(c) Solve the initial value problem:

$$x^2\frac{d^2y}{dx^2} - 2y = 4x - 8,\ y(1) = 4,\ y'(1) = -1. \qquad (6+6)$$

5. (a) Find the partial differential equation satisfied by the following surface

$$z = f(x - y).$$

(b) Find the general solution of the partial differential equation

$$u_x + yu_y = 0.$$

(c) Solve the following Cauchy problem

$$yu_x + xu_y = 0,\ u(0, y) = e^{-y^2}. \qquad (6.5+6.5)$$

6. (a) Apply the method of separation of variables $u(x, y) = f(x) + g(y)$ to solve the following equation

$$u_x^2 + u_y^2 = 1.$$

(b) Find general solution of the following second order partial differential equation with constant coefficients

$$u_{xx} + 4u_{xy} + 4u_{yy} = 0.$$

(c) Classify the following equation and obtain general solution by reducing it to canonical form

$$yu_{xx} - xu_{yy} = 0,\ x > 0,\ y > 0. \qquad (7+7)$$

(2500)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1588

Unique Paper Code : 42357501

Name of the Paper : DSC – Differential Equations

Name of the Course : **B.Sc. (Math Sci)-II B.Sc. (Phy Sci)-II B.Sc. (Life Sci)-II Applied Sciences-II**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Show that the differential equation

$(4x + 3y^2)dx + 2xydy = 0$ is not exact.

Find an integrating factor of form y^n, n being an integer. Multiply by the integrating factor and find the solution of the equation.

(b) Solve the initial value problem

$(2x + 3y + 1)dx + (4x + 6y + 1)dy = 0,\ y(-2) = 2.$

P.T.O.

(c) Solve

$$\left(x\tan\left(\frac{y}{x}\right)+y\right)dx - xdy = 0 . \qquad (6+6)$$

2. (a) Find the orthogonal trajectories of $x^2 + y^2 = 2cx$, c being an arbitrary constant.

(b) Show that e^{2x} and e^{3x} are two linearly independent solutions on $-\infty < x < \infty$ of the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0 .$$

Write the general solution. Find the solution that satisfies the conditions

$$y(0) = 2,\ y'(0) = 3.$$

(d) If $y = (x + 1)$ is a solution of

$$(x+1)^2\frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 3y = 0 ,$$

find a linearly independent solution by reducing the order. Write the general solution. (6+6)

3. (a) Solve the initial value problem

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0 ,\ y(0) = 0,\ y'(0) = 0,$$
$$y''(0) = 2.$$

(b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \frac{1}{1+e^{2x}}$$

using the method of variation of parameters.

(c) Find the general solution of

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = 2x\ln(x) . \qquad (6+6)$$

4. (a) Show that $x = 3e^{7t}$, $y = 2e^{7t}$ and $x = e^{7t}$, $y = -2e^{-t}$ are two linearly independent solutions on every interval $a \le t \le b$ of the linear system

$$\frac{dx}{dt} = 5x + 3y,$$

$$\frac{dy}{dt} = 4x + y .$$

Write the general solution.

(b) Solve the linear system

$$\frac{dx}{dt} + \frac{dy}{dt} + 2y = \sin(t),$$

$$\frac{dx}{dt} + \frac{dy}{dt} - x - y = 0 .$$