

Roll No.....

Unique Paper Code : 32227502

Name of Paper : Advanced Mathematical Physics

Name of Course : **B.Sc. (Hons.) Physics-CBCS_DSE**

Semester : **V-Semester**

Duration: **3 Hours**

Maximum Marks: **75**

All questions carry equal marks.

Attempt four questions in all.

Use of Scientific calculator is allowed.

1 (a) If V is the vector space spanned by the vectors α_1, α_2 and α_3 , then show that the vectors $\beta_1 = \alpha_1, \beta_2 = \alpha_1 + \alpha_2$ and $\beta_3 = \alpha_1 + \alpha_2 + \alpha_3$ also spans V .

(b) A transformation $T: R^2 \rightarrow R^2$ is defined by

$$T[x, y] = \left[\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}, \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \right]$$

Show that T is linear and give the geometrical interpretation of T .

(c) Find the dimension of, and a basis for, the solution space of the system of equations

$$x + 2y - 2z + 2s - t = 0$$

$$x + 2y - z + 3s - 2t = 0$$

$$2x + 4y - 7z + s + t = 0$$

(4.75+7+7)

2(a) Verify Cayley-Hamilton theorem for the given matrix B .

$$B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(b) Find the inverse of matrix B .

(b) What constant should be multiplied to make the given matrix Unitary?

(6+6 + 6.75)

3. (a) Solve the given system of differential equations using matrix method:

$$y_1' = y_1 + y_2$$

$$y_1' = 4y_1 + y_2$$

subject to the initial conditions $y_1(0) = y_2(0) = 1$.

(b) If A is square matrix prove that e^A is nonsingular.

(15+3.75)

4. Explain and prove the Quotient Law. If $A_{ij}B^i = C_j$ are component of covariant vector for all choice of contravariant vector, B^i then A_{ij} is a covariant tensor.

(18.75)

5. (a) Prove that

$$\nabla(\nabla \cdot A) = \nabla \times (\nabla \times A) + \nabla^2 A \text{ using tensors.}$$

(b) Explain the physical significance of diagonal and off-diagonal terms of strain tensor. Also prove that it is symmetric tensor of rank two.

(8.75, 10)

6. (a) Show that one time contraction reduces the order of tensor by 2.

(b) Show that $T_k^{ij} \delta_l^i$ is not a tensor, where T_k^{ij} and δ_l^h are tensor.

(8.75, 10)