[This question paper contains 4 printed pages.]



Your Roll No. ..............



**Sr. No. of Question Paper : 2779** **A**

Unique Paper Code : 62357603

Name of the Paper : Numerical Methods

Name of the Course : **B.A. (Prog.)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All six** questions are compulsory.
3. Attempt any **two** parts from each question.
4. Use of Non – Programmable Scientific Calculator is allowed.



Q-1. (a) Find the real root of the equation $2x^3 - 3x + 1 = 0$ by Regula Falsi method. Perform two iterations. [6]

(b) Use secant method to find root of $3x + \sin(x) - e^x = 0$ in $]0,1[$. Perform two iterations. [6]

(c) Define the floating point representation, Global error and Truncation error with examples. [6]

(d) Obtain the Rate of Convergence of Bisection method. [6]

P.T.O.

Q-2. (a) Find the value of :-

$$sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

with an absolute error smaller than 0.005 for x = 0.2145E0 using Normalized floating point arithmetic with 4 digit mantissa. [6.5]

(b) Evaluate the sum S = √3 + √8 + √10 to four significant digits and find its absolute and relative error. [6.5]

(c) Use Bisection method to find a real root of the equation

f(x) = 2x - √(1+sinx) = 0. [6.5]

(d) Find a real root of the equation 3x = cos x + 2 by Newton- Raphson method. [6.5]

Q-3. (a) The function $y = f(x)$ is given at the point $(7,3),(8,1),(9,1)$ and $(10,9)$. Find $f(8.5)$ using Lagrange's interpolation technique. [6]

(b) Find the inverse of the following matrix using the Gauss-Jordan method:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ [6]

(c) For the following system of equations:

$$4x + y + 2z = 4$$

$$3x + 5y + z = 7$$

$$x + y + 3z = 3.$$

Use Gauss-Jacobi iteration method by performing three iterations. Take the initial approximation as $(x, y, z) = (0,0,0)$. [6]

(d) If $f(x) = \frac{1}{x}$ then evaluate Newton Dividend difference $f[a, b, c, d]$. Also prove the following relation:

$$(1 - \nabla)^{-1} = 1 + \frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$$ [6]





Q-4. (a) Solve the linear system $Ax = b$ using Gauss-Elimination method with row pivoting:

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 4 & -2 & 1 \\ 3 & -1 & 3 \end{bmatrix} \text{ and } b = \begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix}$$ [6.5]

(b) Starting with initial vector $(x, y, z) = (1, 1, 1)$, perform three iterations of Gauss-Seidel method to solve the following system of equation:

$$10x + 2y + z = 9$$

$$2x + 20y - 2z = -44$$

$$-2x + 3y + 10z = 22.$$ [6.5]

(c) Find the cubic polynomial which takes the following values: [6.5]

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$ | 1 | 2 | 1 | 0 |

(d) Apply Gauss-Jordan method to solve:

$$x + 2y + z = 8$$

$$2x + 3y + 4z = 20$$

$$4x + 3y + 2z = 16.$$ [6.5]

Q-5. (a) The velocity $v$(km/min) of a moped which starts from rest, is given at fixed intervals of time $t$ (min) as follows: [6]

| $t$ | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| $v$ | 0 | 10 | 18 | 25 | 29 | 32 | 20 |

Estimate approximately the distance covered in 12 minutes by Simpson $\frac{1}{3}$rd rule.

(b) Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$, at $y = 0.1$ by taking $h = 0.02$ by using Euler's method. [6]

(c) Apply modified Heun's method to calculate $y(1)$, given that

$$\frac{dy}{dx} = x + 2y; y(0) = 0;\ h = 0.5.$$ [6]

(d) Evaluate $I = \int_0^1 x\sqrt{1+x}\,dx$ using Trapezoidal rule with 4 subintervals. [6]

Q-6. (a) Evaluate $\int_0^\pi \sin x\ dx$ using Simpson rule by dividing interval into four equal parts. [6.5]

(b) Calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 5$, given the following table: [6.5]

| x | 2 | 4 | 9 | 10 |
|---|---|---|---|---|
| f(x) | 4 | 56 | 71 | 90 |

(c) Apply modified Euler's method to approximate the solution of the initial value problem and calculate $y(1.3)$ by using $h = 0.1$:

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2},\ y(1) = 1.$$ [6.5]

(d) The following table of values is given:

| $x$ | 0.6 | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 | 1.4 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0.7072 | 0.8599 | 0.9259 | 0.9840 | 1.0337 | 1.0746 | 1.1280 |

using the formula $f'(x_0) = \frac{f(x_0+h)-f(x_0-h)}{2h}$, with $h = 0.4, 0.2, 0.1$ and the Richardson extrapolation, find $f'(1)$. [6.5]