curl $\left(\frac{\vec{a} \times \vec{r}}{r^3}\right) = \frac{3(\vec{a}.\vec{r})\vec{r}}{r^5} - \frac{\vec{a}}{r^3}$ where a is a constant vector and $\vec{r}$ is a position vector.

(b) Find Curl $(\vec{r} f(r))$ where $f(r)$ is differentiable.

(c) If $\vec{v} = (\vec{\omega} \times \vec{r})$, prove $\vec{\omega} = \frac{1}{2} curl(\vec{v})$ where $\vec{\omega}$ is a constant vector. (6+4+5=15)

3. (a) Find the directional derivative of $\varphi = xy^2 - 4x^2y + z^2$ at (1, –1, 2) in the direction $6\hat{i} + 2\hat{j} + 3\hat{k}$.

(b) Find the unit tangent vector to any point on the curve $x = t^2 + 1,\ y = 4t - 3,\ z = 2t^2 - 6t$.

(c) If A and B are differentiable functions of a scalar u, prove

$$\frac{d(\vec{A}.\vec{B})}{du} = \vec{A}.\frac{d\vec{B}}{du} + \frac{d\vec{A}}{du}.\vec{B}$$ (6+5+4=15)

4. (a) (i) Show that $\vec{A} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field.

(ii) Find the scaler potential.

(iii) Find the work done in moving an object in this field from (1,–2,1) to (3,1,4).

(b) Verify Green's theorem in the plane for $\oint_c (xy + y^2)\,dx + x^2\,dy$ where c is the closed curve of the region bounded by y = x and $y = x^2$. (8+7=15)

5. (a) Evaluate $\iint \vec{A}.\hat{n}\, dS$, where $\vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the part of the plane 2x + 3y + 6z = 12 which is located in the first octant.

(b) Prove that a spherical coordinate system is orthogonal. (10+5=15)

P.T.O.