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[This question paper contains 6 printed pages.]

Your Roll No.



Sr. No. of Question Paper : 1116

Unique Paper Code : 32221601

Name of the Paper : Electromagnetic Theory

Name of the Course : B.Sc. (Hons) Physics-CBCS

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question No. 1 is compulsory.
3. Answer any **four** of the remaining **six**.
4. Use of non-programmable calculator is allowed.

1. Attempt any 5 parts of this question. (5x3)

(a) A conductor of square cross section and having conductivity 3.8×10^7 S/m is 50 m long. It measures 0.5 cm on either side. Calculate the skin depth if conductor carries current at 150 kHz.

P.T.O.

- (b) The conduction current density in a material is given as $\vec{J}_C = 0.02 \sin(10^9 t) \text{ A/m}^2$. Find the displacement current density if $\sigma = 10^5 \text{ mho/m}$, $\epsilon_r = 6.5$.
- (c) What is plasma frequency and minimum penetration depth for collision free plasma having $10^{12} \text{ electrons/m}^3$.
- (d) A 300 mm long tube containing 56 cm^3 of sugar solution produces an optical rotation of 12° when placed in a polarimeter. If the specific rotation of sugar solution is 66° , calculate the quantity of sugar contained in the tube.
- (e) For glass air interface ($n_1 = 1.5$, $n_2 = 1.0$), find the reflection and transmission coefficients for normal incidence.
- (f) A left circularly polarized wave ($\lambda = 5893 \text{ \AA}$) is incident normally on calcite crystal of thickness 0.00514 mm . Find the state of polarization of the emergent beam. Take $n_o = 1.65836$ and $n_e = 1.48641$.
- (g) A circularly polarized electromagnetic wave is propagating in the z-direction in free space and is described by the following equation

$$\vec{E} = 5 \cos(\omega t - kz) \hat{x} + 5 \sin(\omega t - kz) \hat{y} \text{ Vm}^{-1}$$

The wavelength is $6 \times 10^{-7} \text{ m}$. Find the corresponding magnetic field and the average of the Poynting vector.

2. (a) State and establish the Poynting theorem for conservation of energy for electromagnetic fields. Explain the physical significance of each term in the equation. (6)
- (b) Using electromagnetic scalar and vector potentials, show that the four Maxwell equations can be written as two coupled second order differential equations. (6)
- (c) What is the ratio of amplitudes of conduction current density and displacement current densities density if applied field is $\vec{E} = \vec{E}_0 e^{-t/\tau}$, where τ is real. (3)
3. (a) Discuss the propagation of high frequency electromagnetic wave in plasma. Show that the critical frequency for the propagation of electromagnetic wave in plasma is given by $f_c = 9\sqrt{n_0}$, where n_0 is the electron density. (8)

- (b) A material has $\sigma = 6.0 \times 10^{-2} \Omega^{-1}/\text{m}$, $\mu = \mu_0$ and $\epsilon_r = 7.0$. A plane wave of frequency 10^9 Hz with amplitude 200V/m is propagating along positive z direction. Find (a) E_x at ($x = 0$ cm, $y = 0$ cm, $z = 3$ cm. $t = 0.16$ ns) (b) H_y at ($x = 0$ cm, $y = 0$ cm, $z = 3$ cm. $t = 0.16$ ns). (7)
4. (a) Derive the Fresnel's equation for reflection and transmission of a plane electromagnetic wave at the boundary separating between two dielectric media when electric field vector is perpendicular to the plane of incidence. (9)
- (b) An electromagnetic wave propagating in a dielectric medium with $\epsilon = 16\epsilon_0$ along the z direction. It strikes another dielectric medium with $\epsilon = 4\epsilon_0$ at $z = 0$. If the incoming wave has a maximum value of 0.2 V/m at the interface, and its angular frequency is 300 M rad/s, determine the power densities of the incident, reflected, and transmitted waves. (6)
5. (a) Derive Fresnel's formula for the propagation of light in Anisotropic crystals. Also explain how this leads to the phenomenon of double refraction. (9)

- (b) Discuss the propagation of light in uniaxial crystals. Explain the difference between a positive uniaxial crystal and a negative uniaxial crystal. Also give one example of each. (6)
6. (a) Explain the construction and working of a Babinet Compensator. How is it used to analyze the elliptically polarized light? (9)
- (b) A right-handed circularly polarized plane wave with electric field magnitude of 3 mV/m is travelling in the +Y direction in a dielectric medium with $\epsilon = 4\epsilon_0$ and $\mu = \mu_0 = 0$. If the frequency is 100MHz, obtain expressions for $\vec{E}(y,t)$ and $\vec{H}(y,t)$. (6)
7. (a) Starting with Maxwell's equations, derive the wave equations for a symmetric planar dielectric waveguide with refractive index profile as :
- $$n = n_1 \quad -d/2 < x < d/2$$
- $$n = n_2 \quad -d/2 > x > d/2$$
- Using the boundary conditions, obtain the eigenvalue equation for symmetric TE modes. (9)

- (b) Consider a symmetric planar waveguide with the following parameters.

$$n_1 = 1.5 ; n_2 = 1.48 ; d = 3.912 \text{ } \mu\text{m}$$

How many TE modes exist for $\lambda_0 = 1 \text{ } \mu\text{m}$?
Determine the corresponding propagation constants. (6)

Value of Constants:

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ farad/m} = \frac{10^{-9}}{36\pi} \text{ farad/m}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ henry/m}$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$\eta_0 = 120 \pi \Omega = 377 \Omega$$

$$\text{Mass of electron} = 9.11 \times 10^{-31} \text{ kg}$$