[This question paper contains 6 printed pages.]

Your Roll No........

Sr. No. of Question Paper : 1361 A

Unique Paper Code : 32221602

Name of the Paper : Department of Physics & Astrophysics

statistical mechanics

Name of the Course : B.Sc. (Hons) Physics – CBCS

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **four** questions in all.
3. **All** questions carry equal marks.
4. Non-programmable Scientific calculators are allowed.

1. (a) Given a system of 5 weakly interacting distinguishable particles which can occupy any of the three energy levels of energy 0, ε, and 2ε. Let the total energy of the system be 5ε. Write

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change in the wavelength after one reflection during adiabatic expansion of blackbody radiation is $d\lambda = (2v\lambda/c)\cos\theta$.

(where c is velocity of light)

(b) Obtain the value of Wien's constant by using the Planck's radiation formula.

(c) A radiating cavity has the maximum of its radiating power per unit area at $(\lambda_1)_{max} = 24\ \mu m$ at temperature T_1. Now the temperature of the cavity is changed to T_2 such that total power radiated per unit area by the cavity is 81 times higher than its previous value. Calculate the wavelength $(\lambda_2)_{max}$ where the maximum emission of radiation occur. (6.75,6,6)

4. (a) At what temperature would you expect a trapped gas of hydrogen atoms with peak density 1.8×10^{14} atoms/cm^3 to show the signs of Bose- Einstein Condensation.

(Given $m_H = 1.66 \times 10^{-27}$ kg)

If the number density of bosons become 8 times of its previous value, find the change in the condensation temperature.

2. Consider an isolated system of N distinguishable particles. Each particle can occupy only one of two energy levels of energy ε_1 and ε_2 (where $\varepsilon_1 < \varepsilon_2$). Particles are distributed in such a way that n_2 particles resides in energy level ε_2 and n_1 particles are present in level of energy ε_1. (Assume N is very large and $N = n_1 + n_2$)

(a) Find the entropy and energy of this system. Show that entropy of such system is maximum when n2 = N/2.

(b) Find the maximum and minimum value of the entropy.

(c) Obtain the general expression of temperature for the above mentioned isolated system and explain how is it possible to attain negative temperature in it. (6.75,6,6)

3. (a) Consider a spherical enclosure whose wall are moving outward with speed v ($v << c$) and are perfectly reflecting. Suppose that an electromagnetic wave of wavelength λ incident at an angle θ to the normal on the wall. Show that

all possible macrostates and their corresponding number of microstates. Find the entropy of this system.

(b) Consider equal amount of two identical ideal gases at the same temperature T but at different pressure P_1 and P_2 in two different containers of volume V_1 and V_2 respectively which are joined by the partition. Starting with Sackur-Tetrode relation, prove that If gases are allowed to mix each other by removing the partition between them, the change in the entropy is given by :

$$\Delta S = Nk \ln [(P_1 + P_2)^2 / (4 P_1 P_2)]$$

where N denotes the number of atoms in each container. Assume that the temperature remain the same after mixing of the ideal gases.

(c) The partition function, Z (V, T), for some physical system is given as:

$$Z(V, T) = \exp[(8\pi^5 k^3 V T^3) / (45 h^3 c^3)]$$

where the symbols have their usual meaning. Calculate the internal energy and pressure for such system. (6.75,6,6)

(b) Consider a photon gas enclosed in a Volume V. The photons are in equilibrium at temperature T. The average number of photons in equilibrium is given as $N = \gamma V^{\alpha} T^{\beta}$. Obtain the value of constants α, β and γ.

(c) Plot the pressure of strongly degenerate bosons with temperature. Show explicitly the $T < Tc$ and $T > Tc$ regions in the graph. Compare it with classical gas. (6.75,6,6)

5. (a) Calculate the internal energy possessed by the non-relativistic and strongly degenerate ($T < T_F$) electrons moving in 3-dimensions.

(b) Derive an expression for Fermi velocity of electrons at T = 0 K and hence show that the de Broglie wavelength associated with the electrons is given by

$$\lambda_{dB} = 2\ (\pi/3n)^{1/3}$$

where n is the number density (N/V) of the electron gas.

(c) Prove that for a system consisting of fermions at temperature T ($T << T_F$), the probability that a filled state ΔE lying above Fermi level is the same as the probability of an empty state ΔE lying below the Fermi level. (6.75,6,6)

Useful constants and Integrals :

$h = 6.6 \times 10^{-23}$ Js

$k_B = 1.38 \times 10^{-23}$ J/K

$c = 3 \times 10^8$ m/s

$\int_0^\infty x^2/(e^x - 1)\, dx = 2.404$