

Unique Paper Code : 62354343

Name of the Course : B.A. (Prog.) Mathematics

Name of the Paper : Analytic Geometry and Applied Algebra

Semester : III (CBCS)

Time : 3 Hours

Maximum Marks :75

- **Attempt any four questions in all.**
- **All questions carry equal marks.**

1. (a) Identify and sketch the curve:

$$y = 4x^2 + 8x + 5$$

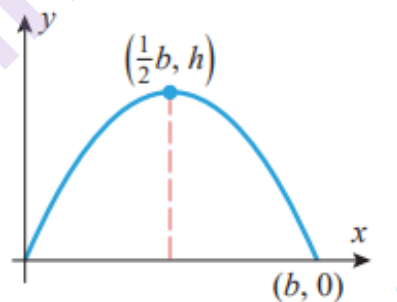
Also label the focus, vertex and directrix

- (b) Describe the graph of the curve:

$$x^2 + 9y^2 + 2x - 18y + 1 = 0$$

Find its foci, vertices and the ends of the minor axis.

- (c) Find an equation for the parabolic arch with base b and height h , shown in the accompanying figure



2. (a) Find the equation of parabola with vertex $(2,4)$ and focus $(3,4)$.
- (b) Find the equation for the ellipse that has ends of major axis $(\pm 6, 0)$ and passes through $(2, 3)$
- (c) Find the equation for a hyperbola that satisfies the given conditions:
Asymptotes $y = 2x + 1$, $y = -2x + 3$ and passes through the origin.
3. (a) Find an equation of the sphere with centre $(2,-1,-3)$ and satisfying
- i) Tangent to the x - y plane.

- ii) Tangent to the x-z plane.
- iii) Tangent to the y-z plane.
- b) Show that the graph of the equation:

$$\sqrt{x} + \sqrt{y} = 1, \quad \forall x \in [0,1], y \in [0,1]$$

is a portion of a parabola.

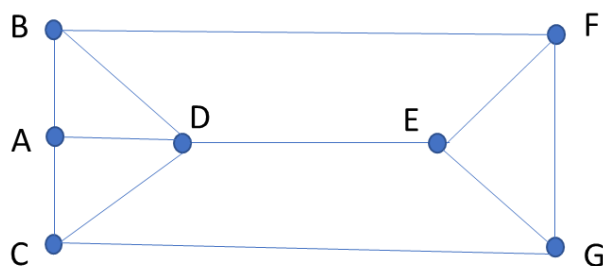
- (c) Describe the surface whose equation is given as

$$x^2 + y^2 + z^2 + 2y - 6z + 5 = 0$$

4. (a) Find \vec{u} and \vec{v} if $5\vec{u} + 2\vec{v} = 6\hat{i} - 5\hat{j} + 4\hat{k}$ and $3\vec{u} - 4\vec{v} = \hat{i} + 2\hat{j} + 9\hat{k}$. Also find a vector of length 3 and oppositely directed to \vec{v} .
- (b) (i) Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$
- (ii) Determine whether $\vec{u} = \langle 6, 1, 3 \rangle$ and $\vec{v} = \langle 4, -6, -7 \rangle$ make an acute angle, an obtuse angle or are orthogonal? Justify your answer.
- (c) Find the volume of the parallelepiped with adjacent edges $\vec{u} = 3\hat{i} + 2\hat{j} + \hat{k}$, $\vec{v} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{w} = \hat{i} + 3\hat{j} + 3\hat{k}$. Also find the area of the face determined by \vec{u} and \vec{v} .
5. (a) Find the distance of the point $P(2, 5, -3)$ from the plane $\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$
- (b) Find the equation of the plane through the points $P_1(2, 1, 4)$, $P_2(1, 0, -3)$ that is perpendicular to the plane $4x + y + 3z = 2$.
- (c) Show that the lines L_1 and L_2 are parallel and find the distance between them

$$L_1 : x = 2 - t, \quad y = 2t, \quad z = 3 + t$$

$$L_2 : x = -1 + 2t, \quad y = 3 - 4t, \quad z = 5 - 2t$$
6. (a) Suppose a job placement agency wants to schedule interviews for candidates Ann, Judy and Carol with interviewers Al, Brian and Carl on Monday, Tuesday and Wednesday in such a way that each candidate gets interviewed by each interviewer. Solve this problem using a Latin Square.
- (b) Find a vertex basis for the following graph:



- (c) For the following graph, find a minimal edge cover and a maximal independent set of vertices.

