(b) Define basis of a vector space over a field F. Prove that every element of a vector space uniquely expressible as a linear combination of elements of the basis. (6.5)

(c) Check whether the vectors (1,-1, 2), (-1,2,-4), (-1,-1,2) form a basis of R^3. (6.5)

6\. (a) Let T: V->U be a Linear Transformation. Define null space N(T) and range R(T) of T Show that N(T) and R(T) are subspaces of V and U respectively. (6.5)

(b) Define Linear Transformation. Prove that there exists a Linear Transformation T: R^2 -> R^3 such that T (1,1) = (1,0,2) and T(2,3) = (1,-1,4). Find T (8, 11). (6.5)

(c) Let T: R^3-> R^3 be a Linear Transformation defined by T (x, y, z) = (x-y+2z, 2x+y, -x- 2y+2z). Find the Range, Rank, Kernal and Nullity of T. Verify the Dimension Theorem. (6.5)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1713 C

Unique Paper Code : 42354302

Name of the Paper : Algebra

Name of the Course : **B.Sc.(Prog)/Mathematical Sciences**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt **any two parts** from each question.
4. All questions are **compulsory**

Unit I

1\. (a) Define the inverse of an element and show that inverse of an element in a group is unique. (6)

(b) Let the element "α" belong to a group and $\alpha^{12} = e$. Express the inverse of each of the elements α, α^6, α^8 and α^{11} in the form α^k for some positive integer k. (6)

(c) Let $\sigma = (1,5,7)(2,5,3)\ (1,6)$. Then find σ^{98}. (6)

2. (a) Let G be a group and let H be a nonempty subset of G. If ab is in H whenever a and b are in H and a^{-1} is in H whenever a is in H, then H is a subgroup of G. (6)

(b) Let G be an Abelian group and H and K be subgroups of G. Then HK = {hk: h ∈ H, k ∈ K} is a subgroup of G. (6)

(c) State and prove Lagrange's Theorem. Is the converse of this theorem true? (6)

3. (a) In a finite cyclic group, the order of an element divides the order of the group. (6)

(b) Find the inverse of $\left\{\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\right\}$ in $(2, Z_{11})$ (6)

(c) Every permutation of a finite set can be written as a cycle or as a product of disjoint cycles. (6)

Unit II

4. (a) State the subring test. Check whether the set

$$\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in Z\right\}$$ is a subring of the ring of all 2 × 2 matrices over Z,the set of integers. (6.5)

(b) Define a field. Prove that a finite commutative ring with unity having no zero divisors is a field. (6.5)

(c) Show that the set $[Q\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in Q\}$ forms a ring. Is it a field? If yes, justify your answer. (6.5)

Unit III

5. (a) Prove that intersection of two subspaces of a vector space V(F) is a subspace of V(F). Is the result true for the union of two subspaces? Justify your answer. (6.5)