

6. Solve the differential equations :

(a) $\frac{dy}{dx} = \frac{x^2 + 2}{y}$

(b) $2xy \, dx + (1+x^2) \, dy = 0$

(c) $\frac{dy}{dx} = \frac{2y^4 + x^4}{xy^3}$ (5+5+5=15)

7. (a) Solve the differential equation by method of undetermined coefficients

$$y'' - y' - 2y = 4x^2; y(0) = 1, y'(0) = 4.$$

(b) Solve the differential equation by method of variation of parameters

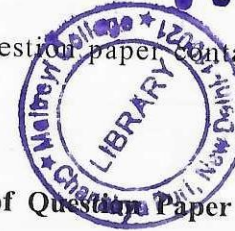
$$y'' - 2y' + y = \frac{e^x}{x}$$

(c) A function $f(x)$ is defined as follows :

$$f(x) = \begin{cases} 6x(1-x), & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

Show that it is a probability density function and find its mean. (7+5+3=15)

[This question paper contains 4 printed pages.]



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Your Roll No.....

Sr. No. of Question Paper : 1069

Unique Paper Code : 32221101

Name of the Paper : Mathematical Physics - I

Name of the Course : B.Sc. (Hons.) Physics - CBCS

Semester : I

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question no. 1 is compulsory.
3. Attempt any **five** questions.
4. Use of non-programmable scientific calculator is allowed.

1. Attempt any **five** questions : (5×3=15)

(a) Determine the order and degree of the following differential equation

$$2x \frac{d^2y}{dx^2} + x^2 \left(\frac{dy}{dx} \right) - (\sin x)y = 0$$

- (b) Define polar and axial vectors. Give one example of each.
- (c) Show that the area bounded by a simple closed curve C is given by $\frac{1}{2} \oint (x dy - y dx)$.
- (d) Find a unit vector normal to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$.
- (e) Determine the constant a so that the vector $\vec{A} = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + az)\hat{k}$ is Solenoidal.
- (f) Prove that a separable equation is always exact.
- (g) Find the Wronskian of the set $\{\sin 3x, \cos 3x\}$.
2. (a) Prove that $\text{curl} \left(\frac{\vec{a} \times \vec{r}}{r^3} \right) = \frac{3(\vec{a} \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{a}}{r^3}$ where \vec{a} is a constant vector and \vec{r} is a position vector.
- (b) Find $\text{Curl}(\vec{r}f(r))$ where $f(r)$ is differentiable.
- (c) If $\vec{v} = (\vec{\omega} \times \vec{r})$, prove $\vec{\omega} = \frac{1}{2} \text{curl}(\vec{v})$ where $\vec{\omega}$ is a constant vector. (6+4+5=15)
3. (a) Find the directional derivative of $\phi = xy^2 - 4x^2y + z^2$ at $(1, -1, 2)$ in the direction $6\hat{i} + 2\hat{j} + 3\hat{k}$.

- (b) Find the unit tangent vector to any point on the curve $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$.
- (c) If A and B are differentiable functions of a scalar u , prove

$$\frac{d(\vec{A} \cdot \vec{B})}{du} = \vec{A} \cdot \frac{d\vec{B}}{du} + \frac{d\vec{A}}{du} \cdot \vec{B} \quad (6+5+4=15)$$

4. (a) (i) Show that $\vec{A} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field.
- (ii) Find the scalar potential.
- (iii) Find the work done in moving an object in this field from $(1, -2, 1)$ to $(3, 1, 4)$.
- (b) Verify Green's theorem in the plane for $\oint_c (xy + y^2) dx + x^2 dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. (8+7=15)
5. (a) Evaluate $\iint \vec{A} \cdot \hat{n} dS$, where $\vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ which is located in the first octant.
- (b) Prove that a spherical coordinate system is orthogonal. (10+5=15)