Find the normal stress and shear stress on the surface defined by

$$3x - 2y + 2z = 10 \quad (3)$$

8. (a) Prove that g_{ij} is a covariant tensor of rank 2. (5)

(b) If $ds^2 = 3(dx^1)^2 + 5(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4\,dx^2dx^3$

Find the matrices

(i) g_{ij} (4)

(ii) g^{ij} (4)

(iii) the product of (g_{ij}) and (g^{ij}) (2)

6 Dec

[This question paper contains 6 printed pages.]

Your Roll No.

Sr. No. of Question Paper : 1140

Unique Paper Code : 32227502

Name of the Paper : Advanced Mathematical Physics (DSE – Paper)

Name of the Course : **B.Sc. (Hons) Physics (CBCS – LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all taking at least **two** questions from each section.
3. **All** questions carry equal marks.

SECTION A

1. (a) Is the set $\{1, -1, i, -i\}$ a group under multiplication? (5)

P.T.O.

(b) Show that W is not a subspace of vector space V where

$$W = \{f : f(7) = 2 + f(1)\}.$$ [5]

(c) Consider the following subspace of R^4:

$$W = \{(a, b, c, d) : b + c + d = 0\}$$

Find the dimension and basis of W. (5)

2. (a) Determine whether the transformation, $T: R^3 \to R^3$ defined by,

$$T(x, y, z) = (x + 2y - 3z, x + y + z, 7x - y + 5z)$$

is linear or not. (5)

(b) Let $T: R^3 \to R^3$ be defined by

$$T(x, y, z) = (x + y - 2z, x + 2y + z, 2x + 2y - 3z).$$

Show that T is a non-singular transformation. (5)

(c) Linear transformation T on R^2 is defined as

$$T(x, y) = (3x - 4y, x + 5y)$$

Find the matrix representation of T relative to the u-basis: $\{u_1 = (1, 3)$ and $u_2 = (2, 5)\}$. (5)

7. (a) Stress tensor (p_{ij}) satisfies the equations $p_{ij} \epsilon_{ijk} = 0$ and $p_{ij} = f_i n_j$, where f_k is the restoring force per unit area along x_k - axis and $\hat{n}$ is the arbitrary unit vector. Prove that stress tensor is a symmetric tensor of order two. (5)

(b) Stress tensor and strain tensor are related as

$$p_{ij} = \omega_{ijks} e_{ks},$$

where, elastic tensor ω_{ijks} is symmetric in i, j and k, s and its general form is

$$\omega_{ijks} = \lambda \delta_{ij} \delta_{ks} + \mu \delta_{ik} \delta_{js} + \gamma \delta_{is} \delta_{jk}.$$

Prove that

(i) $\omega_{ijks} = \lambda \delta_{ij} \delta_{ks} + \mu(\delta_{ik} \delta_{js} + \delta_{is} \delta_{jk})$

(ii) $p_{ii} = (3\lambda + 2\mu) e_{ii}$ (7)

(c) Let the state of stress at a point in a solid body is given by

$$S_{ik} = \begin{bmatrix} 10 & 10 & 20 \\ 10 & 20 & 0 \\ 20 & 0 & 55 \end{bmatrix}$$

SECTION B

5. (a) Prove that $\delta_{pr} \in_{prs} = 0$. (3)

(b) If $B_{ps} = \in_{psk} A_k$, show that

$$A_u = \frac{1}{2} \in_{ups} B_{ps}$$

(5)

(c) If a tensor A_{ijklm} is symmetric with respect to two indices i and k in the coordinate system x_i, then show that it is symmetric with respect to the same indices in any other co-ordinate system $\bar{x}_p$. (7)

6. (a) Prove that

$$\in_{abc} \in_{pkm} = \begin{vmatrix} \delta_{ap} & \delta_{ak} & \delta_{am} \\ \delta_{bp} & \delta_{bk} & \delta_{bm} \\ \delta_{cp} & \delta_{ck} & \delta_{cm} \end{vmatrix}$$

and hence show that

$$\in_{ibc} \in_{ikm} = \delta_{bk}\delta_{cm} - \delta_{bm}\delta_{ck}$$

(8)

(b) Using tensor methods, verify the identity

$$\vec{\nabla}(\vec{A}.\vec{B}) = \vec{A} \times (\vec{\nabla} \times \vec{B}) + \vec{B} \times (\vec{\nabla} \times \vec{A}) + (\vec{A}.\vec{\nabla})\vec{B} + (\vec{B}.\vec{\nabla})\vec{A}$$

(7)

3. (a) Assume that A, $I - A$, $I - A^{-1}$ are all non-singular matrices, show that :

$$(I - A)^{-1} + (I - A^{-1})^{-1} = I$$ (5)

(b) Find the condition for the following matrix to be orthogonal

$$\begin{bmatrix} a+b & b-a \\ a-b & a+b \end{bmatrix}.$$ (5)

(c) Evaluate C^{20}, where $C = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$. (5)

4. (a) Given a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, prove that its eigenvalue equation is given by

$$\lambda^2 - \lambda \mathrm{Tr}(A) + \det(A) = 0.$$ (5)

(b) Solve the following system of differential equations using matrix method

$$\dot{y} = z$$
$$\dot{z} = y$$

where, $y(0) = 4$, $z(0) = 2$. (10)