[This question paper contains 6 printed pages.]



Your Roll No.....



Sr. No. of Question Paper : 1224

Unique Paper Code : 32227626

Name of the Paper : Classical Dynamics

Name of the Course : B.Sc. (Hons) Physics-DSE-4

Semester : VI

Duration : 3 Hours Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question No. 1 which is compulsory.
3. Attempt any three questions from the remaining.

1. Attempt any four of the following : (4×6=24)

   (a) Show that phase of an electromagnetic wave is Lorentz invariant.

   (b) Write the Hamiltonian of a one-dimensional harmonic oscillator.

P.T.O.

(c) Define pathlines, streamlines and streaklines.

(d) A particle of mass m is constrained to move on the boundary of an ellipse $x^2 + 2y^2 = 2$. Identify the generalized coordinates of this system.

(e) An alpha-particle ($q = 3.2 \times 10^{-19}$ C) moves through a uniform magnetic field whose magnitude is 1.5 T. The field is directly parallel to the positive z-axis of the rectangular coordinate system. What is the magnitude and direction of the magnetic force on the alpha- particle when it is moving with a velocity $\vec{v} = (2.0\hat{i} - 3.0\hat{j} + 1.0\hat{k}) \times 10^4\ m/s$.

(f) Given the fluid velocity components

$$v_x = -\alpha y/(x^2 + y^2),\ v_y = \alpha x/(x^2 + y^2),\ v_z = 0.$$

Verify that the fluid is incompressible.

2. (a) A bead of mass m slides freely on a frictionless circular wire of radius b. The wire itself rotates in a horizontal plane about an axis passing through a point on perimeter of the wire with a constant angular velocity $\omega$.

Find the Lagrangian of the system with a suitable choice of generalized coordinates. Deduce the Euler- Lagrange's equation of motion of the bead. Show that the bead oscillates as a pendulum of length $l = g/\omega)^2$. (10)

(b) For a time independent holonomic system, show that the Hamiltonian of the system represents total energy. (7)

3 (a) Using cylindrical coordinates, write the Hamiltonian and Hamilton's equations for a particle of mass m moving on the inside of a frictionless cone $x^2 + y^2 = z^2 \tan^2 \alpha$. (10)

(b) Show that the energy conservation is nothing but a consequence of the time-shifit invariance of a system. (7)

4. (a) An observer A stands 1 light-second away from a tunnel of length L = 1 light-second. A high-speed train speeds through the tunnel at constant velocity $\beta = 0.5$. An observer inside the train measures the length of the train to be L = 2 light-second.

P.T.O.



(i) Draw a spacetime diagram for observer A showing the worldlines of the front and rear of the train and the tunnel. Assume that at t = 0 the rear of the train has just crossed the observer.

(ii) Label the event at which the front end of the train emerge from the tunnel and the rear end of the train enter the tunnel. Using the spacetime diagram, show that the train fits the tunnel as observed by A.

(iii) In the train frame, label the event along the worldline of the rear which is simultaneous to the event when the front end emerges from the tunnel and label the event along the worldline of the front which is simultaneous to the event when the rear end of the train enters the tunnel. (10)

(b) Show that in units such that c = 1, the 4-acceleration is given by $A = \gamma(d\gamma/dt, vd\gamma/dt + \gamma a)$ where a is the 3-acceleration. Prove that $v^\mu a_\mu = 0$ where $v^\mu$ is 4-velocity and $a_\mu$ is 4-acceleration. (7)

5. (a) A particle of mass m moving at speed v collides with another particle of the same mass at rest. They stick together and move with speed V. What is V in terms of v? What is the mass of the final combined particle? (10)

(c) Let an observer B move relative to an observer A with fractional velocity v. For a photon moving in the v direction, show that energy $E' = E\sqrt{(1-\beta)/(1+\beta)}$ where $\beta = v/c$. Also derive the relativistic Doppler shift. (7)

6. (a) Three masses m each, initially located equidistant from one another on a horizontal circle of radius R. They are connected in pairs by three springs of force constant k each and of unstretched length $2\pi R/3$. The spring threads the circular tract so that the mass is constrained to move on the circle. Find the normal modes with their frequencies and normalized coordinates. (10)

(b) Define stable and unstable equilibrium. In case of a simple pendulum, find the point of stable and unstable equilibrium. (7)

P.T.O.



7. (a) Show that motion of a particle with mass m and charge q moving at a speed $\vec{v}$ in a magnetic field of strength $\vec{B}$ is helical. A proton enters a uniform magnetic field of with a speed of $v = 1.0 \times 10^{-4}$ T. At what angle must the magnetic field be from the velocity so that the pitch of the resulting helical motion is equal to the radius of the helix? (10)

(b) Show that the pressure at a point in an inviscid fluid is independent of direction. (7)