

Name of the Department: PHYSICS DEPARTMENT

Name of Course: B.Sc. Hons.–CBCS_Core (NC)

Semester: V- Semester

Name of the Paper: Quantum Mechanics and Applications

Unique Paper Code: 32221501

Time Duration: 3 Hours

Maximum Marks: 75

Attempt four questions out of six. Each question carries equal marks.

1.

- i. A particle is represented at (t=0) by the wave function:

$$\psi(x, 0) = \begin{cases} A(a^2 - x^2) & -a \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$$

Find A and expectation value of x, x^2, p and p^2 . Find uncertainties in position and momentum.

- ii. Show that Divergence of J (probability current density) is zero for stationary states.
- iii. Find the Fourier transform of the wave function e^{-ax^2} .

(14.75+2+2)

2.

- i. State Heisenberg's Uncertainty principle. What is the origin of concept of uncertainty in position and momentum? Derive $\Delta x \Delta p \geq \hbar/2$.
- ii. Verify whether the following operators are linear:

a. $\widehat{f(x)} = \frac{d}{dx} f(x)$

b. $\widehat{f(x)} = \sqrt{f(x)}$

- iii. What is uncertainty in the location of a photon of wavelength 5000 Angstrom which is known to an accuracy of one part in 10^7 ?

(12.75+3+3)

3.

- i. Solve Schrodinger's equation for the potential energy $V = (1/2)kx^2$ and show that the energy eigenvalue are $E_n = (n + \frac{1}{2})\hbar\omega$.
- ii. Which of the following wave functions

$$(i) e^{-ax^2} \quad (ii) \sin(kx)$$
 are eigenvalues of operator (a) p and (b) p^2 .
- iii. Find the locations of classical turning points for a One Dimensional Harmonic Oscillator in its ground state.

(12.75+4+2)

4.

- i. Describe the Stern-Gerlach Experiment and its theory. Discuss the significance of the experiment. Why is an inhomogeneous magnetic field required?
- ii. A beam of silver atoms moving with a velocity of 10^5 cm/s passes through a magnetic field of gradient $0.50 \text{ Wb/m}^2/\text{cm}$ for a distance of 10 cm . Determine acceleration of Ag atoms, time spent by atoms in the field and displacement of Ag atoms along z-direction as it comes out of the magnetic field (along z-axis).
- iii. Show that $\frac{d}{dt} \int_{-\infty}^{\infty} \psi_1^* \psi_2 dx = 0$ for any two (normalizable) solutions to Schrodinger's equation, ψ_1 and ψ_2 .

(12.75+3+3)

5.

- i. Derive an expression for energy difference ΔE between doublets due to Spin-Orbit coupling. How does ΔE depend on quantum numbers n and l ?
- ii. Show that the angle between angular momentum (\mathbf{L}) and z-axis is given by $\theta_{m_l} = \cos^{-1}(\frac{m_l}{\sqrt{l(l+1)}})$. Find the values of angle θ_{m_l} for $l=2$.
- iii. Calculate the probability of finding the electron in the region $\frac{a_0}{2} < r < 2a_0$ in a hydrogen atom in ground state given that wave function for

the ground state of Hydrogen atom is $\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{\frac{-r}{a_0}}$, where $a_0 =$

$$\frac{4\pi\epsilon_0\hbar^2}{me^2}.$$

(12.75+3+3)

6.

- i. Solve 1-D time independent Schrodinger's equation for a particle having energy E for a square well of finite depth V_0 ($E < V_0$). Show graphically existence of bound states.
- ii. An electron moves in 1-D potential well of width 8 Angstrom and depth 12eV. Find the number of bound states?
- iii. Assuming LS coupling scheme, list the possible total angular momentum and spectral terms for three electron having configuration 2p 3p 4d.

(12.75+3+3)

Physical Constants:

Mass of Ag atom = 107.87amu,

Charge of electron = 1.6×10^{-19} C,

$h = 6.626 \times 10^{-34}$ Js

Mass of electron = 9.1×10^{-31} Kg.