[This question paper contains 4 printed pages]

Your Roll No...........

Sr. No. of Question Paper : 2819 **A**

Unique Paper Code : 62354443

Name of the Paper : Analysis (LOCF)

Name of the Course : **B.A. (Prog.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.

1. (a) Let $S = \{x \in \mathbb{R} : x \geq 0\}$. Show in detail that the set S has lower bounds, but no upper bounds. Show that inf S=0. Verify your answer.

   (b) Define continuity of a real valued function at a point .

   Show that the function defined as $f(x) = \begin{cases} \dfrac{x^2-9}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$

   is continuous at $x = 3$.

   (c) Let S be a non empty bounded set in $\mathbb{R}$. Let $a > 0$, and let $aS = \{as: s \in S\}$. Prove that $\inf aS = a \inf S$, $\sup aS = a \sup S$.

P.T.O.

(d) Test for convergence the series those nth term is $\left(\frac{\sqrt{n+1}-\sqrt{n-1}}{n}\right)$.

2. (a) A function f is defined by

$$f(x)=\begin{cases}\frac{1}{2}-x, & if\ 0<x<\frac{1}{2}\\ \frac{3}{2}-x, & if\ \frac{1}{2}\le x<1\end{cases}$$

Evaluate $\lim_{x\to\frac{1}{2}} f(x)$

(b) Define order completeness property of real numbers. State and prove Archimedean Property of real numbers.

(c) Show that the function f defined by $f(x)=x^3$ is uniformly continuous in the interval [0, 3].

(d) Prove that a necessary and sufficient condition for a monotonically increasing sequence to be convergent is that it is bounded above.

3. (a) State Cauchy's second Theorem on Limits. Prove that

$$\lim_{n\to\infty}\left[\frac{(2n)!}{(n!)^2}\right]^{1/n}=4$$

(b) Test for convergence the series whose nth term is $u_n=\frac{n^{n^2}}{(n+1)^{n^2}}$

(c) State Cauchy's general principle of convergence. Apply it to prove that the sequence $\langle a_n\rangle$ defined by

$$a_n=1+\frac{1}{4}+\frac{1}{7}+\dots+\frac{1}{3n-2}$$ is not convergent.

(d) Prove that a sequence of real numbers converges if and only if it is a Cauchy sequence.

4 (a) State D'Alembert's ratio test for the convergence of a positive term series. Use it to test for convergence the series $\sum_{n=1}^{\infty}\frac{2^{n-1}}{3^n+1}$.

(b) A sequence $\langle a_n\rangle$ is defined as follows:

$$a_1=1,\quad a_{n+1}=\frac{4+3a_n}{3+2a_n},\quad n\ge 1$$

Show that sequence $\langle a_n\rangle$ converges and find its limit.

(c) Show that the series

$$\sum_{n=1}^{\infty}\frac{2.4.6....2n}{1.3.5.....(2n+1)}$$ diverges.

(d) Prove that if a function f is continuous on a closed and bounded interval [a, b], then it is uniformly continuous on [a, b].

5 (a) State Leibnitz test for convergence of an alternating series of real numbers. Apply it to test for convergence the series $\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{7}}+\dots$

(b) Show that the function f defined by

$$f(x)=\begin{cases}0, & \text{when } x \text{ is rational}\\ 1, & \text{when } x \text{ is irrational}\end{cases}$$

is not integrable on any interval.

(c) Test for convergence and absolute convergence of the following series

$$\frac{1}{1.2}-\frac{1}{3.4}+\frac{1}{5.6}-\frac{1}{7.8}+\dots$$

(d) Show that the sequence defined by $\langle a_n\rangle=\left\langle\frac{n}{n+1}\right\rangle$ is a Cauchy sequence.

6 (a) Show that every Monotonic function on [a, b] is integrable on [a, b]

(b) Test the convergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n\alpha}{n^p}$, p > 0. Is this series absolutely convergent.

(c) Show that the function $f(x) = [x]$, where $[x]$ denotes the greatest integer not greater than x, is integrable over [0, 3] and $\int_0^3 [x]dx = 3$

(d) Show that the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \tan \frac{1}{n}$ is convergent.