[This question paper contains 6 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1311

Unique Paper Code : 32227626

Name of the Paper : Classical Dynamics

Name of the Course : **B.Sc. (Hons) Physics-DSE-4**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question No. **1** which is compulsory.
3. Attempt any **three** questions from the remaining.

1. Attempt any **four** of the following : (4×6=24)

   (a) Write the Lagrangian and Hamiltonian of a free particle.

   (b) A muon at rest lives 10^{-6} s and its rest mass is 100 MeV/c^2. How energetic must a muon be to

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reach the earth surface if it is produced high in the atmosphere at 10^4 m up?

(c) What is spacetime interval? Components of a 4-vector A^μ are $(3, 1, \sqrt{2}, 3)$ with the first component being the time component. Determine whether this vector is timelike, spacelike or null.

(d) Consider motion of a particle in plane with coordinates (r, θ) in the presence of a radial potential $V(r)$. Identify the conserved quantities.

(e) Given the fluid velocity components

$$v_x = -\alpha y/(x^2 + y^2),\ v_y = \alpha x/(x^2 + y^2),\ v_z = 0.$$

Find the equation of stream lines.

(f) What is Reynold's number? Give its importance in viscous fluid flow.

2. (a) The point of support of a simple pendulum moves vertically according to $y = h(t)$ where $h(t)$ is some function of time.

(i) Find the Lagrangian of the system with a suitable choice of generalized coordinates.

(ii) Deduce the Euler-Lagrange equation of motion.

(iii) Show that for $h(t) = vt$ the system executes simple oscillations with angular frequency $\sqrt{g/l}$. (10)

(b) Derive Hamilton's canonical equations from variational principle. (7)

3. (a) Consider a particle of charge q and mass m moving with velocity $\vec{v}$ in an electromagnetic field with the potential energy $V = q(\phi - \vec{v} \cdot \vec{A})$ where $\vec{A}$ and ϕ are vector and scalar potentials. Show that Hamiltonian of the particle is

$$H = \frac{1}{2m}(\vec{p} - q\vec{A})^2 + q\phi$$

Also find the equation of motion. (10)

(b) Show that the linear momentum conservation is nothing but a consequence of the spatial-translation invariance of a system. (7)

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4. (a) A train of rest length $L_0 = 1$ light-second travels at speed $\beta = 0.5$ in the positive x-direction of Lata. As the front of the train passes Lata at $t = 0$, a light signal is sent from the front of the train to the rear. (i) Draw a spacetime diagram showing the worldlines of the front and rear of the train and the photon in Lata's reference frame. Using the diagram or otherwise, (ii) find when does the rear of the train pass Lata and (iii) when does the signal reach the rear of the train according to Lata? (10)

(b) Two particles with the same mass m are emitted in the same direction with momenta 5mc and 10mc respectively. As seen fron the slower one, what is the speed of the faster particle. (7)

5. (a) A meson of mass π comes to rest and disintegrates into a meson of mass μ and a neutrino of zero mass. Show that the kinetic energy of motion of the μ meson (i.e. without the rest mass energy) is $c^2(\pi - \mu)^2/2\pi$. (10)

(c) Show that for a space-like interval between two events in Minkowski space, there does not exist any frame in which the two events are co-located. Can the two events be simultaneous in any inertial frame? Explain your answer. (7)

6. (a) Consider a pair of identical pendulums consisting of a pair of massless rigid rods of length l each at the end of each of which a mass m is attached. The mid-points of the rods are connected by a spring of force constant k. In the absence of coupling, each oscillator has a frequency $\omega = \sqrt{g/l}$. Find the normal modes with their frequencies and normalized coordinates. (10)

(b) Calculate the gyro radius, gyro frequency and velocity of the motion of a negatively charged particle $-q$ of mass m in a uniform electric field $E = E_0\hat{x}$ and uniform magnetic field $B = B_0\hat{z}$, which are perpendicular to each other. (7)

7. (a) What is a streamline flow? Derive the equation of streamline. Under what condition a streamline flow change to a turbulent flow. (10)

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(b) Consider the steady and two-dimensional flow field given by $V = x\hat{i} - y\hat{j}$. Find the shape of the streamlines. Find the shape of the pathlines.

(7)