[This question paper contains 10 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 1211 **A**

Unique Paper Code : 32357614

Name of the Paper : DSE-3 MATHEMATICAL FINANCE

Name of the Course : **B.Sc. (Hons) Mathematics CBCS (LOCF)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory and carry equal marks.
4. Use of Scientific calculator, Basic calculator and Normal distribution tables all are allowed.

1. (a) Explain Duration of a zero-coupon bond. A 4-year bond with a yield of 10% (continuously compounded) pays a 9% coupon at the end of each year.

(i) What is the bond's price?

(ii) Use duration to calculate the effect on the bond's price of a 0.3% decrease in its yield?

(You can use the exponential values: $e^x = 0.9048$, 0.8187, 0.7408, and 0.6703 for $x = -0.1, -0.2, -0.3$, and -0.4, respectively)

(b) Explain Continuous Compounding. Suppose R_c denotes rate of interest with continuous compounding and R_m denotes equivalent rate with compounding m times per annum. Find the relation between R_c and R_m.

(c) An investor receives ₹ 1100 in one year in return for an investment of ₹ 1000 now. Calculate the percentage return per annum with :

(i) Annual compounding

(ii) Semi-annual compounding

(iii) Continuous compounding.

(You can use: $\ln(1.1) = 0.953$)

(d) Define Bond Yield and Par Yield. Suppose that the 6-month, 12-month, 18-month, and 24-month zero rates are 5%, 6%, 6.5% and 7% respectively. What is the 2-year par yield? (You can use the exponential values: $e^x = 0.9753$, 0.9418, 0.9071, 0.8694 for $x = -0.025, -0.06, -0.0975, -0.14$, respectively.)

2. (a) Explain Hedging. A United States company expects to pay 1 million Canadian dollars in 6 months. Explain how the exchange rate risk can be hedged using

(i) A Forward Contract

(ii) An Option.

(b) (i) What is the difference between the over-the counter market and the exchange-traded market?

(ii) An investor enters a short forward contract to sell 175,000 British pounds for US dollars at an exchange rate of 1.900 US dollars per pound. How much does the investor gain or lose if the exchange rate at the end of the contract is 2.420?

(c) A 1-year forward contract on a non-dividend paying stock is entered into when the stock price is ₹ 40, and the risk-free rate of interest is 10% per annum with continuous compounding. What is the forward price? Justify using no arbitrage arguments. ($e^{0.1} = 1.1052$)

(d) (i) A trader writes an October call option with a strike price of ₹ 35. The price of the option is ₹ 6. Under what circumstances does the trader make a gain,

(ii) Suppose that you own 6,000 shares worth ₹ 75 each. How can put options be used to provide an insurance against a decline in the value of the holding over the next 4 months?

3. (a) Draw the diagrams illustrating the effect of changes in stock price, strike price, and expiration date on European call and put option prices when

$S_0 = 50$, $K = 50$, $r = 5\%$, $\sigma = 30\%$, and $T = 1$.

(b) Derive the put-call parity for European options on a non-dividend-paying stock. Use put-call parity to derive the relationship between the delta of a European call and the delta of a European put on a non-dividend-paying stock.

(c) An investor sells a European call on a share for ₹ 4. The stock price is ₹ 47 and the strike price is ₹ 50. Under what circumstances does the investor make a profit? Under what circumstances will the option be exercised? Draw a diagram showing the variation of the investor's profit with the stock price at the maturity of the option.

(d) Define upper bound and lower bound for European options on a non-dividend- paying stock. What is a lower bound for the price of a 3-month European put option on a non-dividend-paying stock when the stock price is ₹38, the strike price is ₹40, and the risk-free interest rate is 10% per annum? Justify using no arbitrage arguments. ($e^{-0.04} = 0.9753$)

4. (a) A 4-month European call option on a dividend-paying stock is currently selling for ₹ 50. The stock

price is ₹ 640, the strike price is ₹ 600, and a dividend of ₹ 8 is expected in 1 month. The risk-free interest rate is 12% per annum for all maturities. What opportunities are there for an arbitrageur? ($e^{-0.04} = 0.9608$)

(b) Consider a one-period binomial model where the stock can either go up from S_0 to S_0u $(u > 1)$ or down from S_0 to S_0d $(d < 1)$. Suppose we have an option with payoff f_u if the stock moves up and payoff f_d if the stock moves down. By considering a portfolio consisting of long position in Δ shares of stock and a short position in the option, find the price of the option. Explain how the price can be expressed as an expected payoff discounted by the risk-free interest rate.

(c) A stock price is currently ₹ 50. It is known that at the end of two months it will be either ₹ 53 or ₹ 48. The risk-free interest rate is 12% per annum with continuous compounding. What is the value of a two-month European call option with a strike price of ₹ 49? Use no-arbitrage arguments. ($e^{0.02} = 1.0202$)

(d) Consider a two-period binomial model with current stock price S_0 = ₹ 100, the up factor $u = 1.3$, the down factor $d = 0.8$, $T = 1$ year and each period being of length six months. The risk-free interest rate is 5% per annum with continuous compounding. Construct the two-period binomial tree for the stock. Find the price of an American put option with strike K = ₹ 95 and maturity $T = 1$ year. ($e^{-0.025} = 0.9753$)

5. (a) Stock price in the Black-Scholes model satisfies

$$\ln S_T \sim \phi\left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma^2 T\right]$$

where $\phi(m, v)$ denotes a normal distribution with mean m and variance v. Find $\mathrm{Var}[S_T]$.

(b) What is the price of a European put option on a non-dividend-paying stock when the stock price is ₹ 69, the strike price is ₹ 70, the risk-free interest rate is 5% per annum, the volatility is 35% per annum, and the time to maturity is six months?

(You can use exponential values: $e^{-0.0144} = 0.9857$, $e^{-0.025} = 0.9753$)

(c) Let V be a lognormal random variable with ω being the standard deviation of ln V. Prove that

$$E[\max(V - K, 0)] = E(V)N(d_1) - KN(d_2)$$

where

$$d_1 = \frac{\ln\left[\frac{E(V)}{K}\right] + \frac{\omega^2}{2}}{\omega}, \quad d_2 = \frac{\ln\left[\frac{E(V)}{K}\right] - \frac{\omega^2}{2}}{\omega}$$

and E denotes the expected value. Use this result to derive the Black-Scholes formula for the price of a European call option on a non-dividend paying stock.

(d) A stock price is currently ₹ 50. Assume that the expected return from the stock is 18% and its volatility is 30%. What is the probability distribution for the stock price in 2 years? Calculate the mean and standard deviation of the distribution. ($e^{0.18}$ =1.1972)

6. (a) Discuss gamma of a portfolio of options and calculate the gamma of a European call option on a non-dividend-paying stock where the stock price is ₹ 49, the strike price is ₹ 50, the risk-free

interest rate is 5% per annum and the time to maturity is 20 weeks, and the stock price volatility is 30% per annum. (ln(49/50) = –0.0202)

(b) What is the relationship between delta, theta and gamma of an option? Show by substituting for various terms in this relationship that it is true for a single European put option on a non-dividend-paying stock.

(c) Find the payoff from a bear spread created using put options. Also draw the profit diagram corresponding to this trading strategy.

(d) Companies X wishes to borrow US dollars at a fixed interest rate. Company Y wishes to borrow Indians rupees at a fixed rate of interest. The amounts required by the two companies are roughly the same at the current exchange rate. The companies have been quoted the following interest rates, which have been adjusted for the impact of taxes :

	Rupees	Dollars
Company X	9.6%	6.0%
Company Y	11.1%	6.4%

Design a swap that will net a bank, acting as intermediary, 50 basis points per annum. Make the swap equally attractive to the two companies and ensure that all foreign exchange risk is assumed by the bank.