Name of the Course:	**LOCF B.Sc. (H)Mathematics**
Unique Paper Code:	**32357501**
Name of the Paper:	**DSE-1 (i) Numerical Analysis**
Semester:	**V**
Duration:	**3 hours**
Maximum Marks:	**75 Marks**

Attempt any four questions. All questions carry equal marks.

1. (a) Find the smallest positive root of the given equation by performing three iterations of the Bisection method

$f(x) = e^x - 3x = 0$.

(b) Apply four iterations of the Fixed Point Iteration Method to find an approximate root of the following equation by taking the initial approximation as p_0=0

$$f(x) = 3x - (1 + \cos x) = 0.$$

Also represent the root graphically.

(c) Find LU decomposition for the matrix

$$A=\begin{pmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & -2 & -1 \end{pmatrix}$$

and then solve the system

$$3x + y + 2z = 4$$
$$2x - 3y - z = -6$$
$$x - 2y - z = -4$$

2. (a)Set up the Gauss-Seidel iteration scheme for the following system of equations

$$2x - y + 2z = 3$$
$$x + 3y + 3z = -1$$
$$x + 2y + 5z = 1$$

and iterate three times starting with the initial vector $X^{(0)} = (0,0,0.5)^T$.

(b) Apply Secant method to find a root of the equation

$\ln(1+x)-\frac{1}{2}\cos x = 0$

(0,1). Perform three iterations.What order of convergence do you expect?

(c)Solve the following system of equations using SOR iteration method

$$5x_1 + x_2 - 2x_3 = 2$$

$$3x_1 + 4x_2 - x_3 = -2$$
$$2x_1 - 3x_2 + 5x_3 = 10$$

Take $w = 0.9$ with $X^{(0)} = (0,0,0)^T$ and iterate three times.

3. (a) Find a polynomial of degree 3 or less passing through the points (-1,9), (0,5),(1,3) and (2,1) using Lagrange interpolation. Use this polynomial to estimate the ordinate for $x = 1.5$.

(b) Use central difference formula to approximate $f'(0.5)$ for the function $f(x) = 4e^{-2x}$ by taking two step sizes $h = 0.1$ and 0.05. What is the order of approximation?

4. (a) Derive the following approximation of function $f'(x_0)$ for an arbitrary function f(x):

$$f'(x_0) = \frac{-3f_0 + 4f_1 - f_2}{2h}$$

Hence show that the above approximation provides exact value of the first order derivatives for $f(x) = 1, x \ and \ x^2$but not for $f(x) = x^3$.

(b) Determine the step size h in an equidistant table for $f(x) = sinx$ in $[0, \pi/4]$, if the error in magnitude in quadratic interpolation is less than or equal to 5×10^{-8} in magnitude.

(c) Obtain the piecewise linear interpolating polynomials for the function f(x) defined by the given data .

X	0.5	1.5	2.5
f(x)	0.125	3.375	15.625

Interpolate at $x = 1.0 \ and \ 2.0$.

5. (a) Approximate the value of the given integral by Simpson's 1/3 rule and the Trapezoidal rule

$$\int_0^3 \frac{1}{4+x^2} dx.$$

Calculate the difference between the actual value and the approximate value.Also find the error term for both.

(b)Apply the Newton Raphson Method to find a root of the equation

f(x)=$x^3 - x^2 - 10x + 6 = 0$. Perform three iterations with x_0=1.

6 (a) Construct the Richardson extrapolation table to find the derivative of the function $f(x) = \tan^{-1} x$ at $x_0 = 3$ using the first-order forward difference approximation

$$D_h^{(1)} = \frac{f(x_o + h) - f(x_o)}{h}$$

taking h=1,0.5,0.25,0.125.

(b) Apply the Modified Euler method to find approximate solution of the following initial value problem with four numbers of steps

$\frac{dy}{dx} = xy, \qquad (1 \leq x \leq 2), \quad y(1) = 1.$

Also compare with the exact solution.