Name of Course	: B.Sc. (H) Mathematics CBCS (LOCF)
Unique Paper Code	: 32351303
Name of Paper	: BMATH307 - Multivariate Calculus
Semester	: III
Duration	: 3 hours
Maximum Marks	: 75 Marks

Attempt any four questions. All questions carry equal marks.

1. Let
$$f(x, y) = \begin{cases} \frac{xy + y^3}{x^2 + y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$$

- (a) Show that $f_x(0,0)$ and $f_y(0,0)$ both exist but f is not differentiable at (0,0).
- (b) Find the rate of change of f at the point P(2,0) in the direction from P to $Q\left(\frac{1}{2},2\right)$.
- (c) Find the direction in which f decreases most rapidly and increases most rapidly at the point (2,1).
- (d) Find the tangent plane to the given surface z = f(x, y) at the point (0, 2, 2).

2. Let $f(x, y) = x^2 - 4xy + y^3 + 4y$.

- (a) Find the absolute extrema of f(x, y) on the closed bounded rectangular region $0 \le x \le 3, \ 0 \le y \le 2$
- (b) Use the method of constrained optimization to find the largest and the smallest value of f(x, y) subject to the constraint x + y = 2.
- (c) Compare the results obtained in parts (a) and (b) and interpret it.

- 3. (a) Let *R* be the region which lies between two squares of sides 2 and 4 with center at the origin and sides parallel to the coordinate axes. Compute the double integral $\iint_{x} e^{x+y} dA$.
 - (b) Evaluate

$$\iint_D \sin^2(x+y) dy dx$$

where D is the parallelogram with vertices $(0,\pi), (\pi,0), (\pi,2\pi), (2\pi,\pi)$.

- (c) Compute $\iiint_{D} \frac{z}{(x-1)^{2} + (y-1)^{2}} dV$, where D is the solid bounded by the surface $(x-1)^{2} + (y-1)^{2} = 2z$ and the plane z = 2.
- 4. Let the vector field \vec{F} be given as

$$\vec{F}(x, y, z) = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$$

- (a) Show that \vec{F} is conservative.
- (b) Find the scalar potential for \vec{F} .
- (c) What will be the work done in moving an object in this force field from the point P(0,1,-1) to the point $Q\left(\frac{\pi}{2},-1,2\right)$.
- (d) If the path P to Q is traversed through ten additional in between points then what will be the effect on the work done. Give reason for your answer.
- (e) Evaluate $\int_C \vec{F} \cdot d\vec{R}$, where *C* is the line segment x = 1, y = 1, $0 \le z \le 1$.
- 5. Let D be a ball of radius 3 with center at the origin and \vec{F} be the vector field given by

$$\vec{F}(x, y, z) = 2xy\,\hat{i} + yz^2\,\hat{j} + xz\,\hat{k}\,.$$

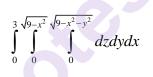
- (a) Use spherical coordinates to compute $\iiint_D \nabla \cdot \vec{F} \, dV$.
- (b) If S denotes the upper half of the ball D, then verify Stokes' Theorem for \vec{F} .
- 6. (a) Let the force field \vec{F} be given by

$$\vec{F}(x, y, z) = xyz(\hat{i} + \hat{j} + \hat{k}).$$

If \vec{N} is the outward drawn normal, then use Divergence theorem to evaluate $\iint_{S} \vec{F} \cdot \vec{N} dS$,

where S is the surface of the box: $0 \le x \le 2$, $0 \le y \le 1$, $0 \le z \le 4$.

(b) Evaluate



using

- (i) Cylindrical coordinates
- (ii) Spherical polar coordinates.