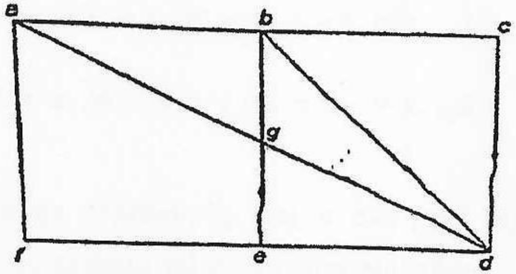
(c) For the following graph, find a minimal edge cover and a maximal independent set of vertices.



[This question paper contains 6 printed pages.]

Your Roll No. ....

Sr. No. of Question Paper : 2142 C

Unique Paper Code : 62354343

Name of the Paper : Analytic Geometry and Applied Algebra

Name of the Course : **B.A. (Prog.)**

Semester : III CBCS (LOCF)

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each questions.
4. Each question carries **12.5** marks.

1. (a) Identify and sketch the curve :

$$y = 4x^2 + 8x + 5$$

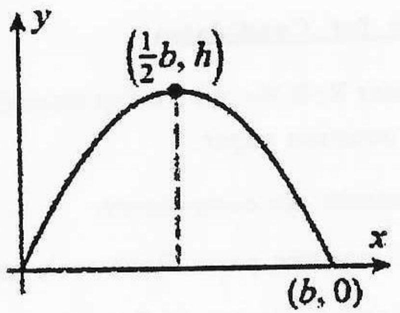
Also label the focus, vertex and directrix.

(b) Describe the graph of the curve:

$$x^2 + 9y^2 + 2x - 18y + 1 = 0$$

Find its foci, vertices and the ends of the minor axis.

(c) Find an equation for the parabolic arch with base b and height h, shown in the accompanying figure



2. (a) Find the equation for the parabola that has axis $y = 0$ and passes through (3, 2) and (2, −3).

(b) Find the equation for the ellipse that has foci (1,2) and (1,4) and minor axis of length 2.

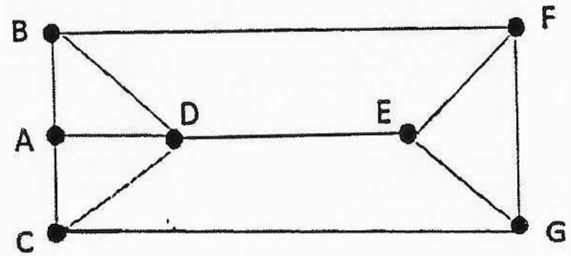
(c) Show that the lines L_1 and L_2 are parallel and find the distance between them

$L_1: x = 2 - t, y = 2t, z = 3 + t$

$L_2: x = -1 + 2t, y = 3 - 4t, z = 5 - 2t$

6. (a) Suppose a job placement agency wants to schedule interviews for candidates Ann, Judy and Carol with interviewers Al, Brian and Carl on Monday, Tuesday and Wednesday in such a way that each candidate gets interviewed by each interviewer. Solve this problem using a Latin Square.

(b) Find a vertex basis for the following graph:



4. (a) Consider the equation $x^2 - xy + y^2 + 12 = 0$. Rotate the coordinate axes to remove xy-terms. Then identify and sketch the curve.

(b) Let an $x'y'$-coordinate system be obtained by rotating an xy -coordinate system through an angle of $\theta = 45°$.

(i) Find the $x'y'$-coordinates of the point whose xy-coordinates are $\left(\sqrt{2}, \sqrt{2}\right)$.

(ii) Find an equation of the curve $x^2 + xy + 2y^2 + 6 = 0$ in $x'y'$-coordinates.

(c) Describe the surface whose equation is given as

$$x^2 + y^2 + z^2 + 2y - 6z + 5 = 0$$

5. (a) Find the distance from the point P(2, 5, -3) to the plane

$$\bar{r}.\left(6\hat{i} - 3\hat{j} + 2\hat{k}\right) = 4$$

(b) Find the equation of the plane through the points $P_1(2, 1, 4)$, $P_2(0, 0, -3)$ that is perpendicular to the plane $4x + y + 3z = 2$.

(c) Describe the graph of the hyperbola :

$$x^2 - 4y^2 + 2x + 8y - 7 = 0$$

Also sketch its graph.

3. (a) If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors, then prove that

$$\|\vec{a} + \vec{b} + \vec{c}\| = \sqrt{3}$$

(b) Express $\vec{v}$ as the sum of a vector parallel to $\vec{b}$ and a vector orthogonal to $\vec{b}$ where

$$\vec{v} = 3\hat{i} + \hat{j} + 2\hat{k} \quad \text{and} \quad \vec{b} = 2\hat{i} + \hat{k}$$

(c) (i) Using vectors, find the area of triangle with vertices P(2, 2, 0), Q(1, 4, –5) and R(7, 2, 9).

(ii) Use scalar triple product to determine whether the vectors

$$\vec{u} = \langle 5, -2, 1 \rangle, \vec{v} = \langle 4, -1, 2 \rangle \quad \text{and} \quad \vec{w} = \langle 1, -1, 0 \rangle$$

are co-planar.