Roll No.....…

Unique Paper Code : 32227502

Name of Paper : Advanced Mathematical Physics

Name of Course : **B.Sc. (Hons.) Physics-CBCS_DSE**

Semester : **V-Semester**

Duration: **3 Hours** Maximum Marks: **75**

All questions carry equal marks.

Attempt four questions in all.

Use of Scientific calculator is allowed.

1. (a) For which value of k will the vector $[1,-2,k]$ belong to the subspace of R^3spanned by the vectors $[3,0,-2]$ and $[2,-1,-5]$?

(b) Let V be denoted byP_2, the vector space consisting of all polynomials of degree ≤ 2,and the zero polynomial. Let$S=\{P_1(x),P_2(x),P_3(x)\}$, where

$$P_1(x)=x^2+1, P_2(x)=x^2+x, P_3(x)=x+1$$

Does SspanP_2?

(c) Consider the subset of R^4 given by

$$S=\{(1,-2,0,0),(0,2,1,0),(-1,0,1,1),(0,0,2,1)\}$$

Find a basis of span $S$and extend it to a basis ofR^4.

(4.75, 7, 7)

2. (a) Verify Cayley-Hamilton theorem for the given matrix B.

$$B=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$$

(b) Find the inverse of matrix$B$.

(c) What constant should be multiplied to make the given matrix orthogonal?

$$\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ -2 & 2 & -1\end{bmatrix}$$

(6+6 + 6.75)

3. (a) Solve the given system of differential equations using matrix method:

$$y_1' = -y_1 + 4y_2$$

$$y_1' = 3y_1 - 2y_2$$

subject to the initial conditions $y_1(0) = 3, y_2(0) = 4$.

(b) If A is square matrix prove that $(e^A)^{-1} = e^{-A}$.

(15 + 3.75)

4. (a) Show that

$\varepsilon_{ijm}\varepsilon_{klm} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}$.

(b) Show that there is no isotropic tensor of rank one except the null vector.

(10, 8.75)

5. (a) Prove that

$(A \times B) \times (C \times D) = B[A.(C \times D)] - A[B.(C \times D)]$ using tensors.

(b) Show that moment of inertia is a symmetric tensor of rank two.

(10, 8.75)

6. Show that dx^i is a contravariant vector and grad $\emptyset$ is a covariant vector. If $\emptyset(x^i)$ is scalar valued function of general coordinate (x^i), prove that $\frac{\partial^2 \emptyset}{\partial x^i \partial x^j}$ does not form a tensor. Also find the coordinate tensor in which they form as second order tensor.

(18.75)