[This question paper contains 6 printed pages.]

Your Roll No. ....

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Sr. No. of Question Paper : 1152

Unique Paper Code : 32221401

Name of the Paper : Mathematical Physics – III

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : IV

Duration : 3 Hours — Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question No. **1** is compulsory.

1. Attempt any **five** parts : (5×3=15)

   (a) Find the cube root of $z = 1 + i$ and locate them in the plane.

   (b) Show that $u(x, y) = x^2 - y^2$ is a harmonic function in the whole complex plane, find its harmonic conjugate, $v(x, y)$.

P.T.O.

(c) Evaluate

$$\oint_C \frac{e^{3z}}{(z - i\,\pi)}\,dz \qquad C : |z - 1| = 4.$$

(d) If a complex function f(z) is analytic in a domain D and |f(z)| = Const. K in D, then show that f(z) is also constant in D.

(e) Show that the Laplace Transform of Dirac delta function is 1, ie, $L\{\delta(t)\} = 1$.

(f) If Laplace Transform of a function $L\{f(t)\} = F(s)$, show that

$$L\{t^n f(t)\} = (-1)^n F^{(n)}(s),$$

where $F^{(n)}$ represents n-th derivative of F(s).

(g) If Fourier Transform of f(x) is F(ω), find Fourier Transform of f(x) cos ax, where a > 0.

(h) Evaluate the following integrals

(i) $\int_0^\infty e^{3t}\,\delta(t-4)\,dt$

(ii) $\int_0^\infty \sin 2t\,\delta(t-\pi/4)\,dt.$

2. (a) Find all values of $\sin^{-1} 2$. (5)

(b) Expand $f(z) = e^{z/(z-2)}$ in the Laurent series about z = 2 and determine the region of convergence of this series. Also classify the singularity. (6)

(c) Evaluate (4)

$$\oint_C \frac{z}{z^2+9}\,dz, \quad \text{where} \quad C : |z - 2i| = 4$$

3. Using Contour Integration, solve any **two** of the followings : (7.5×2=15)

(a) $\int_0^\infty \frac{dx}{x^4+1}$

(b) $\int_0^\pi \frac{a}{a^2+\sin^2\theta}\,d\theta \quad a > 0$

(c) $\int_0^\infty \frac{\cos x\,dx}{(x^2+a^2)(x^2+b^2)} \quad a \,\&b > 0$

(d) $\int_0^\infty \frac{\sin^2 x}{x^2}\,dx$

4. (a) Obtain Fourier Integral representation of the function (4)

$$f(x) = \begin{cases} 0 & x < 0 \\ a & 0 \le x \le 3 \\ 0 & x > 3 \end{cases}$$

(b) Find the Fourier Transform of the function (4)

$$f(x) = \frac{x}{x^2+1}$$

(c) Find Fourier sine transform of e^{-mx}, $m > 0$ and hence evaluate the integral (7)

$$\int_0^\infty \frac{\omega \sin \omega x}{a^2+\omega^2}\, d\omega.$$

5. (a) For a periodic function f(t) having periodicity T, such that $f(t+T) = f(t)$, show that the Laplace Transform is given by (7)

$$L\{f(t)\} = \frac{\int_0^T e^{-st}\, f(t)\, dt}{1-e^{sT}}.$$

(b) If a function is piece-wise continuous on $0 < t \le T$ and is of exponential order for $t > T$ then show that (8)

$$\lim_{s\to\infty} L\{f(t)\} = \lim_{s\to\infty} F(s) = 0,$$

and hence further show that

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

where F(s) represents the Laplace Transform of f(t).

6. (a) Plot the given function (5)

$$f(x) = \begin{cases} 1 & |x| < 2 \\ 0 & |x| > 2. \end{cases}$$

Finding its Fourier Transform, F(s), plot it.

(b) Solve the differential equation

$y''(t) + 4y(t) = 9t$ with initial condition $y(0) = 0$ and $y'(0) = 7$. (10)

7. (a) Show that the Dirac delta function can be expressed as the derivative of Heaviside's unit step function. (5)

(b) For the Dirac delta function $\delta(x)$, prove that

$$\delta(x^2-a^2) = \frac{1}{2|a|}\left[\delta(x+|a|)+\delta(x-|a|)\right]. \quad (5)$$

(e) If a continuous function f(t) is an even function, then show that its Fourier Transform F(ω) will also be an even function. (5)

8. (a) Find the Fourier Transform of the function

$$f(x) = e^{-\alpha|x|}, \qquad \alpha > 0$$

and hence show that

$$\int_{-\infty}^{\infty} \frac{e^{-ikx}}{(\alpha^2 + k^2)}\, dk = \frac{\pi}{\alpha}\, e^{-\alpha|x|}. \quad (7)$$

(b) For a function

$$h(t) = \begin{cases} e^{-xt} g(t) & t > 0 \\ 0 & t < 0, \end{cases}$$

show that F{h(t)} = L{g(t)}. (3)

(c) For the equation $z^4 - 3z^2 + 1 = 0$, find the sum of its roots. (5)