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| **Name of the Department** | **:** | **Physics** |
| **Name of the Course** | **:** | **B.Sc. (Hons.) Physics (CBCS)** |
| **Name of the Paper** | **:** | **Mathematical Physics-II** |
| **Semester** | **:** | **III** |
| **Unique Paper Code** | **:** | **32221301** |
| **Question Paper** | **:** | **Set-C** |

**Duration : 3 Hours** **Maximum Marks : 75**

Attempt any **four** questions. All questions carry equal marks.

**Q1.**Using method of separation of variables, solve 2-D equation $\frac{\partial^2 u}{\partial t^2} = c^2(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2})$ subjected to the conditions

$u(a, \theta, t) = 0$,

$u(r, \theta, 0) = 0$ and

$(\frac{\partial u}{\partial t})_{t=0} = g(r, \theta)$ (18.75)

**Q2. (a)** Using one dimensional heat equation $\frac{\partial V}{\partial t} = h^2 \frac{\partial^2 V}{\partial x^2}$, find the temperature V (x, t) in a bar of length which is perfectly insulated and whose ends are kept at temperature zero and the initial temperature is

$f(x) = x$ when $0 < x < \frac{L}{2}$

$= L - x$ when $\frac{L}{2} < x < L$ (10.75)

**(b)** Show that $\int_0^a x^{m-1}(a-x)^{n-1}\,dx = a^{m+n-1}\beta(m, n)$ (4)

**(c)** Show that the relation between beta and gamma function is

$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (4)

**Q3.**Given, f(x) = x for $0 < x < 2$

**(a)** Find the Fourier cosine series of the function in half range. (10.75)

**(b)** Sketch the function. (3)

**(c)** Using Parseval's identity deduce that $\frac{\pi^4}{96} = \sum_1^\infty \frac{1}{n^4}$ (5)

**Q4. (a)** Find the complex form of the Fourier series of f(x) = exp(-x) for $-1 \leq x \leq 1$ (10.75)

**(b)** Show that

**(i)** $(x^2 - 1)\, P'_n(x) = n\,(x\, P_n(x) - P_{n-1}(x))$

**(ii)** $x\, J'_n(x) = -n\, J_n(x) + x\, J_{n-1}(x)$ (4, 4)

**Q5. (a)** Discuss the nature of singularity at x=1 of the differential equation

$(x^2\text{-}1)\, y'' + x\, y' - y = 0$ (5)

**(b)** Solve the differential equation $(x - x^2)\frac{d^2y}{dx^2} + (1 - 5x)\frac{dy}{dx} - 4y = 0$ using Frobenius method about x=0. (13.75)

**Q6. (a)** Solve the differential equation in power series, $y'' + x\, y' + y = 0$. (10.75)

**(b)** Show that $\int_{-1}^{+1}[\mathrm{Pn(x)}\ \mathrm{Pm(x)}]\, dx = \frac{2}{2n+1}\delta_{mn}$ (8)