

Name of the Course	: Generic Elective
Unique Paper Code	: 32355301
Name of the Paper	: GE-3 Differential Equations
Semester	: III
Duration	: 3 Hours
Maximum Marks	: 75 Marks

Attempt any four questions. All questions carry equal marks.

1. (i) Solve

$$x dy - y dx = \sqrt{x^2 + y^2} dx.$$

(ii) Solve the initial value problem

$$(x^2 + y^2 + x) dx + xy dy = 0, \quad y(1) = 1.$$

(iii) Solve the initial value problem

$$x \frac{dy}{dx} + y = y^2 \log x, \quad y(1) = -1.$$

2. (i) Find the orthogonal trajectories of the family of curves $3xy = x^3 - a^3$, a being parameter of the family.

(ii) Find a family of oblique trajectories that intersect the family of circles $x^2 + y^2 = c^2$ at angle 45° .

(iii) Solve

$$\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y}.$$

3. (i) Solve by method of variation of parameters

$$y'' + y = \operatorname{cosec} x.$$

(ii) Solve by method of undetermined coefficients

$$y'' + 1.44y = 24 \cos 1.2 x.$$

(iii) Solve

$$y''' - 2y'' + 4y' - 8y = 0, \quad y(0) = -1, \quad y'(0) = 30, \quad y''(0) = 28.$$

4. (i) Show that $\{e^{-x}, e^{3x}, e^{4x}\}$ forms a basis of the solution set of the equation

$$\frac{d^3 y}{dx^3} - 6 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 12y = 0.$$

- (ii) Solve the initial value problem

$$x^2 y'' - 2x y' - 10y = 0, \quad y(1) = 5, \quad y'(1) = 4.$$

- (iii) Solve the linear system

$$y_1' = 2y_1 + 5y_2$$

$$y_2' = 5y_1 + 12.5y_2$$

5. (i) Find the partial differential equation arising from the surface

$$z = xy + f(x^2 + y^2).$$

- (ii) Find the general solution of the partial differential equation

$$u_x + 2xy^2 u_y = 0.$$

- (iii) Apply the method of separation of variables $u(x, y) = f(x)g(y)$ to solve

$$y u_x + x u_y = 0 \quad \text{on} \quad u(0, y) = y^2.$$

6. Reduce each of the following equations into canonical form and find the general solution:

(i) $u_x - u_y = u, \quad u(x, 0) = 4e^{-3x}.$

(ii) $u_{xx} + 6u_{xy} + 9u_{yy} + 3y u_y = 0.$

(iii) $u_{xx} - 3u_{xy} + 2u_{yy} = 0.$